

Mathematics and Biostatistics



LECTURE 7 Probability concepts

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OUTLINE

- Probability concepts :
- Properties of probability
- Set theory and set notation (basic notation)
- counting techniques- permutations and combinations
- calculating the probability of an events
- probability distribution of discrete variable
- binomial distribution, Poisson distribution
- continues probability distribution and normal distribution, review questions and exercises

PROBABILITY CONCEPTS IN BIOSTATISTICS

 Probability is a fundamental concept in biostatistics, providing the foundation for statistical inference and decision-making in medical and biological research. Understanding probability allows researchers to quantify uncertainty, make predictions, and draw conclusions from data.

1. PROPERTIES OF PROBABILITY

- Probability is a measure of the likelihood that an event will occur. It is quantified as a number between 0 and 1, where:
- **0** indicates impossibility (the event will not occur).
- **1** indicates certainty (the event will occur).
- Key Properties:
- **1.** Non-negativity: For any event A, $P(A) \ge 0$
- Additivity: If two events AA and BB are mutually exclusive (cannot occur simultaneously), then P(AUB)=P(A)+P(B).
- **3.** Normalization: The probability of the entire sample space S is 1, i.e., P(S)=1.

2.SET THEORY AND SET NOTATION

- Set theory is used to describe collections of outcomes or events. Basic notation includes:
- Sample Space (S): The set of all possible outcomes.
- Event (A): A subset of the sample space.
- Union (AUB): The event that either A or B occurs.
- Intersection $(A \cap B)$: The event that both A and B occur.
- **Complement (A**^c) : The event that A does not occur.

3. COUNTING TECHNIQUES: PERMUTATIONS AND COMBINATIONS

- Counting techniques are used to determine the number of ways events can occur.
- Permutations:
- The number of ways to arrange r objects from a set of n objects, where order matters:

$$P(n,r) = rac{n!}{(n-r)!}$$

- Combinations:
- The number of ways to choose r objects from a set of n objects, where order does not matter:

$$C(n,r) = rac{n!}{r!(n-r)!}$$

CALCULATING THE PROBABILITY OF AN EVENT

The probability of an event A is calculated as:

 $P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

- Example in Biostatistics:
- In a study of 100 patients, 20 have a specific gene mutation. The probability of randomly selecting a patient with the mutation is:

$$P(A) = \frac{20}{100} = 0.20$$

5. PROBABILITY DISTRIBUTION OF A DISCRETE VARIABLE

- A probability distribution describes the probabilities of all possible outcomes for a discrete random variable.
- Example in Biostatistics:
- Let X be the number of defective genes in a patient. Suppose:
- P(X=0)=0.5, P(X=1)=0.3, P(X=2)=0.2

This defines the probability distribution of X.

6. BINOMIAL DISTRIBUTION

- The binomial distribution models the number of successes in n independent trials, each with probability p of success.
- Formula:
- $P(X=k) = C(n,k) \cdot p^{k} \cdot (1-p)^{n-k}$

Example in Biostatistics:

- In a clinical trial, the probability of a patient responding to a treatment is p=0.6. If 5 patients are treated, the probability that exactly 3 respond is:
- $P(X=3) = C(5,3) \cdot 0.6^3 \cdot 0.4^2 = 0.3456$

7. POISSON DISTRIBUTION

- The Poisson distribution models the number of rare events occurring in a fixed interval of time or space.
- Formula:

$$P(X=k)=rac{\lambda^k e^{-\lambda}}{k!}$$

• where λ is the average rate of occurrence.

Example in Biostatistics:

Suppose the average number of hospital admissions per day for a rare disease is 2. The probability of exactly 3 admissions in a day is:

$$P(X=3) = rac{2^3 e^{-2}}{3!} = 0.1804$$

8. CONTINUOUS PROBABILITY DISTRIBUTION AND NORMAL DISTRIBUTION

- Continuous distributions describe variables that can take any value within an interval. The normal distribution is the most important continuous distribution in biostatistics.
- Normal Distribution:
- Symmetric, bell-shaped curve.
- Defined by mean (μ) and standard deviation (σ).
- Probability density function:

$$f(x)=rac{1}{\sigma\sqrt{2\pi}}e^{-rac{(x-\mu)^2}{2\sigma^2}}$$

Example in Biostatistics:

Suppose the blood pressure of adults follows a normal distribution with μ=120 mmHg and σ=10 mmHg. The probability that a randomly selected adult has a blood pressure between 110 and 130 mmHg can be calculated using the standard normal table.

9. REVIEW QUESTIONS AND EXERCISES

• Example:

- In a study of 100 patients, 30 have hypertension, and 20 have diabetes. If 10 patients have both conditions, what is the probability that a randomly selected patient has hypertension given that they have diabetes?
- **Solution**:

$$P(ext{Hypertension}| ext{Diabetes}) = rac{P(ext{Hypertension} \cap ext{Diabetes})}{P(ext{Diabetes})} = rac{10/100}{20/100} = 0.5$$

• Example:

- In a study, you need to select a committee of 3 members from a group of 10 researchers. How many different committees can be formed?
- Solution:

$$C(10,3) = rac{10!}{3!7!} = 120$$

• Example:

- In a population of 1,000 individuals, 150 have a specific genetic mutation. What is the probability that a randomly selected individual has the mutation?
- **Solution**:

$$P(\text{Mutation}) = \frac{150}{1000} = 0.15$$



- Thanks for lessening ...
- Any questions?