1 Introduction

When the armature rotates in a magnetic field, a conductor of armature coil cut the magnetic flux and EMF get induced in it. The induced EMF in a coil depend upon the speed of armature and the flux produced by field winding.

In case of generator, the EMF induced in rotating armature called as generated EMF and it is denoted by Eg. But in case of motor EMF induced in rotating coil is known as Back EMF and it is denoted by Eb.

2 EMF equation in generator

When the armature of a DC generator rotates in magnetic field, an emf is induced in the armature winding, this induced EMF is known as generated EMF. It is denoted by Eg.

Let

$\emptyset$ = Magnetic flux per pole in Wb.

Z = Total number of armature conductors.

P = Number of poles in the machine.

A = Number of parallel paths.

N = Speed of armature in RPM.

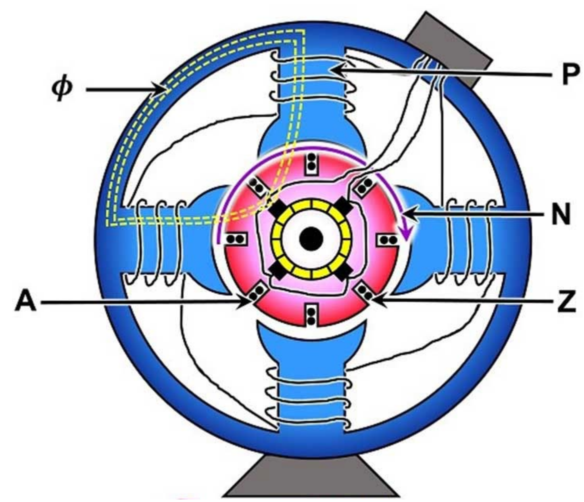

Fig 1. Elements of EMF equation.

Eg = Generated E.M.F = E.M.F per parallel path. (1)

Therefore, the magnetic flux cut by one conductor in one revolution of the armature can be defined as follows:

Flux cut by one conductor

$$d\emptyset = P\emptyset \quad (2)$$

Time taken to complete one revolution is given as:

$$dt = \frac{60}{N} \quad (3)$$

Therefore, the average induced E.M.F in one conductor will be:

$$Eg,\ Per\ conductor = \frac{d\emptyset}{dt} = \frac{P\,\emptyset}{t} = \frac{P\,\emptyset\,N}{60}\ volt \quad (4)$$

The number of conductors connected in series in each parallel path = **Z/A**.

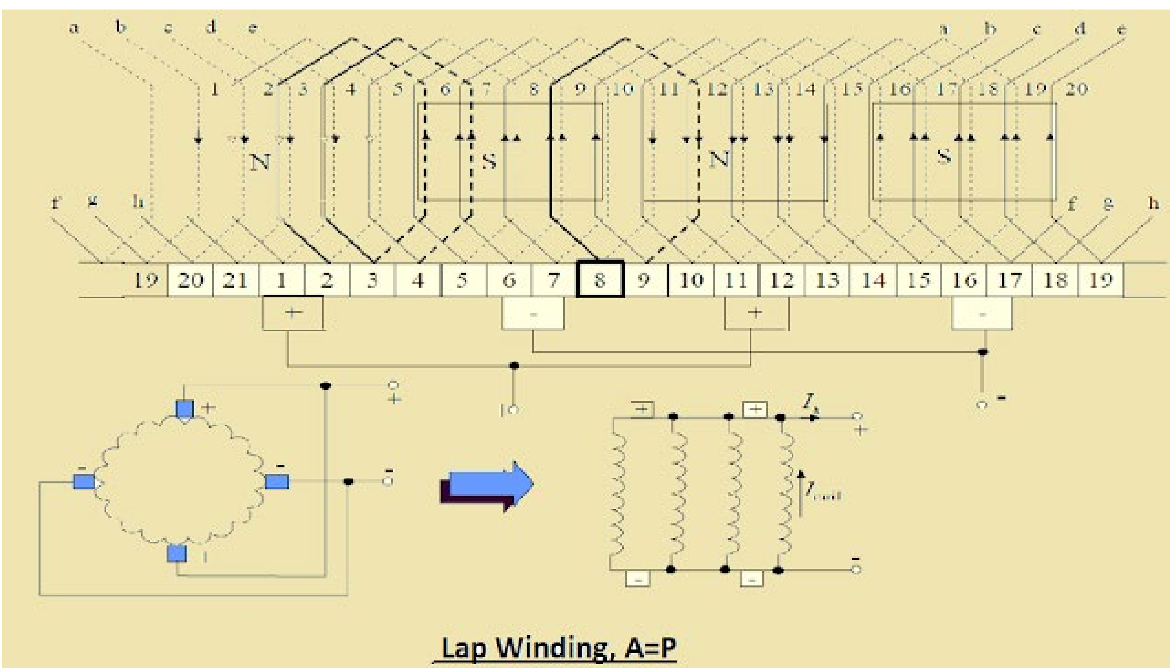

Fig 2. Paths in lap winding DC machine.

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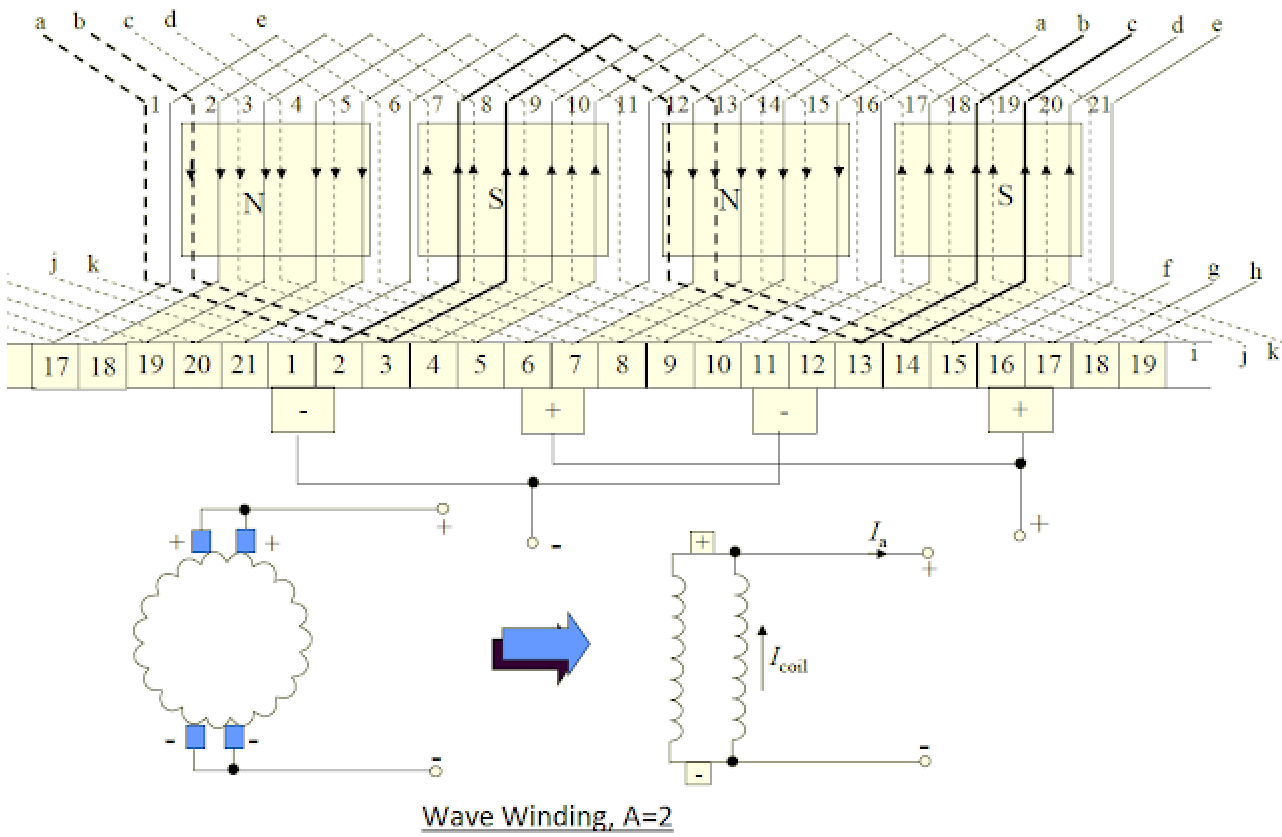


Fig 3. Path in wave winding DC machine.

Therefore, the average induced EMF across each parallel path or the armature terminals is given by the equation shown below:

$$Eg = \frac{Z\ P\ \emptyset\ N}{A\ 60}\ volt \tag{5}$$

If the angular velocity were been given then: -

$$Eg = \frac{2\pi}{2\pi} \times \frac{Z\ P\ \emptyset\ N}{A\ 60} = \frac{2\pi N}{60} \times \frac{Z\ P\ \emptyset}{2\pi\ A} = \frac{\omega\ Z\ \emptyset\ P}{2\pi\ A}\ volt \tag{6}$$

Where ω is the angular velocity in radians/second.

Example 1: A four-pole generator, lap-wound armature winding has 51 slots, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 rpm assuming the flux per pole to be 7.0 mWb?

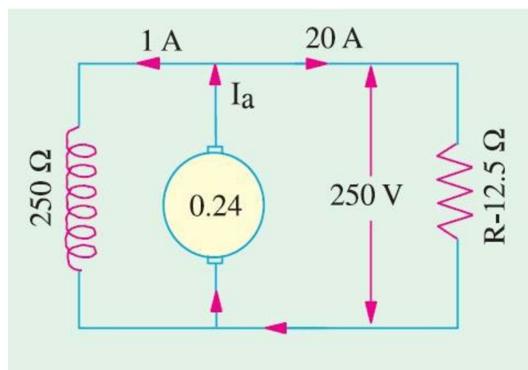
Solution

$\emptyset = 7 \times 10^{-3}$ Wb. $Z = 51 \times 20 = 1020$. $A = P = 4$. $N = 1500$ rpm

$$Eg = \frac{Z\ P\ \emptyset\ N}{A\ 60} = \frac{1020 \times 7 \times 10^{-3} \times 1500}{60} = \mathbf{178.5\ V}$$

Example 2: An 8-pole DC shunt generator with 778 wave-connected armature conductors and running at 500 rpm supplies a load of 12.5 Ω resistance at terminal voltage of 250 V. The armature resistance is 0.24 Ω and the field resistance is 250 Ω. Find the armature current, the induced EMF and the flux per pole?

Solution



Load current = V/R = 250/12.5 = **20 A**

Shunt current = 250/250 = **1 A**

Armature current = 20 + 1 = **21 A**

Induced EMF = 250 + (21 × 0.24) = **255.04 V**

$$Eg = \frac{Z\ P\ \emptyset\ N}{A\ 60}$$

$$255.04 = \frac{778 \times 8 \times \emptyset \times 500}{2 \times 60}$$

$$\emptyset = \mathbf{9.83\ mWb}$$

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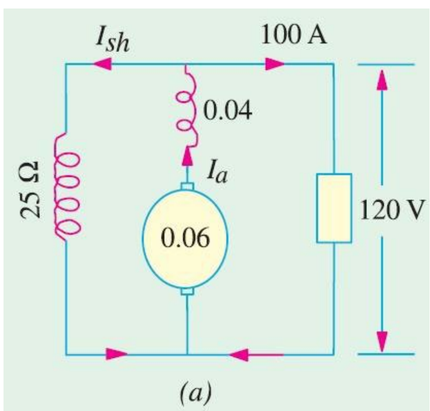
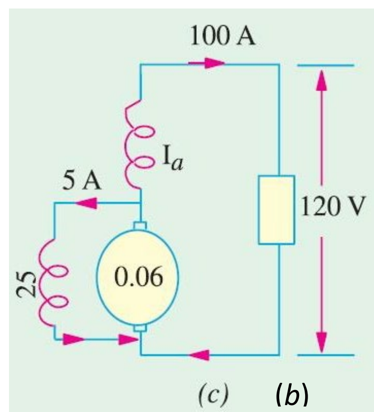
Example 3: In a 120 V compound generator, the resistances of the armature shunt and series windings are 0.06 Ω, 25 Ω and 0.04 Ω respectively. The load current is 100 A at 120 V. Neglect brush contact drop and ignore armature reaction. Evaluate the following:

(i) The induced EMF and the armature current when the machine is connected as long-shunt.

(ii) The induced EMF and the armature current when the machine is connected as short-shunt.





Solution

(i) Long Shunt [Fig.(*a*)]

I_{sh} = 120/25 = **4.8 A**, I = **100 A**,

$I_a = I + I_{sh}$ = **104.8 A**

Voltage drop in series winding = 104.8 × 0.04 = **4.19 V**

Armature voltage drop = 104.8 × 0.06 = **6.29 V**

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$\therefore Eg$ = 120 + 4.19 + 6.29 = **13.5 V**

(*ii*) Short Shunt [Fig. (*b*)]

Voltage drop in series winding = 100 × 0.04 = **4 V**

Voltage across shunt winding = 120 + 4 = **124 V**

$\therefore I_{sh}$ = 124/25 = **5 A**; $\therefore I_a$ = 100 + 5 = **105 A**

Armature voltage drop = 105 × 0.06 = **6.3 V**

Eg = 120 + 5 + 4 = **129 V**.

3 Losses in DC machines

In a practical machine, whole of the input power cannot be converted into output power as some power is lost in the conversion process. This causes the **efficiency of the machine** to be reduced. Efficiency is the ratio of output power to the input power. Thus, in order to design rotating dc machines (or any electrical machine) with higher efficiency, it is important to study the losses occurring in them. **Various losses in a rotating DC machine (DC generator or DC motor)** can be characterized as in fig 4.

3.1 Copper losses

These losses occur in armature and field copper windings. Copper losses consist of Armature copper loss, Field copper loss and loss due to brush contact resistance.

Armature copper loss $= I_a{}^2 R_a$ (where, I_a = Armature current and R_a = Armature resistance).

This loss contributes about 30 to 40% to full load losses. The armature copper loss is variable and depends upon the amount of loading of the machine.

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Field copper loss in shunt winding = ${I_{sh}}^2 R_{sh}$.

Field copper loss in series winding = ${I_{se}}^2 R_{se}$.

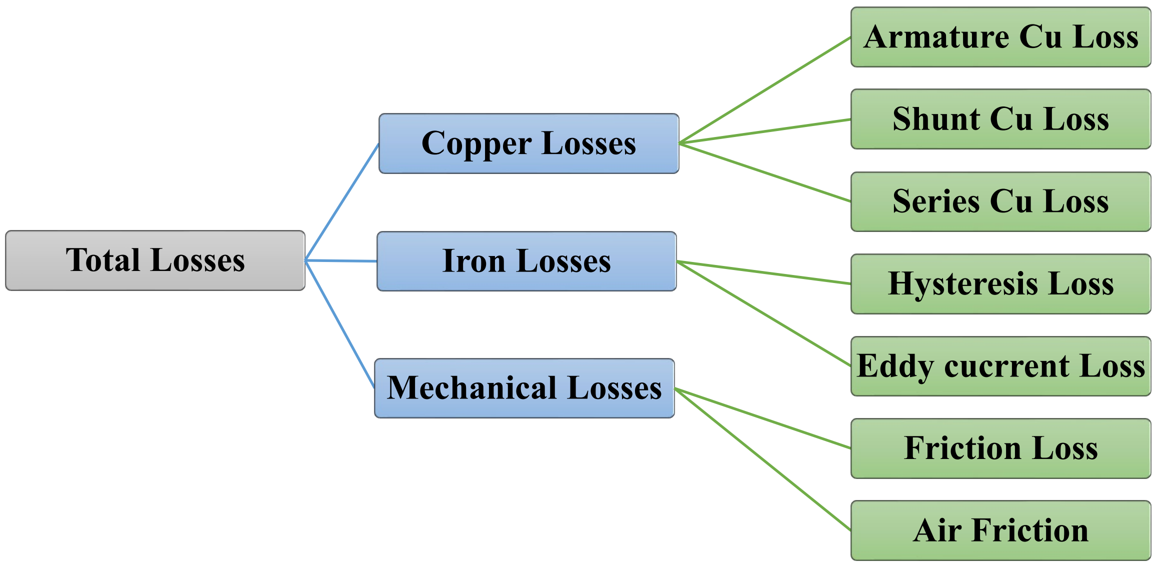

Fig 4. Losses in DC Machine.

Brush contact resistance also contributes to the copper losses. Generally, this loss is included into armature copper loss.

3.2 Iron losses (Core losses)

As the armature core is made of iron and it rotates in a magnetic field, a small current gets induced in the core itself too. Due to this current, **eddy current loss** and **hysteresis loss** occur in the armature iron core. Iron losses are also called as **Core losses or magnetic losses.**

Hysteresis loss: $W_h \propto (B^{1.6} max)\ f$.

Eddy current loss: $W_e \propto (B^2 max)\ f^2$.

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Mechanical losses consist of the losses due to friction in bearings and commutator. Air friction loss of rotating armature also contributes to these. These losses are about 10 to 20% of full load losses. Iron and mechanical losses are collectively known as **Stray (Rotational) losses W_{co}.**

4 Power Stages and Efficiency

Various power stages in the case of a DC generator is shown below:

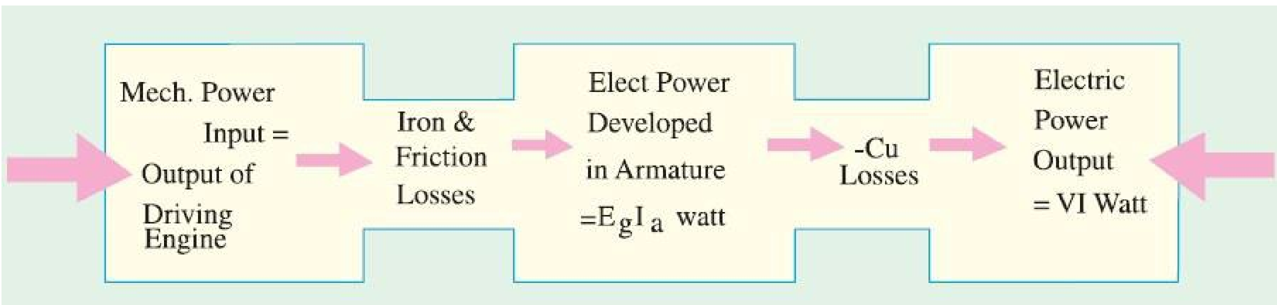

Fig 5. Power stages in DC machine.

Following are the three generator efficiencies:

1- Mechanical Efficiency:

$$\eta_m = \frac{B}{A} = \frac{\text{total watts generated in armature}}{\text{mechanical power supplied}} = \frac{Eg\, I_a}{\text{output of driving engine}} \quad (7)$$

2- Electrical Efficiency:

$$\eta_e = \frac{C}{B} = \frac{\text{watts available in load circuit}}{\text{total watts generated}} = \frac{VI}{Eg\, I_a} \quad (8)$$

3- Overall or Commercial Efficiency:

$$\eta_c = \frac{C}{A} = \frac{\text{watts available in load circuit}}{\text{mechanical power supplied}} \quad (9)$$

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It is obvious that overall efficiency $\boldsymbol{\eta_c = \eta_m \times \eta_e}$. For good generators, its value may be as high as 95%.

5 Condition for Maximum Efficiency

The maximum Efficiency in DC Shunt generator can be evaluated as follows: -

$$\textbf{Generator output (at load)} = VI \quad (10)$$

$$\textbf{Generator input} = \textbf{output} + \textbf{losses}$$

$$= VI + {I_a}^2 R_a + W_{co} = VI + (I + I_{sh})^2 R_a + W_{co} \quad (11)$$

However, if I_{sh} is negligible as compared to load current, then $I_a = I$ (approx.)

$$\eta = \frac{\textbf{Output}}{\textbf{Input}} = \frac{VI}{VI + I^2 R_a + W_{co}} = \frac{1}{1 + \left(\frac{IR_a}{V} + \frac{W_{co}}{VI}\right)} \quad (12)$$

Now, efficiency is maximum when denominator is minimum i.e. when

$$\frac{d}{dI}\left(\frac{IR_a}{V} + \frac{W_{co}}{VI}\right) = 0$$

Which leads to the following conclusion: -

$$I^2 Ra = W_{co} \quad (13)$$

Example 4: A 10 kW, 250 V, DC, 6-pole shunt generator runs at 1000 RPM. when delivering full-load. The armature has 534 lap-connected conductors. Full-load Cu loss is 0.64 kW. The total brush drop is 2 volts. Determine the flux per pole. Neglect shunt current.

Solution

Since shunt current is negligible, there is no shunt Cu loss. The copper loss occurs in armature only.

$$I = I_a = \frac{10,000}{250} = \mathbf{40\ A}$$

Armature losses $= {I_a}^2 R_a \Longrightarrow\ 40^2 \times R_a =\ 0.64 \times 10^3\ \Longrightarrow\ R_a = \mathbf{0.4\ \Omega}$

$I_a R_a = 0.4 \times 40 =$ **16 V;**

Brush drop = **2 V**

∴ Generated EMF $Eg = 250 + 16 + 2 =$ **268 V**

$$Eg = \frac{Z\ P\ \emptyset\ N}{A\ 60} \Longrightarrow\ 268 = \frac{\emptyset \times 534 \times 1000}{60}$$

$$\emptyset = 30\ \times 10^{-3}\ Wb = \mathbf{30\ mWb}$$

Example 5: A shunt generator delivers 195 A, 250 V. The armature resistance and shunt field resistance are 0.02 Ω and 50 Ω respectively. The iron and friction losses equal 950 W. Find **(a)** EMF generated **(b)** Cu losses **(c)** output of the prime motor **(d)** commercial, mechanical and electrical efficiencies.

Solution

(a) $I_{sh} =\ 250/50\ =\ \mathbf{5\ A},\ \ I_a = 195\ + 5 = \mathbf{200\ A.}$

Armature voltage drop $= I_a R_a\ = 200 \times 0.02 = \mathbf{4\ V.}$

∴ Generated EMF = 250 + 4 = **254 V**

(*b*) Armature Cu loss $= {I_a}^2 R_a\ = 200^2 \times 0.02 = \mathbf{800\ W}$

Shunt Cu loss $= V I_{sh}\ =\ 250\ \times 5\ =\ \mathbf{1250\ W}$

∴ Total Cu loss = 1250 + 800 = **2050 W**

(*c*) Stray losses = **950 W**; Total losses = 2050 + 950 = **3000 W**

Output = $V\,I$ = 250 × 195 = **48750 W**; Input = 48750 + 3000 = **51750 W**

∴ Output of prime mover = **51,750 W**

(*d*) Generator input = **51750 W**; Stray losses = **950 W**

Electrical power produced in armature = 51750 − 950 = **50800 W**

$$\eta_m = \frac{\text{total watts generated in armature}}{\text{mechanical power supplied}} = \left(\frac{50800}{51750}\right) \times 100 = \mathbf{98.2\%}$$

Electrical or Cu losses = **2050 W**

$$\eta_e = \frac{\text{watts available in load circuit}}{\text{total watts generated}} = \frac{48750}{48750 + 2050} \times 100 = \mathbf{95.9\%}$$

$$\boldsymbol{\eta_c} = \eta_m \times \eta_e = \mathbf{94.2\%}$$

Example 6: A shunt generator has a Full Load current of 196 A at 220 V. The stray losses are 720 W and the shunt field coil resistance is 55 Ω. If it has a Full Load efficiency of 88%, find the armature resistance. Also, find the load current corresponding to maximum efficiency.

Solution

Output (load) = 220 × 196 = **43120 W**; η_e = **88%**

Electrical input = 43120/0.88 = **49000 W**

Total losses = 49000 − 43120 = **5880 W**

Shunt field current = 220/55 = **4 *A***

∴ I_a = 196 + 4 = **200 A**

∴ Shunt Cu loss = 220 × 4 = **880 W**; Stray losses = **720 W**

Constant losses = 880 + 720 = **1600 W**

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∴ Armature Cu loss = 5880 − 1600 = **4280 W**

$\therefore\ I_a{}^2 R_a = 4280\ \mathrm{W} \implies R_a = \mathbf{0.107\ \Omega}$

For maximum efficiency,

$I_a{}^2 R_a =$ constant losses = 1600 W $\implies I = \sqrt{1600/0.107} = \mathbf{122.34\ A}$.