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Mathematics 1

Lecture 4

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الرياضيات الاساسية: المرحلة الاولى

مادة الرياضيات 1

المحاضرة الرابعة

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1 The Derivative

1.1 Derivative of a Function

The derivative of a function $f(x)$ with respect to x is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

where h represents a small change in x, and the limit represents the instantaneous rate of change of the function.

The derivative can also be denoted as:

$$f'(x), \quad \frac{dy}{dx}, \quad \frac{d}{dx}f(x), \quad D[f(x)].$$

Example: For the function $f(x) = x^2$, the derivative is calculated as:

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0}(2x+h) = 2x.$$

Thus, the derivative of $f(x) = x^2$ is $f'(x) = 2x$.

1.2 Rules of Derivation

The following are the basic rules for finding derivatives of functions, along with examples:

1. **Constant Rule:** The derivative of a constant is zero.

$$\frac{d}{dx}[c] = 0, \quad \text{where } c \text{ is a constant.}$$

Example:

$$\frac{d}{dx}[5] = 0.$$

2. **Power Rule:** The derivative of x^n, where n is a real number, is given by:

$$\frac{d}{dx}[x^n] = nx^{n-1}.$$

Example:

$$\frac{d}{dx}[x^3] = 3x^2.$$

3. **Sum and Difference Rule:** The derivative of the sum or difference of two functions is the sum or difference of their derivatives.

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x).$$

Example:

$$\frac{d}{dx}[x^2 + 3x] = 2x + 3.$$

4. **Product Rule:** The derivative of the product of two functions is:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x).$$

Example: Let $f(x) = x^2$ and $g(x) = \sin(x)$. Then:

$$\frac{d}{dx}[x^2 \sin(x)] = 2x \sin(x) + x^2 \cos(x).$$

5. **Quotient Rule:** The derivative of the quotient of two functions is:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}, \quad g(x) \neq 0.$$

Example: Let $f(x) = x^2$ and $g(x) = x + 1$. Then:

$$\frac{d}{dx}\left[\frac{x^2}{x+1}\right] = \frac{2x(x+1) - x^2(1)}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}.$$

6. **Chain Rule:** The derivative of a composite function $f(g(x))$ is:

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x).$$

Example: Let $f(x) = \sin(x^2)$. Then:

$$\frac{d}{dx}[\sin(x^2)] = \cos(x^2) \cdot 2x = 2x\cos(x^2).$$

Derivatives of Trigonometric Functions

Function	**Derivative**
$\sin(ax)$	$a\cos(ax)$
$\cos(ax)$	$-a\sin(ax)$
$\tan(ax)$	$a\sec^2(ax), \quad ax \neq \frac{\pi}{2} + n\pi,\, n \in \mathbb{Z}$
$\cot(ax)$	$-a\csc^2(ax), \quad ax \neq n\pi,\, n \in \mathbb{Z}$
$\sec(ax)$	$a\sec(ax)\tan(ax), \quad ax \neq \frac{\pi}{2} + n\pi,\, n \in \mathbb{Z}$
$\csc(ax)$	$-a\csc(ax)\cot(ax), \quad ax \neq n\pi,\, n \in \mathbb{Z}$

Derivatives of Hyperbolic Functions

Function	**Derivative**
$\sinh(ax)$	$a\cosh(ax)$
$\cosh(ax)$	$a\sinh(ax)$
$\tanh(ax)$	$a\,\text{sech}^2(ax)$
$\coth(ax)$	$-a\,\text{csch}^2(ax), \quad ax \neq 0$
$\text{sech}(ax)$	$-a\,\text{sech}(ax)\tanh(ax)$
$\text{csch}(ax)$	$-a\,\text{csch}(ax)\coth(ax)$

Derivatives of Inverse Trigonometric Functions

Function	Derivative
$\sin^{-1}(ax)$	$\frac{a}{\sqrt{1-(ax)^2}}, \quad \lvert ax\rvert < 1$
$\cos^{-1}(ax)$	$-\frac{a}{\sqrt{1-(ax)^2}}, \quad \lvert ax\rvert < 1$
$\tan^{-1}(ax)$	$\frac{a}{1+(ax)^2}$
$\cot^{-1}(ax)$	$-\frac{a}{1+(ax)^2}$
$\sec^{-1}(ax)$	$\frac{a}{\lvert ax\rvert\sqrt{(ax)^2-1}}, \quad \lvert ax\rvert > 1$
$\csc^{-1}(ax)$	$-\frac{a}{\lvert ax\rvert\sqrt{(ax)^2-1}}, \quad \lvert ax\rvert > 1$

Derivatives of Inverse Hyperbolic Functions

Function	Derivative
$\sinh^{-1}(ax)$	$\frac{a}{\sqrt{1+(ax)^2}}$
$\cosh^{-1}(ax)$	$\frac{a}{\sqrt{(ax)^2-1}}, \quad ax > 1$
$\tanh^{-1}(ax)$	$\frac{a}{1-(ax)^2}, \quad \lvert ax\rvert < 1$
$\coth^{-1}(ax)$	$\frac{a}{1-(ax)^2}, \quad \lvert ax\rvert > 1$
$\text{sech}^{-1}(ax)$	$-\frac{a}{(ax)\sqrt{1-(ax)^2}}, \quad 0 < ax \leq 1$
$\text{csch}^{-1}(ax)$	$-\frac{a}{\lvert ax\rvert\sqrt{1+(ax)^2}}$

Derivative of Logarithmic Functions

The derivatives of the following logarithmic functions are given below:

$$\frac{d}{dx}\ln(x) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}\ln(x^n) = \frac{nx^{n-1}}{x^n}, \quad x > 0,\, n \in \mathbb{R}$$

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}, \quad f(x) \neq 0$$

Derivative of Exponential Functions

The derivatives of the following exponential functions are:

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x), \quad f(x) \text{ is differentiable}$$