Routh-Hurwitz Criterion

This represents a method of determining the location of poles of a characteristic equation with respect to the left half and right half of the s-plane without actually solving the equation.

The T.F. of any linear closed loop system can be represented as,

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_n} = \frac{B(s)}{F(s)}$$

where `a' and `b' are constants.

To find closed loop poles we equate F(s) = 0. This equation is called **characteristic equation** of the system.

i.e. $$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots\dots + a_n = 0$$

Thus the roots of the characteristic equation are the closed loop poles of the system which decide the stability of the system.

Hurwitz's Criterion

The necessary and sufficient condition to have all roots of characteristic equation in left half of s-plane is that the sub-determinants D_K , K = 1, 2, ...n obtained from Hurwitz's determinant `H' must all be positive.

Email (abdulrahim.thiab.humod@uomus.edu.iq)

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots\ldots + a_n = 0$$

Method of forming Hurwitz determinant :

$$H = \begin{vmatrix} a_1 & a_3 & a_5 & \ldots\ldots & a_{2n-1} \\ a_0 & a_2 & a_4 & \ldots\ldots & a_{2n-2} \\ 0 & a_1 & a_3 & \ldots\ldots & a_{2n-3} \\ 0 & a_0 & a_2 & \ldots\ldots & a_{2n-4} \\ 0 & 0 & a_1 & \ldots\ldots & a_{2n-5} \\ . & . & . & \ldots\ldots & . \\ . & . & . & \ldots\ldots & . \\ 0 & . & . & \ldots\ldots & a_n \end{vmatrix}$$

$$D_1 = |a_1| \qquad D_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} \qquad D_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} \qquad D_K = |H|$$

For the system to be stable, all the above determinants must be positive.

Example : *Determine the stability of the given characteristic equation by Hurwitz's method.*

$F(s) = s^3 + s^2 + s^1 + 4 = 0$ is characteristic equation.

Solution : $a_0 = 1,\ a_1 = 1,\ a_2 = 1,\ a_3 = 4,\ n = 3$

$$H = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix}$$

$$D_1 = |1| = 1$$

$$D_2 = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -3$$

$$D_3 = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 4 - 16 = -12$$

As D_2 and D_3 are negative, given system is unstable.

Examples on Hiwartz Criterion (Multiple Choice Questions)

1- D1 of the Hiwartz stability methods of the given characteristic equation

F(s) = $S^3 + S^2 + S + 4 = 0$ is

A) 1 B) -3 C) -12 D) 4

2- D2 of the Hiwartz stability methods of the given characteristic equation

F(s) = $S^3 + S^2 + S + 4 = 0$ is

A) 1 B) -3 C) -12 D) 4

3- D3 of the Hiwartz stability methods of the given characteristic equation

F(s) = $S^3 + S^2 + S + 4 = 0$ is

A) 1 B) -3 C) -12 D) 4

4- the Hiwartz stability methods of the given characteristic equation

F(s) = $S^3 + S^2 + S + 4 = 0$ is

A) Stable , B) unstable , C) Marginally , D) Critically

5- For the Hiwartz stability we use .... as the T.F

A) G(s) H(s) B) 1+ G(s)H(s) C) G(s) D) G(s)/ H(s)

Email (abdulrahim.thiab.humod@uomus.edu.iq)

6- According to Hurwitz criterion, the characteristic equation.

$$S^4 + 8S^3 + 18S^2 + 16S + 5 = 0$$

a) The system is stable

b) The system is marginally stable

c) The system is unstable

d) None of these

$$H = \begin{vmatrix} 8 & 16 & 0 & 0 \\ 1 & 18 & 5 & 0 \\ 0 & 8 & 16 & 0 \\ 0 & 1 & 18 & 5 \end{vmatrix}$$

$$D_1 = |8| = 8$$

$$D_2 = \begin{vmatrix} 8 & 16 \\ 1 & 8 \end{vmatrix} = 8 \times 18 - 16 = 128$$

$$D_3 = \begin{vmatrix} 8 & 16 & 0 \\ 1 & 18 & 5 \\ 0 & 8 & 16 \end{vmatrix} = 8 \times \begin{vmatrix} 18 & 5 \\ 8 & 16 \end{vmatrix} - 16 \times \begin{vmatrix} 1 & 5 \\ 0 & 16 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & 18 \\ 0 & 8 \end{vmatrix}$$

$$= 1984 - 256 - 0 = 1728$$

$$D_4 = \begin{vmatrix} 8 & 16 & 0 & 0 \\ 1 & 18 & 5 & 0 \\ 0 & 8 & 16 & 0 \\ 0 & 1 & 18 & 5 \end{vmatrix} = 5 \times \begin{vmatrix} 8 & 16 & 0 \\ 1 & 18 & 5 \\ 0 & 8 & 16 \end{vmatrix} = 5 \times 1728 = 8640$$

D_1, D_2, D_3 and D_4 are positive

7- in Hurwitz criterion, the system to be sable if all determinate must me ---------

A) Zero b) 1 c) negative d) positive

8- in Hurwitz criterion, if all determinates are positive then the system will be ------

A) Stable b) unstable c) marginally stable d) conditionally stable

Email (abdulrahim.thiab.humod@uomus.edu.iq)

9- in Hurwitz criterion, if all determinates are positive then the system will be ------

A) Stable b) unstable c) marginally stable d) conditionally stable

10- in Hurwitz criterion, if two determents are negative then the system will be -------

A) Stable b) unstable c) marginally stable d) conditionally stable

11- in Hurwitz criterion, if all determents are negative then the system will be ------

A) Stable b) unstable c) marginally stable d) conditionally stable

12. For the characteristic equation:

s^3 + 7s^2 + 14s + K = 0

What is the range of (K) for which the system is stable?

A) K > 0 B) K < 98 C) 0 < K < 98 D) K > 98

Answer: C) 0 < K < 98

13. For the characteristic equation:

s^3 + 3s^2 + 3s + K = 0

What is the range of (K) for which the system is stable?

A) (K > 0) B) (K < 9) C) (0 < K < 9) D) (K > 9)

Answer: C) (0 < K < 9)

Examples on pole -zero (Multiple Choice Questions)

1- ……………………The value of "s" , which make the T.F infinite after substitution in the denominator of a T.F ,

A) Pole , B) Zero , C) Stability , D) Characteristic Equation

2- …………………….The value of "s" , which make the T.F zero after substitution in the numerator of a T.F ,

A) Pole , B) Zero , C) Stability , D) Characteristic Equation

3- The S- plane can be divided in to three distinct zone which is stable, unstable and ………..

A) Pure stable , B) unstable , C) Marginally stable D) absolutely stable

4- The system is (unstable) if the Poles are in

A) LHS B) RHS C) on Imj axis D) two non-repeated pair on Imj axis

Email (abdulrahim.thiab.humod@uomus.edu.iq)

59-The value of (s) which make the transfer function ------------------after the substitution in the denominator of a T.F are called "poles" of that T.F

a) Zero b) infinite c) 1 d) -1

60-The value of (s) which make the transfer function ------------------after the substitution in the nominator of a T.F are called "zeros" of that T.F

a) Zero b) infinite c) 1 d) -1

$$61 - T(s) = \frac{2(S+1)^2(S+2)(S^2+2S+2)}{S^3(S^2+6S+25)(S+4)}$$, the simple zero of this equation is

a) S= -1 b)S =-2 c) S= -1± j d) s = 1

$$62 - T(s) = \frac{2(S+1)^2(S+2)(S^2+2S+2)}{S^3(S^2+6S+25)(S+4)}$$, the repeated zero of this equation is

a) S= -1 b)S =-2 c) S= -1± j d) s = 1

$$63 - T(s) = \frac{2(S+1)^2(S+2)(S^2+2S+2)}{S^3(S^2+6S+25)(S+4)}$$, the complex zero of this equation is

a) S= -1 b)S =-2 c) S= -1± j d) s = 1

$$64 - \frac{(S+2)}{S\ \ (S^2+2S+2)(S^2+7S+12)}$$ in this equation, the number of zeros is:

a) 2 b) 1 c) 3 d) 5

$$65 - \frac{(S+2)}{S\ \ (S^2+2S+2)(S^2+7S+12)}$$ in this equation, the number of poles is:

b) 2 b) 1 c) 3 d) 5

$$67 - \frac{(S+2)}{S\ \ (S^2+2S+2)(S^2+7S+12)}$$ in this equation, the simple pole is:

a) S = 0 b) S= -3 C) S=-4 d) s=-1 ±j

Email (abdulrahim.thiab.humod@uomus.edu.iq)

$68 - \frac{(S+2)}{S\ (S^2+2S+2)(S^2+7S+12)}$ in this equation, the complex pole is:

A) S = 0 b) S= -3 C) S=-4 d) s=-1 ±j

$69 - \frac{(S+2)}{S\ (S^2+2S+2)(S^2+7S+12)}$ in this equation, the repeated pole is:

B) S = 0 b) S= -3 C) S=-4 d) None of them

70- Control system can be -----------

a) Stable b) unstable c) marginally stable d) all of them

71-If the real negative poles located in the L.H.S of S-plane, that's means the stability condition is -----------

A) Stable b) unstable c) marginally stable d) all of them

72-Complex conjugate with the negative real part poles located in L.H.S of S-plane, which means the stability condition is -------------

A) Stable b) unstable c) marginally stable d) all of them

73- Real positive poles located in the R.H.S of S-plane, which means the system is---------

A) Stable b) unstable c) marginally stable d) all of them

74- complex conjugate with the positive real part poles located in the R.H.S of S-plane, which means the stability condition---------------

A) Stable b) unstable c) marginally stable d) all of them

75- non-repeated pair pole on Imaginary axis with no real part on the R.H.S of S-plane , which means the stability condition is ---------

A) Stable b) unstable c) marginally stable d) all of them

Email (abdulrahim.thiab.humod@uomus.edu.iq)

76- non-repeated pair pole on Imaginary axis with no real part on the R.H.S of S-plane , which means the stability condition is --------

A) Stable b) unstable c) conditionally stable d) critically stable

77-repeated pair pole on the Imaginary axis without any pole in the R.H.S of the S-plane, which means the stability condition is ---------

A) Stable b) unstable c) marginally stable d) all of them

78- in Hurwitz criterion, the system to be sable if all determinate must me ---------

A) Zero b) 1 c) negative d) positive

79 - in Hurwitz criterion, if all determinates are positive then the system will be ------

A) Stable b) unstable c) marginally stable d) conditionally stable

80 - in Hurwitz criterion, if one determent is negative then the system will be ------

A) Stable b) unstable c) marginally stable d) conditionally stable

81- in Hurwitz criterion, if two determents are negative then the system will be -------

A) Stable b) unstable c) marginally stable d) conditionally stable

82 - in Hurwitz criterion, if all determents are negative then the system will be ------

A) Stable b) unstable c) marginally stable d) conditionally stable

83 - in Routh criterion, if there is any sign change, then the system will be------

A) Stable b) unstable c) marginally stable d) conditionally stable

84 - in the Routh criterion, the number of sign changes is ------------------ the number of the roots lying in the RHS of the S-plane

A) greater than b) equal c) less than d) none of them

85- in the Routh criterion, if there is no sign change in the first column, then the system is

A) Stable b) unstable c) marginally stable d) conditionally stable

Email (abdulrahim.thiab.humod@uomus.edu.iq)

85- in the Routh criterion, if there is a sign change in the first column, then the system is

A) Stable b) unstable c) marginally stable d) conditionally stable

86- in the Routh criterion, if there is one sign change in the first column, then the system is

A) Stable b) unstable c) marginally stable d) conditionally stable

87- in the Routh criterion, if there is one sign change in the first column, then there is ------ pole in the RHS of the S-plane

a) one b) two c) there d) zero

88- in the Routh criterion, if there are two sign changes in the first column, then there is ------ pole in the RHS of the S-plane

a) one b) two c) there d) zero

Email (abdulrahim.thiab.humod@uomus.edu.iq)