جامعة المستقبل
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كلية العلوم

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# Mathematics 1

## Lecture 5

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الرياضيات الاساسية: المرحلة الاولى

**مادة الرياضيات 1**

المحاضرة الخامسة

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# Contents

# 1 Examples of Derivative

**Example 1.1.** Find $y'$

1. $y = 3x + 5\sin(x)$

$$y' = 3 + 5\cos(x)$$

2. $y = \sqrt{x}\sec(x) + 3$

$$\frac{d}{dx}(3) = 0$$

$$\frac{d}{dx}\left(\sqrt{x}\sec(x)\right) = \frac{d}{dx}\left(x^{1/2} \cdot \sec(x)\right)$$

$$\frac{d}{dx}\left(\sqrt{x}\sec(x)\right) = \frac{1}{2}x^{-1/2}\sec(x) + x^{1/2}\sec(x)\tan(x)$$

3. $y = \frac{\cos(x)}{1+\sin(x)}$

$$\frac{d}{dx}\left(\frac{\cos(x)}{1+\sin(x)}\right) = \frac{(-\sin(x))(1+\sin(x)) - \cos(x)\cdot\cos(x)}{(1+\sin(x))^2}$$

$$y' = \frac{-\sin(x)(1+\sin(x)) - \cos^2(x)}{(1+\sin(x))^2}$$

$$y' = \frac{-\sin(x) - \sin^2(x) - \cos^2(x)}{(1+\sin(x))^2}$$

Using the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$, we simplify further:

$$y' = \frac{-\sin(x) - 1}{(1+\sin(x))^2}$$

$$y' = \frac{-(\sin(x) + 1)}{(1+\sin(x))^2}$$

4. $y = 2x + 5\cosh(x)$

$$y' = 2 + 5\sinh(x)$$

5. $y = \frac{\sinh(t)}{t+1}$

$$\frac{d}{dt}\left(\frac{\sinh(t)}{t+1}\right) = \frac{(\cosh(t))(t+1) - (\sinh(t))(1)}{(t+1)^2}$$

$$y' = \frac{\cosh(t)(t+1) - \sinh(t)}{(t+1)^2}$$

6. $y = \tan^{-1}(xe^{2x})$

$$\frac{d}{du}\left(\tan^{-1}(u)\right) = \frac{1}{1+u^2}$$

Let $u(x) = xe^{2x}$. Using the chain rule:

$$y' = \frac{1}{1+(xe^{2x})^2} \cdot \frac{d}{dx}\left(xe^{2x}\right)$$

$$\frac{d}{dx}\left(xe^{2x}\right) = e^{2x} + 2xe^{2x}$$

Thus, the derivative is:

$$y' = \frac{e^{2x} + 2xe^{2x}}{1+(xe^{2x})^2}$$

7. $y = \ln(\cos^{-1}(x))$

$$y' = \frac{1}{\cos^{-1}(x)} \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

Thus, the derivative of $y = \ln(\cos^{-1}(x))$ is:

$$y' = -\frac{1}{\cos^{-1}(x)\sqrt{1-x^2}}$$

## Theorem: Derivative of $\sin^{-1}(u)$

We are tasked to prove that:

$$\frac{d}{dx}\left(\sin^{-1}(u)\right) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

Proof:

Let $y = \sin^{-1}(u)$. This implies:

$$\sin(y) = u$$

Now,

$$\frac{d}{dx}(\sin(y)) = \frac{d}{dx}(u)$$

$$\cos(y) \cdot \frac{dy}{dx} = \frac{du}{dx}$$

Next, we solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{1}{\cos(y)} \cdot \frac{du}{dx}$$

To express $\cos(y)$ in terms of $u$, we use the identity $\sin^2(y) + \cos^2(y) = 1$. Since $\sin(y) = u$, we have:

$$u^2 + \cos^2(y) = 1$$

Thus:

$$\cos^2(y) = 1 - u^2$$

Taking the square root of both sides:

$$\cos(y) = \sqrt{1 - u^2}$$

Therefore, the derivative becomes:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - u^2}} \cdot \frac{du}{dx}$$

Hence, we have proven that:

$$\frac{d}{dx}\left(\sin^{-1}(u)\right) = \frac{1}{\sqrt{1 - u^2}} \cdot \frac{du}{dx}$$