Department of biology

((GENERAL MATHEMATICS))

1st stage

Week 6- lecture 6

Chain Rule

قاعدة السلسلة

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Chain Rule:-

The **chain rule** is used to differentiate composite functions. If a function $f(x)$ can be written as $f(x) = g(h(x))$, where g and h are two functions, then the derivative of $f(x)$ is given by:

$$f'(x) = g'(h(x)) \cdot h'(x)$$

1- Steps to Apply the Chain Rule:-

1. Differentiate the outer function $g(h(x))$ while keeping the inner function $h(x)$ unchanged.
2. Multiply the result by the derivative of the inner function $h(x)$.

❖ Examples:-

1. $f(x) = (2x + 3)^8$

$$f'(x) = 8(2x + 3)^7 \cdot 2$$

$$f'(x) = 16(2x + 3)^7$$

2. $f(x) = e^{3x^2+x}$

$$f'(x) = e^{3x^2+x} \cdot (6x + 1)$$

$$f'(x) = (6x + 1)e^{3x^2+x}$$

3. $f(x) = \ln(x^4 + 7)$

$$f'(x) = \frac{1}{x^4 + 7} \cdot 4x^3$$

$$f'(x) = \frac{4x^3}{x^4 + 7}$$

4. $f(x) = \cos(5x^3 + x^2)$

$$f'(x) = -\sin(5x^3 + x^2) \cdot (15x^2 + 2x)$$

$$f'(x) = -(15x^2 + 2x)\sin(5x^3 + x^2)$$

5. $f(x) = \sqrt{4x^2 + 1}$

$$f'(x) = \frac{1}{2\sqrt{4x^2 + 1}} \cdot 8x$$

$$f'(x) = \frac{4x}{\sqrt{4x^2 + 1}}$$

6. $f(x) = (x^3 + 2x + 1)^{10}$

$$f'(x) = 10(x^3 + 2x + 1)^9 \cdot (3x^2 + 2)$$

$$f'(x) = 10(3x^2 + 2)(x^3 + 2x + 1)^9$$

7. $f(x) = \tan(3x^2 + 2x)$

$$f'(x) = \sec^2(3x^2 + 2x) \cdot (6x + 2)$$

$$f'(x) = (6x + 2)\sec^2(3x^2 + 2x)$$

8. $f(x) = \sin^2(x^3 + x)$

$$f'(x) = 2\sin(x^3 + x)\cos(x^3 + x) \cdot (3x^2 + 1)$$

$$f'(x) = 2(3x^2 + 1)\sin(x^3 + x)\cos(x^3 + x)$$

9. $f(x) = \sqrt{\ln(x^2+1)}$

$$f'(x) = \frac{1}{2\sqrt{\ln(x^2+1)}} \cdot \frac{1}{x^2+1} \cdot 2x$$

$$f'(x) = \frac{x}{(x^2+1)\sqrt{\ln(x^2+1)}}$$

10. $f(x) = e^{x^4+3x^2+1}$

$$f'(x) = e^{x^4+3x^2+1} \cdot (4x^3+6x)$$

$$f'(x) = (4x^3+6x)e^{x^4+3x^2+1}$$

11. $f(x) = \sin(\ln(3x^2+1))$

$$f'(x) = \cos(\ln(3x^2+1)) \cdot \frac{1}{3x^2+1} \cdot 6x$$

$$f'(x) = \frac{6x\cos(\ln(3x^2+1))}{3x^2+1}$$

12. $f(x) = (5x^4+2x)^7$

$$f'(x) = 7(5x^4+2x)^6 \cdot (20x^3+2)$$

$$f'(x) = 7(20x^3+2)(5x^4+2x)^6$$

13. $f(x) = e^{\sin(x^3)}$

$$f'(x) = e^{\sin(x^3)} \cdot \cos(x^3) \cdot 3x^2$$

$$f'(x) = 3x^2 \cos(x^3) e^{\sin(x^3)}$$

14. $f(x) = \ln(\sqrt{4x^2 + 3})$

$$f'(x) = \frac{1}{\sqrt{4x^2 + 3}} \cdot \frac{1}{2} \cdot 8x$$

$$f'(x) = \frac{4x}{4x^2 + 3}$$

15. $f(x) = (x^2 + 1)^5 \cdot \cos(x^3 + 2x)$

$$f'(x) = 5(x^2 + 1)^4 \cdot 2x \cdot \cos(x^3 + 2x) - (x^2 + 1)^5 \cdot \sin(x^3 + 2x) \cdot (3x^2 + 2)$$

$$f'(x) = 10x(x^2 + 1)^4 \cos(x^3 + 2x) - (x^2 + 1)^5 (3x^2 + 2) \sin(x^3 + 2x)$$

16. $f(x) = \ln(\sin(2x))$

$$f'(x) = \frac{1}{\sin(2x)} \cdot \cos(2x) \cdot 2$$

$$f'(x) = \frac{2\cos(2x)}{\sin(2x)}$$

17. $f(x) = \sqrt{\tan(x^2+3)}$

$$f'(x) = \frac{1}{2\sqrt{\tan(x^2+3)}} \cdot \sec^2(x^2+3) \cdot 2x$$

$$f'(x) = \frac{x\sec^2(x^2+3)}{\sqrt{\tan(x^2+3)}}$$

18. $f(x) = \cos(\ln(x^2+4))$

$$f'(x) = -\sin(\ln(x^2+4)) \cdot \frac{1}{x^2+4} \cdot 2x$$

$$f'(x) = \frac{-2x\sin(\ln(x^2+4))}{x^2+4}$$

19. $f(x) = e^{x^2+\sin(x)}$

$$f'(x) = e^{x^2+\sin(x)} \cdot (2x+\cos(x))$$

$$f'(x) = (2x+\cos(x))e^{x^2+\sin(x)}$$

20. $f(x) = \sin^3(x^2+1)$

$$f'(x) = 3\sin^2(x^2+1) \cdot \cos(x^2+1) \cdot 2x$$

$$f'(x) = 6x\sin^2(x^2+1)\cos(x^2+1)$$