Root Locus Technique.

Root Locus

It is a graphical method, in which movement of poles in the s-plane is sketched when a particular parameter of system is varied from zero to infinity. For root locus method, gain (K) is assumed to be a parameter which is to be varied from zero to infinity.

Example 1: Consider the closed loop control system shown in figure (1)

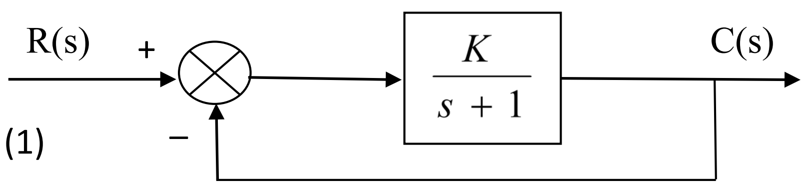


Figure (1)

$$\left(\boldsymbol{G(s) = \frac{K}{s+1}}, \quad \boldsymbol{H(s) = 1} \right)$$

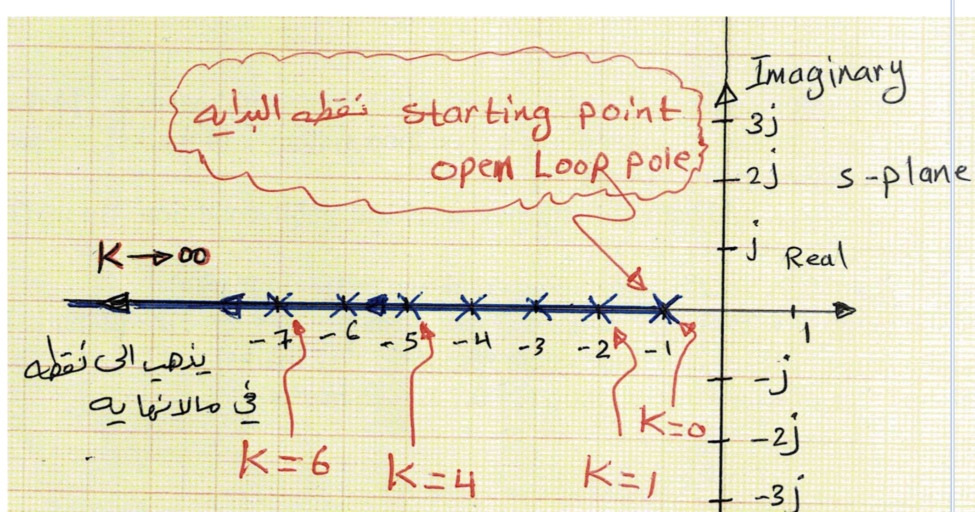
When the transfer function is expressed with the coefficients of the highest powers of s in both the numerator (K) and the denominator (s+1) equal to unity the value of K is defined as the **static loop sensitivity**

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s+1}}{1+\frac{K}{s+1}} = \frac{\frac{K}{s+1}}{\frac{s+1+k}{s+1}} = \frac{K}{s+1+K}$$

The problem is to determine the roots of the characteristic equation $(s + 1 + K = 0)$ for all the values of K and plot the root in the s-plane. The root is given by

$$s = -1 - K$$

K	$s = -1 - K$	
0	-1	Open loop pole
1	-2	Closed loop pole
2	-3	Closed loop pole
3	-4	Closed loop pole
4	-5	Closed loop pole
5	-6	Closed loop pole
6	-7	Closed loop pole



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Example 2: Consider the closed loop control system shown in figure (2)

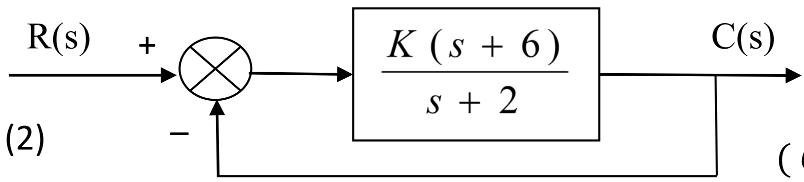


Figure (2)

$$\left(G(s) = \frac{K(s+6)}{s+2}\ ,\quad H(s) = 1 \right)$$

When the transfer function is expressed with the coefficients of the highest powers of s in both the numerator (K(s+6)) and the denominator (s+2) equal to unity the value of K is defined as the **static loop sensitivity**

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K(s+6)}{s+2}}{1+\frac{K(s+6)}{s+2}} = \frac{\frac{K}{s+2}}{\frac{s+2+ks+6K}{s+2}} = \frac{K}{s+2+Ks+6K} = \frac{K}{(1+K)s+2+6K}$$

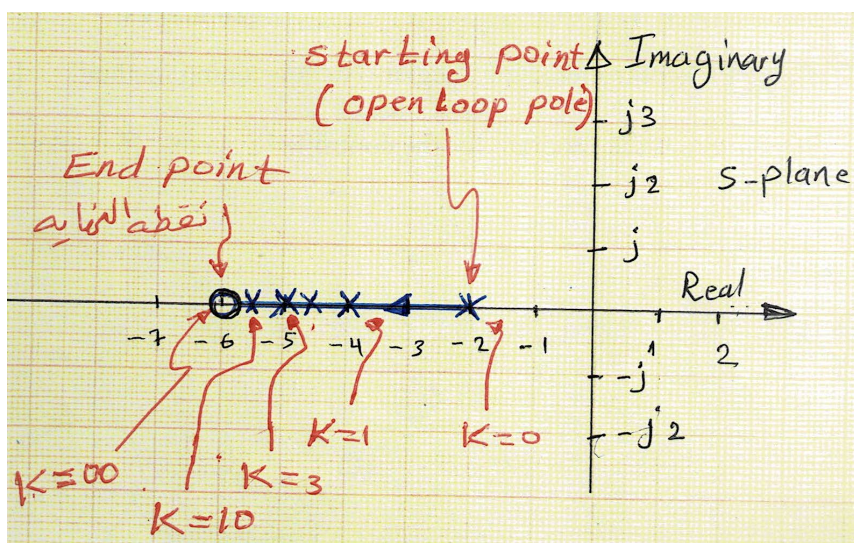
The problem is to determine the roots of the characteristic equation

$$(1+K)s + 2 + 6K = 0$$

for all the values of K and plot the root in the s-plane. The root is given by

$$s = \frac{-2-6K}{1+K}$$

K	$s = \frac{-2-6K}{1+K}$	
0	-2	Open loop pole
1	-4	Closed loop pole
2	-4.66	Closed loop pole
3	-5	Closed loop pole
10	-5.6	Closed loop pole
100	-5.96	Closed loop pole
1000	-5.996	Closed loop pole



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Example 3: Consider the closed loop control system shown in figure (3)

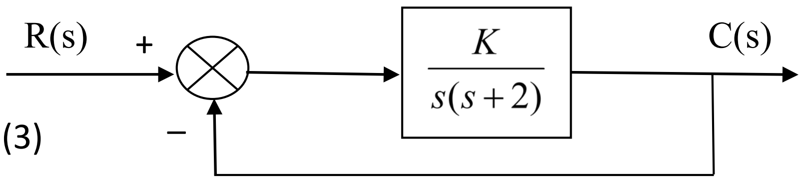

Figure (3)

When the transfer function is expressed with the coefficients of the highest powers of s in both the numerator and the denominator equal to unity the value of K is defined as the **static loop sensitivity**

$$G(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K}{s(s+2)+K} = \frac{K}{s^2+2s+K}$$
$$= \frac{{\omega_n}^2}{s^2+2\varepsilon\omega_n+{\omega_n}^2}$$

The problem is to determine the roots of the characteristic equation $(s^2+2s+K=0)$ for all the values of K and plot these roots in the s-plane. The roots are given by

$$s_1, s_2 = -1 \pm \sqrt{1-K} = -1 \pm \sqrt{-1}\sqrt{K-1} = -1 \pm j\sqrt{K-1}$$

K	$s_1 = -1 - j\sqrt{K-1}$	$s_2 = -1 + j\sqrt{K-1}$	
0	0	-2	Open loop poles
0.5	-0.293	-1.707	Closed loop poles
0.75	-0.5	-1.5	Closed loop poles
1.0	-1	-1	Closed loop poles
2.0	-1+j1	-1-j1	Closed loop poles
3.0	-1+j1.414	-1-j1.414	Closed loop poles
50	-1+j7	-1-j7	Closed loop poles

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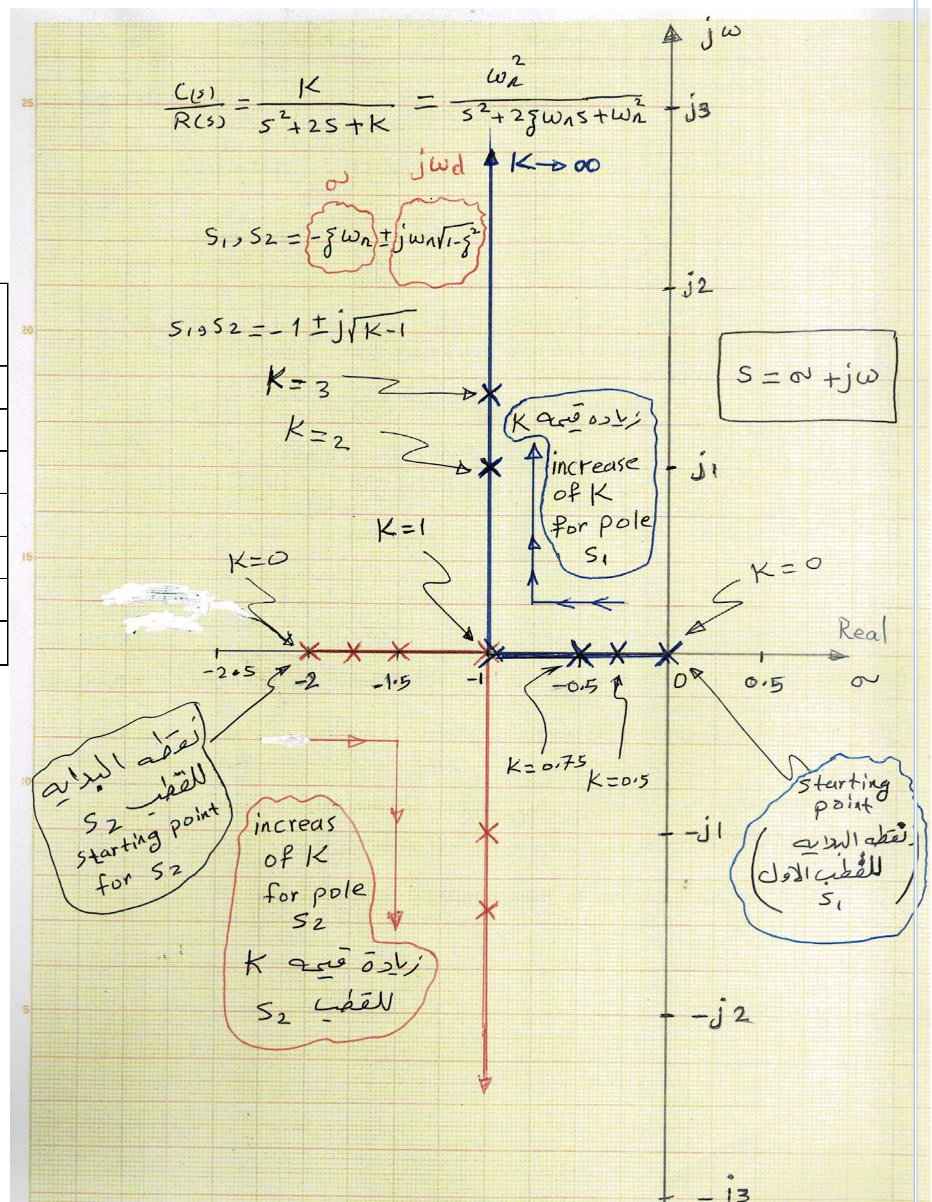
50	-1+j7	-1-j7



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Rules of Construction of Root Locus

The following rules are applicable in sketching the root locus plot.

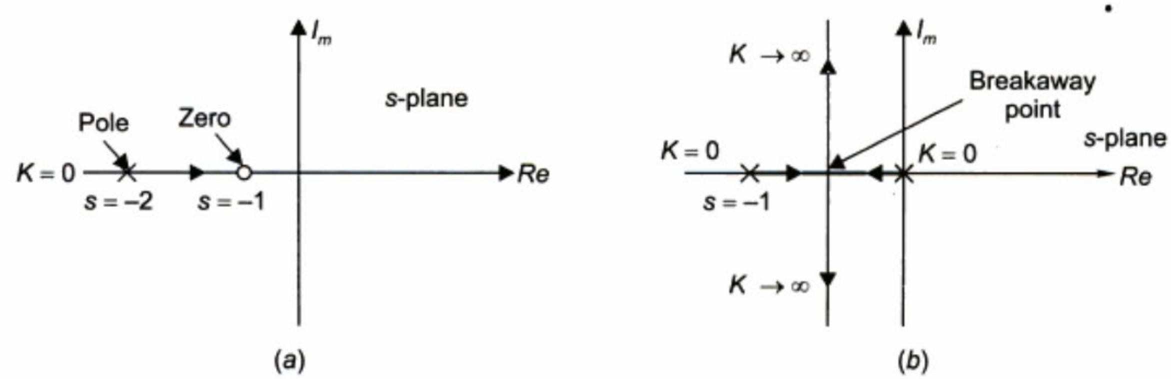
Rule 1: Symmetry of root locus—Any root locus must be symmetrical about the real axis,

Rules 2 and 3: Starting and termination of root loci—Root locus will start from an open-loop pole with gain $K = 0$ and terminate either on an open-loop zero or to infinity with $K = \infty$.

Let us illustrate these rules with an example. Let open-loop transfer functions of control systems are

(1) $G(s)H(s) = \dfrac{K(s+1)}{(s+2)}$

(2) $G(s)H(s) = \dfrac{K}{s(s+1)}$



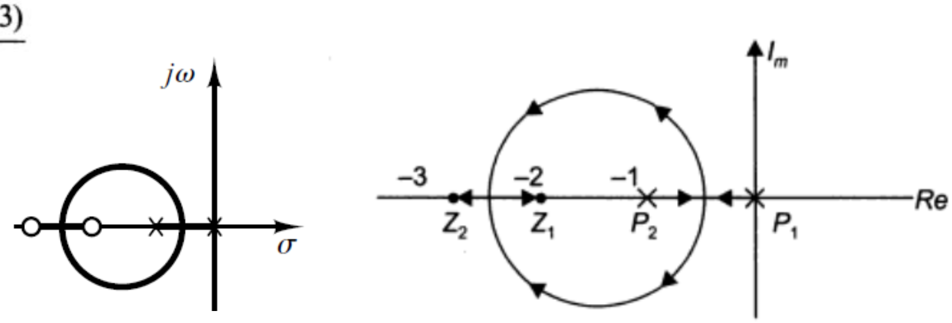
Rule 4: Number of root loci—If P is the number or poles and Z is the number of zeros in the transfer function $G(s)H(s)$, the number of root loci N will be as follows:

$$N = P \text{ if } P > Z$$
$$N = P = Z \text{ if } P = Z$$

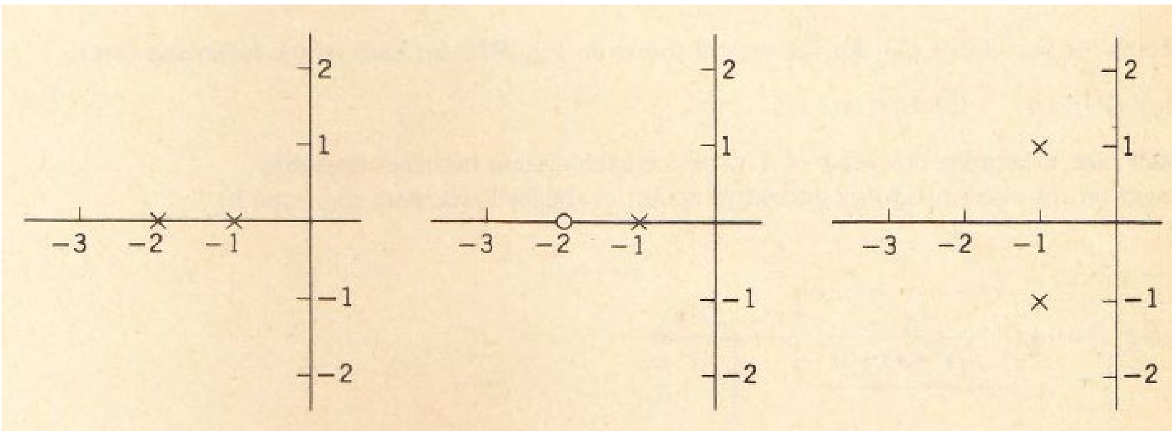
Rule 5: Root loci on the real axis—The root locus on the real axis will lie in a section of the real axis to the left of an odd number of poles and zeros.

This rule is illustrated through the following examples:

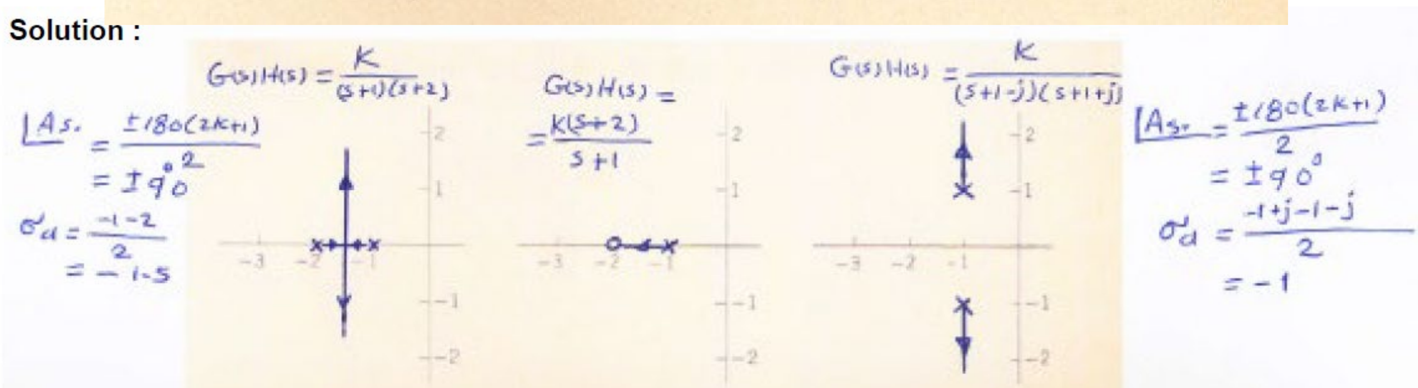
(1) $G(s)H(s) = \dfrac{K(s+2)(s+3)}{s(s+1)}$



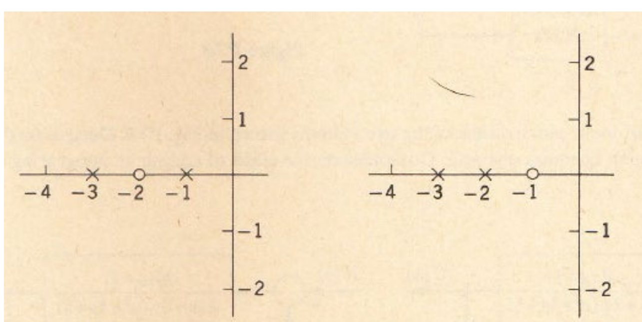
Example: sketch the root locus plot for each system of the following figuers



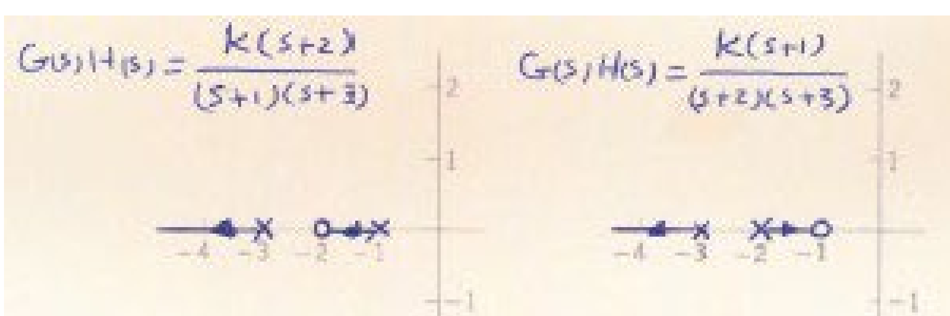
Solution :



Example:



solution



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Rule 6: The number of asymptotes and their angles with the real axis—As the value of K is increased to ∞, some branches of root locus from the real axis approach infinity along some asymptotic lines. These asymptotic lines are straight lines originating from the real axis make certain angles with the real axis. The total number of asymptotic lines and the angles they would make are calculated as follows:

Number of asymptotic lines asymptotes

$$= P - Z$$

where P is the number of poles and Z is the number of zeros of the open-loop transfer function, $G(s)H(s)$.

The angle of asymptotes with the real axis is

$$\phi_A = \frac{(2q+1)180^\circ}{P-Z} \quad \text{where } q = 0, 1, 2, \ldots \ \text{p-z-1}$$

Let us consider,

$$G(s)H(s) = \frac{K}{s(s+2)}$$

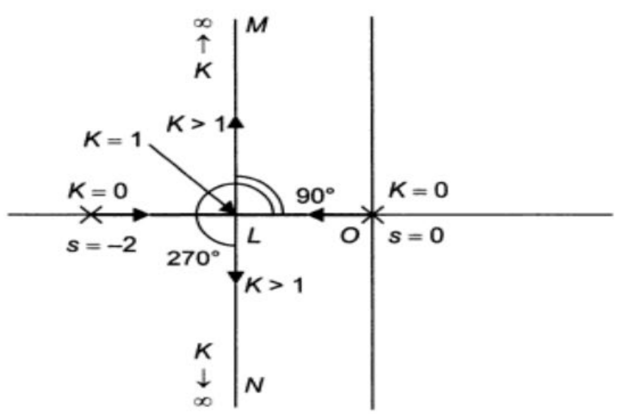
Here, the number of poles $P = 2$; they are at $s = 0$ and $s = -2$ and number of zeros $Z = 0$.

Number of asymptotes $\quad = P - Z = 2 - 0 = 2$

Let the angle of two asymptotes be ϕ_A and ϕ'_A respectively. Then,

$$\phi_A = \frac{(2 \times 0 + 1)180^\circ}{2-0} = 90^\circ \quad \text{for } q = 0$$

$$\phi'_A = \frac{(2 \times 1 + 1)180^\circ}{2-0} = 270^\circ \quad \text{for } q = 1$$



MCQ question

1- What is the primary purpose of the root locus technique in control systems?

- A) To analyze the frequency response of a system
- B) To determine the stability of a system as a function of a parameter (usually gain)
- C) To design a PID controller
- D) To calculate the steady-state error of a system

Answer: B) To determine the stability of a system as a function of a parameter (usually gain)

2- Which of the following is a rule for sketching the root locus?

- A) The root locus starts at the zeros and ends at the poles of the open-loop transfer function
- B) The root locus starts at the poles and ends at the zeros of the open-loop transfer function
- C) The root locus is always symmetric about the real axis
- D) Both B and C

Answer: D) Both B and C

3- The root locus plot is symmetric about:

- A) The imaginary axis- - B) The real axis- C) The origin D) The diagonal axis

Answer: B) The real axis

4- Which technique gives quick transient and stability response?

a) Root Locus b) Bode c) Nyquist d) None of these

5- The root locus of the feedback control system having the real characteristic equation $s^2 + 6Ks + 2s + 5 = 0$ where $K > 0$, enters into the real axis at

a) S = -1 b) $S = -\sqrt{5}$ c) S = 5 d) $S = \sqrt{5}$

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6- Root locus always starts from:

a) The open loop poles and terminate at the open loop zeros

b) The open loop zeros and terminate at the open loop poles

c) The close loop poles and terminate at the open loop zeros

d) The close loop poles and terminate at the close loop zeros

7- An open loop transfer function with negative unity feedback is given as

$\frac{K(S+1)}{S(S+2)(S+5)}$ The number of asymptotes will be

a) 3 b) 4 c) 2 d) 5

8- The root locus plot of a system with open-loop transfer function **G(s)H(s) = K/s(s+2)** will have:

- A) One pole at the origin and one pole at s = -2
- B) One zero at the origin and one pole at s = -2
- C) Two poles at the origin
- D) Two zeros at s = -2

Answer: A) One pole at the origin and one pole at s = -2

9- The number of branches in the root locus plot is equal to:

- A) The number of poles of the open-loop transfer function
- B) The number of zeros of the open-loop transfer function
- C) The number of poles or zeros, whichever is greater
- D) The number of poles minus the number of zeros

Answer: A) The number of poles of the open-loop transfer function

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10- The root locus plot crosses the imaginary axis at:

- A) The gain margin
- B) The phase margin
- C) The point where the system becomes marginally stable
- D) The point where the system becomes unstable

Answer: C) The point where the system becomes marginally stable

11- The asymptotes of the root locus are determined by:

- A) The number of poles and zeros
- B) The difference between the number of poles and zeros
- C) The sum of the poles and zeros
- D) The centroid of the poles and zeros

Answer: B) The difference between the number of poles and zeros

12- The root locus plot is used to analyze the effect of:

- A) System gain on stability
- B) System gain on steady-state error
- C) System gain on frequency response
- D) System gain on phase margin

Answer: A) System gain on stability

13- The root locus plot of a system with no open-loop zeros will:

- A) End at infinity
- B) End at the origin
- C) End at the poles
- D) Not exist

Answer: A) End at infinity

14- The root locus plot is applicable to:

- A) Linear time-invariant systems
- B) Nonlinear systems
- C) Time-varying systems
- D) Discrete-time system

Answer: A) Linear time-invariant systems

15- The root locus plot is used to determine:

- A) The range of gain for stability
- B) The range of gain for instability
- C) The range of gain for optimal performance
- D) The range of gain for maximum overshoot

Answer: A) The range of gain for stability