1- Trigonometric Functions :- الدوال المثلثية

There are six trigonometric functions; they are;

1- $\sin\theta = \frac{y}{1} = y$

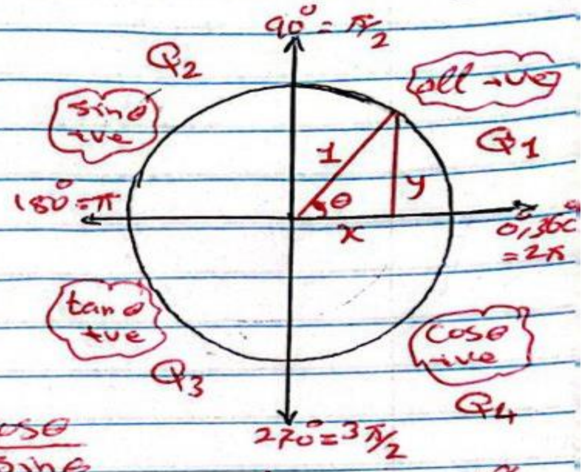
2- $\cos\theta = \frac{x}{1} = x$

3- $\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

4- $\csc\theta = \frac{1}{y} = \frac{1}{\sin\theta}$

5- $\sec\theta = \frac{1}{x} = \frac{1}{\cos\theta}$

6- $\cot\theta = \frac{x}{y} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$



"unit circle"

We have two angle representatives;

1- Degree.

2- Radian.

Degree	Radian	Degree	Radian
30°	$\pi/6$	300°	$5\pi/3$
45°	$\pi/4$	315°	$7\pi/4$
60°	$\pi/3$	330°	$11\pi/6$
90°	$\pi/2$	360°	2π
120°	$2\pi/3$	0°	$0\pi = 0$
135°	$3\pi/4$		
150°	$5\pi/6$		
180°	π		
210°	$7\pi/6$		
225°	$5\pi/4$		
240°	$4\pi/3$		
270°	$3\pi/2$		

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The unit circle has four quarters, The sine & cosine func in these quarters are:

① Q_1

Angle	Sine	Cosine
0	0	1
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
60°	$\sqrt{3}/2$	$1/2$
90°	1	0

② Q_2

Angle	Sine	Cosine
180°	0	-1
150°	$\frac{1}{2}$	$-\sqrt{3}/2$
135°	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
120°	$\sqrt{3}/2$	$-\frac{1}{2}$

③ Q_3

Angle	Sine	Cosine
210°	$-\frac{1}{2}$	$-\sqrt{3}/2$
225°	$-\frac{1}{\sqrt{2}}$	$-1/\sqrt{2}$
240°	$-\sqrt{3}/2$	$-\frac{1}{2}$
220°	-1	0

④ Q_4

Angle	Sine	Cosine
300°	$-\sqrt{3}/2$	$\frac{1}{2}$
315°	$-\frac{1}{\sqrt{2}}$	$1/\sqrt{2}$
330°	$-\frac{1}{2}$	$\sqrt{3}/2$

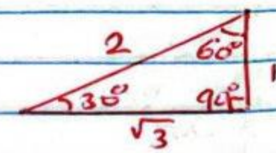
How To Evaluate The Trigonometric Funs.

① $30^\circ \Longrightarrow$ (Q1)

$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\tan 30 = \frac{1}{\sqrt{3}}$

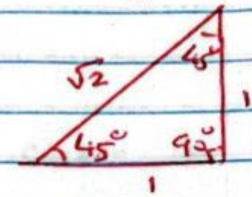


② $45^\circ \Longrightarrow$ (Q1)

$\sin 45 = \frac{1}{\sqrt{2}}$

$\cos 45 = \frac{1}{\sqrt{2}}$

$\tan 45 = 1$

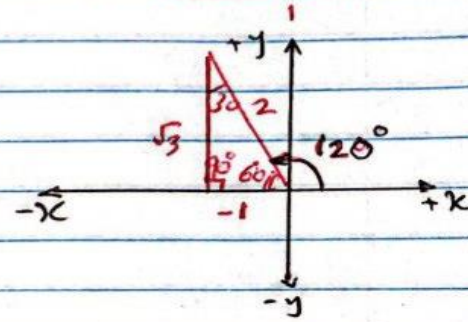


③ $120^\circ \longrightarrow$ (Q2)

$\sin 120 = \frac{\sqrt{3}}{2}$

$\cos 120 = -1/2$

$\tan 120 = \sqrt{3}/_{-1} = -\sqrt{3}$

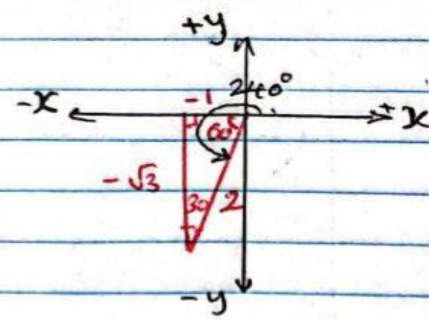


④ $240^\circ \longrightarrow$ (Q3)

$\sin 240 = -\sqrt{3}/2$

$\cos 240 = -1/2$

$\tan 240 = -\sqrt{3}/_{-1} = \sqrt{3}$

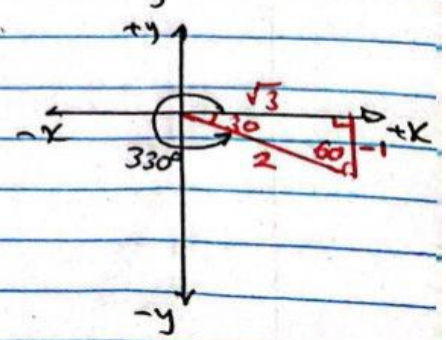


⑤ $330^\circ \longrightarrow$ (Q4)

$\sin 330 = -1/2$

$\cos 330 = \sqrt{3}/2$

$\tan 330 = -1/\sqrt{3}$



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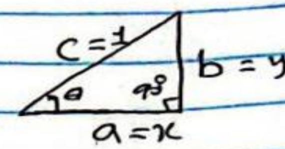
Trigonometric Relations :- العلاقات المثلثية

- Pythagorean Theorm نظرية فيثاغورس



$a^2 + b^2 = c^2$

If $a = x$, $b = y$, $c = 1$

$\therefore x^2 + y^2 = 1$

If we know from the unit circle that

$\sin\theta = y$

$\cos\theta = x$

$\therefore \sin^2\theta + \cos^2\theta = 1$ —— ①

- if we divided eq=1 by $\sin^2\theta$; yields,

$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \Rightarrow 1 + \cot^2\theta = \csc^2\theta$ —— ②

- If we divided eq=1 by $\cos^2\theta$; yields,

$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta} \Rightarrow 1 + \tan^2\theta = \sec^2\theta$ —— ③

Even & Odd Trigonometric Functions :

الدوال المثلثية الزوجية والفردية

Even Funs	Odd Funs
- $\cos(-\theta) = \cos\theta$	$\sin(-\theta) = -\sin\theta$
- $\sec(-\theta) = \sec\theta$	$\csc(-\theta) = -\csc\theta$
	$\tan(-\theta) = -\tan\theta$
	$\cot(-\theta) = -\cot\theta$

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Trigonometric Functions are equal at:

الدوال المثلثية تتساوي عند الزوايا التالية :-

$\cos\theta = \sin(90-\theta)$

$\sin\theta = \cos(90-\theta)$

$\csc\theta = \sec(90-\theta)$

$\cot\theta = \tan(90-\theta)$

Ex)

$\cos 0 = \sin 90 = 1$

$\cos 10 = \sin 80 =$

$\cos 20 = \sin 70$

$\cos 30 = \sin 60 = \sqrt{3}/2$

$\cos 45 = \sin 45 = \frac{1}{\sqrt{2}}$

$\cos 60 = \sin 30 = \frac{1}{2}$

$\cos 90 = \sin 0 = 0$

(A) Identity of the Trigonometric Functions :-

هوية الدوال المثلثية

(1) Double Angle Identity :-

* $\sin 2\theta = 2\sin\theta\cos\theta$
* $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
* $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(2) Half Angle Identity :-

$$\sin\frac{\theta}{2} = \mp\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \mp\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

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Ex) proof that $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

(Solution)

From the half angle identity,

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} * \sqrt{\frac{1-\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

From eqn ① $\Longrightarrow \sin^2\theta + \cos^2\theta = 1 \rightarrow 1-\cos^2\theta = \sin^2\theta$

$$\therefore \tan\frac{\theta}{2} = \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} = \frac{1-\cos\theta}{\sin\theta}$$

O.K

Ex) proof that $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$

(solution)

From the half angle identity,

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} * \sqrt{\frac{1+\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos^2\theta}{(1+\cos\theta)^2}}$$

From previous ex. $\Longrightarrow 1-\cos^2\theta = \sin^2\theta$

$$\therefore \tan\frac{\theta}{2} = \sqrt{\frac{\sin^2\theta}{(1+\cos\theta)^2}} = \frac{\sin\theta}{1+\cos\theta}$$

O.K

③ Sum & Difference Identity :- هوية الجمع والطرح

* $\sin(\alpha \mp \beta) = \sin\alpha\cos\beta \mp \cos\alpha\sin\beta$
* $\cos(\alpha \mp \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$
* $\tan(\alpha \mp \beta) = \dfrac{\tan\alpha \mp \tan\beta}{1 \pm \tan\alpha\tan\beta}$

i.e;

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

(4) The Power Reducing Formulas: علاقات تخفيض القوة (الاس)

* $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

* $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

* $\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$

(5) The Product To Sum Formulas: علاقات الضرب الى الجمع

* $\sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

* $\cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

* $\sin\alpha \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

* $\cos\alpha \sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$

(6) Sum to Product Formulas: علاقات الجمع الى الضرب

* $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

* $\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

* $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

* $\cos\alpha - \cos\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

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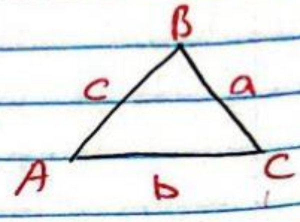
(7) Law of Sines :- قانون الجيوب

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



(8) Law of Cosines :- قانون الجيوب تمام

$$c^2 = a^2 + b^2 - 2ab\cos C$$

To calculate the area of the triangle

$$\text{Area} = A = \frac{1}{2}ab\sin C$$

or

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

(9) Law of Tangents :- قانون الظلال

It's no longer using, due to its complication;

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$$

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2- Inverse Trigonometric Functions
الدوال المثلثية العكسية

Inverse trigonometric Functions are defined as the inverse functions of the basic trigonometric Funs, which are sine, Cosine, tangent, Cotangent, secant & cosecant.

(1) Evaluate $\sin^{-1}$ Fun

The Range of $\sin^{-1}$ Fun is between $-\frac{\pi}{2}$ & $+\frac{\pi}{2}$

$\therefore$ Range $\Rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

So, it is only exist in this range.

Ex)

- $\sin^{-1} 0 = 0$
- $\sin^{-1} 1 = \pi/2$
- $\sin^{-1}(-1) = -\pi/2$
- $\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$
- $\sin^{-1} \frac{\sqrt{3}}{2} = \pi/3 = 60$
- $\sin^{-1}(-\frac{1}{2}) = -\pi/6$
- $\sin^{-1}(-\frac{1}{\sqrt{2}}) = -\pi/4$
- ~~sin~~

(2) Evaluate $\cos^{-1}$ Fun :-

The Range of $\cos^{-1}$ Fun is between zero & π

$\therefore$ Range $\Rightarrow [0, \pi]$, so $\cos^{-1}$ is only exist in this range

Ex)

- $\cos^{-1} \frac{1}{2} = \frac{\pi}{3}$
- $\cos^{-1}(-\frac{\sqrt{3}}{2}) = 150 = \frac{5\pi}{6}$

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- $\cos^{-1}(-\frac{1}{\sqrt{2}}) = 135 = \frac{3\pi}{4}$
- $\cos^{-1}(0) = \frac{\pi}{2}$
- $\cos^{-1}(1) = 0$
- $\cos(-1) = \pi$

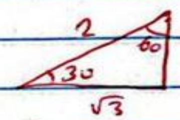
③ Evaluate $\tan^{-1}$ Fun

The Range of $\tan^{-1}$ Fun is similar to $\sin^{-1}$, which is between $-\pi/2$ to $\pi/2$

$\therefore$ Range $\longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

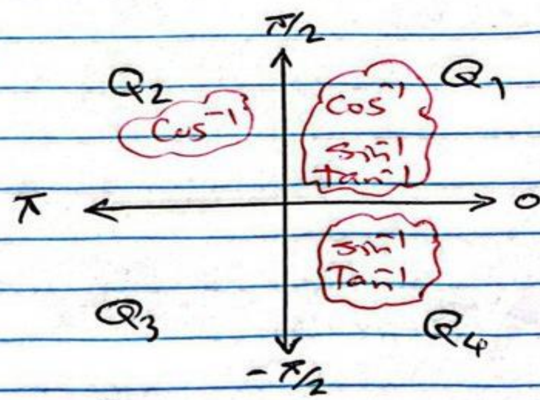
Ex/

- $\tan^{-1}(0) = 0$
- $\tan^{-1}(1) = \pi/4$
- $\tan^{-1}(-1) = -\pi/4$
- $\tan^{-1}(\sqrt{3}) = \pi/3 = 60$
- $\tan^{-1}(-\frac{1}{\sqrt{3}}) = -\pi/6 = -30$



$\tan 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$

Summary



Arc sine	Range
$\cos^{-1}x$ $\longrightarrow$	$[0, \pi]$
$\left.\begin{matrix}\sin^{-1}x\\ \tan^{-1}x\end{matrix}\right\}$ $\longrightarrow$	$[-\pi/2, \pi/2]$

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