Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

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Lecture No.:- 2

Lecture Title: Coordinates, lines, distance, circles, and parabola functions

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2.1 Cartesian Coordinates in the Plane

In the previous section, we learned how to identify points on a line using real number coordinates. Now, we extend this concept to the plane by using ordered pairs of real numbers.

Coordinate Axes:

We draw two perpendicular lines (axes) intersecting at the origin, labeled O (0, 0).

The horizontal line is the x-axis, where values increase to the right.

The vertical line is the y-axis, where values increase upward.

Positive and Negative Directions:

Moving right and up are considered positive directions.

Moving left and down is negative.

Locating Points:

A unique ordered pair can identify any point P in the plane (a, b).

To find this pair, draw perpendicular lines from P to the axes; these lines will intersect the axes at points with coordinates a (x-coordinate) and b (y-coordinate).

Points on the y-axis have an x-coordinate of 0, and points on the x-axis have a y-coordinate of 0.

The Origin:

The origin is the point where both coordinates are zero: (0, 0).

We can start with an ordered pair (a, b) and identify it with a point P in the plane, often denoted as P(a, b). The context will clarify whether (a, b) refers to a point or an interval on the real line.

Rectangular Coordinate System:

- This system, also known as the Cartesian coordinate system, uses two perpendicular axes to define points in a plane.
- The axes divide the plane into four regions called quadrants, numbered counterclockwise.

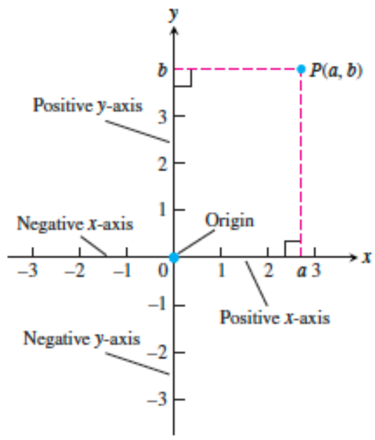


FIGURE 2.1 Cartesian coordinates in the plane is based on two perpendicular axes intersecting at the origin.

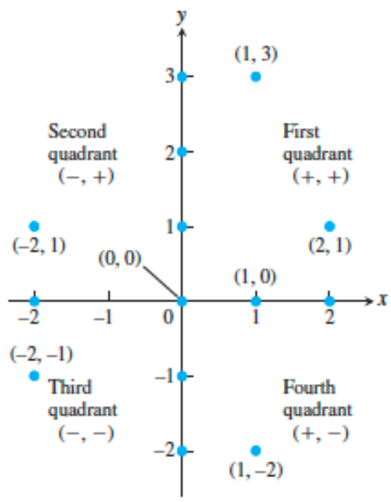


FIGURE 2.2 "Points labeled in the x-y coordinate or Cartesian plane. The points on the axes all have coordinate pairs but are usually labeled with single real numbers, (so (1, 0) on the x-axis is labeled as 1). Notice the coordinate sign patterns of the quadrants.".

2.2 Increments and Straight Lines

When a particle moves from one point in the plane to another, the net changes in its coordinates are called increments. They are calculated by subtracting the coordinates of the starting point from the coordinates of the ending point. If x changes from X1 to X2 the increment in x is:

$$\Delta x = x_2 - x_1 .$$

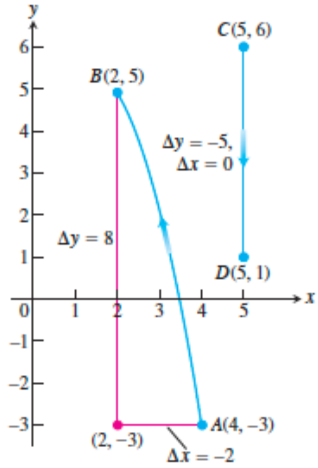


FIGURE 2.3 Coordinate increments may be positive, negative, or zero (Example 1).

EXAMPLE 1 In going from the point $As4, -3d$ to the point $B(2, 5)$ the increments in the x- and y-coordinates are

$$\Delta x = 2 - 4 = -2, \qquad \Delta y = 5 - s-3d = 8.$$

From $C(5, 6)$ to $D(5, 1)$ the coordinate increments are

$$\Delta x = 5 - 5 = 0, \qquad \Delta y = 1 - 6 = -5.$$

Any nonvertical line in the plane has the property that the ratio

$$m = \frac{\Delta y}{\Delta x} = \frac{y2 - y1}{x2 - x1}$$

has the same value for every choice of the two points $P_1\ (x_1, y_1)$ and $P_2\ (x_2, y_2)$ on the line (Figure 2.4). This is because the ratios of corresponding sides for similar triangles are equal.

The slope tells us the direction (uphill, downhill) and steepness of a line.

- A line with a positive slope rises uphill to the right;
- one with a negative slope falls downhill to the right

The direction and steepness of a line can also be measured with **an angle**

$$m = \tan \phi .$$

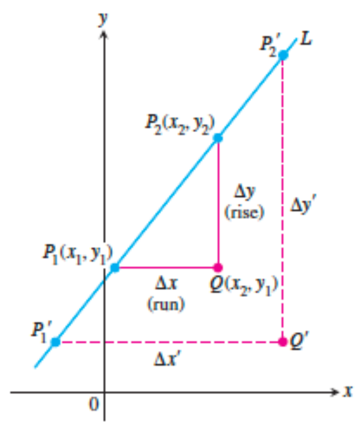


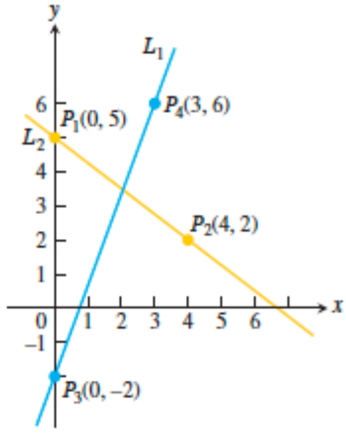
FIGURE 2.4 Triangles P1QP2 and P1’ Q’P2’ are similar, so the ratio of their sides has the same value for any two points on the line. This common value is the line’s slope."

EXAMPLE 2 : Find the slop from the following figure

The slope of L_1 is

$$m = \frac{\Delta y}{\Delta x} = \frac{6 - (-2)}{3 - 0} = \frac{8}{3}$$

That is, y increases 8 units every time x increases 3 units.



The slope of L_2 is

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - 5}{4 - 0} = \frac{-3}{4}$$

That is, y decreases 3 units every time x increases 4 units.

We can write an equation for a nonvertical straight line L if we know its slope m and the coordinates of one-point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is *any* other point on L, then we can use the two points P_1 and P to compute the slope,

$$m = \frac{y - y1}{x - x1}$$

So that

$$y - y_1 = m(x - x_1) \quad \text{or} \quad y = y_1 + m(x - x_1).$$

The equation

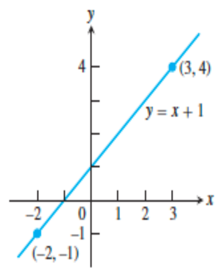
$$y = y_1 + m(x - x_1)$$

is the **point-slope equation** of the line that passes through the point sx_1, y_1d and has slope m.

EXAMPLE 3 : Write an equation for the line through the point (2, 3) with slope -3/2

Solution We substitute $x_1 = 2, y_1 = 3$, and $m = -3/2$ into the point-slope equation and obtain

$$y = 3 - \frac{3}{2}(x - 2), \qquad or\ \ y = 3 - \frac{3}{2}x + 6$$



EXAMPLE 4: A Line Through Two Points

Write an equation for the line through (−2, −1) and (3, 4).

Solution: The line's slope is

$$m = \frac{-1 - 4}{-2 - 3} = 1$$

We can use this slope with either of the two given points in the point-slope equation:

With (-2, -1)

y = -1+1*(x-(-2))

y =-1+x+2

y = x+1

With (3, 4)

y = 4+1*(x-3)

y = 4+x-3

y = x+1

=

Either way, $y = x + 1$ is an equation for the line

EXAMPLE 5: Finding the Slope and *y*-Intercept

Find the slope and *y*-intercept of the line $8x + 5y = 20$.

The equation $y = mx + b$ is called the **slope-intercept equation** of the line with slope *m* and *y*-intercept *b*.

Solution: Solve the equation for *y* to put it in slope-intercept form:

$$8x + 5y = 20$$

$$5y = -8x + 20$$

$$y = -\frac{8}{5}x + 4$$

The slope is $m = -8/5$. The *y*-intercept is $b = 4$.

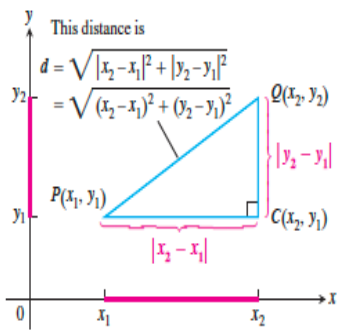
2.3 Distance and Circles in the Plane

The distance between points in the plane is calculated with a formula that comes from the Pythagorean theorem (Figure 2.5)

Distance Formula for Points in the Plane

The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$



EXAMPLE 6: Calculating Distance

The distance between P(-1,2) and *Q* (3, 4) is

$$\sqrt{(3-(-1))^2+(4-2)^2}=2\sqrt{5}$$

By definition, a **circle** of radius *a* is the set of all points *P*(*x*, *y*) whose distance from some center *C*(*h*, *k*) equals *a* (Figure 2.5). From the distance formula, *P* lies on the circle if and only if

$$\sqrt{(x-h)^2+(y-k)^2}=a$$

So

$$(x-h)^2+(y-k)^2 = a^2 \ldots\ldots\ldots\ldots\ldots (eq.1)$$

Equation (1) is the **standard equation** of a circle with center (*h*, *k*) and radius *a*.
The circle of radius $a = 1$ and centered at the origin is the **unit circle** with equation

$$x^2 + y^2 = 1.$$

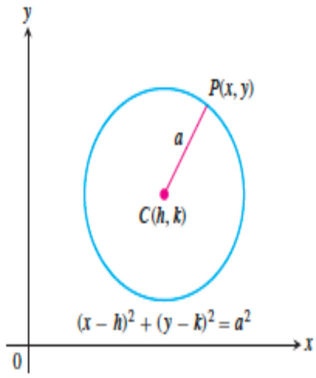


FIGURE 2.5 A circle of radius *a* in the XY-plane, with center at (*h*, *k*).

EXAMPLE 7

(a) The standard equation for the circle of radius 2 centered at (3, 4) is

$$(x-3)^2+(y-4)^2 = 2^2=4$$

(b) The circle

$$(x-1)^2+(y-5)^2 =3$$

has : h = 1, k = -5 , and a= $\sqrt{3}$. The center is the point $(h, k) = (1, -5)$ and the radius is a= $\sqrt{3}$

If an equation for a circle is **<u>not in standard form</u>**, we **can find the circle's center and radius by first converting the equation to standard form**.

EXAMPLE 8: Finding a Circle's Center and Radius

Find the center and radius of the circle

$X^2 + y^2 + 4x - 6y - 3 = 0$

Solution We convert the equation to standard form by completing the squares in *x* and *y*:

$(x^2 + 4x + 4 - 4) + (y^2 - 6y - 9 + 9) = 3$

$(x^2 + 4x + 4) + (y^2 - 6y - 9) = 3 + 4 + 9$

$(x^2 + 2)^2 + (y^2 - 3)^2 = 16$

The center is (-2,3), and the radius $a = 4$.

2.4 Parabolas

we look at parabolas arising as the graphs of equations of the form $y = ax^2 + bx + c$.

EXAMPLE 9: The Parabola $y = x^2$

Consider the equation $y = x^2$. Some points whose coordinates satisfy this equation are (0, 0), (1, 1), (3/2, 9/4), (−1, 1), (2, 4), and (−2, 4). These points (and all others satisfying the equation) make up a smooth parabola curve (Figure 2.6).

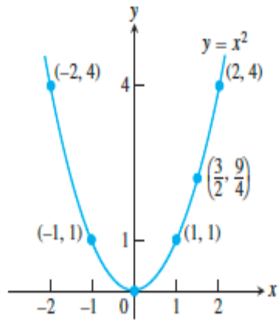


FIGURE 2.6 The parabola $y = x^2$ (Example 9).

The graph of an equation of the form

$$y = ax^2$$

is a **parabola** whose **axis** (axis of symmetry) is the *y*-axis. The parabola's **vertex** (point where the parabola and axis cross) lies at the origin. The parabola opens upward if $a > 0$ and downward if $a < 0$.

The Graph of y = ax^2 + bx + c, a ≠ 0

The graph of the equation y = ax^2 + bx + c, a ≠ 0, is a parabola. The parabola opens upward if a > 0 and downward if a < 0. The axis is the line

$$x = -\frac{b}{2a} \qquad (2)$$

EXAMPLE 10: Graphing a Parabola

Graph the equation $y = -1/2\, x^2 - x + 4$.

Solution Comparing the equation with $y = ax^2 + bx + c$ we see that

a = - ½ , b = -1 , c = 4

Since $a < 0$, the parabola opens downward. From Equation (2) the axis is the vertical line

$$x = -\frac{b}{2a} = -\frac{(-1)}{2\left(-\frac{1}{2}\right)} = -1$$

When x = -1 , we have

$$y = -\frac{1}{2}(-1)^2 - (-1) + 4 = \frac{9}{2}$$

The vertex is (-1 , 9/2) .

The x-intercepts are where y = 0

$$-\frac{1}{2}(x)^2 - x + 4 = 0$$

$$x^2 + 2x - 8 = 0$$
$$(x-2)(x+4) = 0$$
$$x = 2 \,, x = -4$$

We plot some points, sketch the axis, and use the opening direction to complete the graph in Figure 2.7.

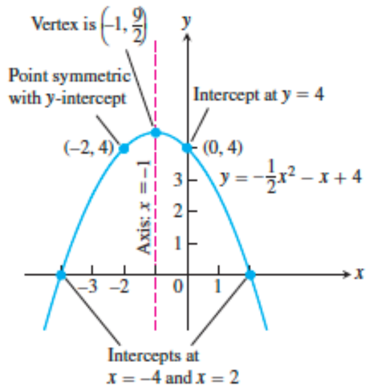


FIGURE 2.7 The parabola in (Example 10).

EXERCISES 1.2

Increments and Distance

In Exercises 1–4, a particle moves from A to B in the coordinate plane. Find the increments in the particle's coordinates. Also, find the distance from A to B.

1. A (-3,2) , B(-1,-2)
2. A(-1,-2) ,B (-3,2)
3. A(-3.2 ,-2) , B (-8.1, -2)
4. A($\sqrt{2}$, 4) , B $(0, 1.5)$

Slopes, Lines, and Intercepts

Plot the points in Exercises 5–8 and find the slope (if any) of the line they determine.

5. A (-1,2) , B(-2,-1)
6. A (-2,1) , B(2,-2)
7. A (2,3) , B(-1,3)
8. A (-2,0) , B(-2,-2)

In Exercises 9–17, write an equation for each line described.

9. Passes through (-1, 1) with slope -1
10. Passes through (2, -3) with slope ½
11. Passes through (3, 4) and (-2, 5)
12. Passes through (-8, 0) and (-1, 3)
13. Has slope -5/4 and y-intercept 6
14. Passes through (-12 , -9) and has slope 0
15. Has x-intercept 4 and y-intercept -1
16. Passes through (5 , -1) and is parallel to the line 2x + 5y = 15
17. Passes through (4, 10) and is perpendicular to the line 6x-3y =5

In Exercises 18–20, find the line's x- and y-intercepts and use this information to graph the line.

18. 3 x +4 y = 12
19. X + 2y = -4
20. 1.5x- y = -3

Circles

In Exercises 21–23, find an equation for the circle with the given center C(h, k) and radius a. Then sketch the circle in the xy-plane. Include the circle's center in your sketch. Also, label the circle's x- and y-intercepts, if any, with their coordinate pairs.

21. C(0,2) , a = 2
22. C(-1,5) , a = $\sqrt{10}$
23. C(3, 1/2) , a = 5

Graph the circles whose equations are given in Exercises 24–27. Label each circle's center and intercepts (if any) with their coordinate pairs.

24. $x^2 + y^2 + 4x - 4y + 4 = 0$
25. $x^2 + y^2 - 8x + 4y + 16 = 0$
26. $x^2 + y^2 + -3y - 4 = 0$
27. $x^2 + y^2 - 4x + 4y = 0$

Parabolas

Graph the parabolas in Exercises 28–31. Label the vertex, axis, and intercepts in each case.

28. $y = x^2 - 2x - 3$
29. $y = -x^2 + 4x$
30. $y = -\frac{1}{4} x^2 + 2x + 4$
31. $y = - x^2 - 6x - 5$