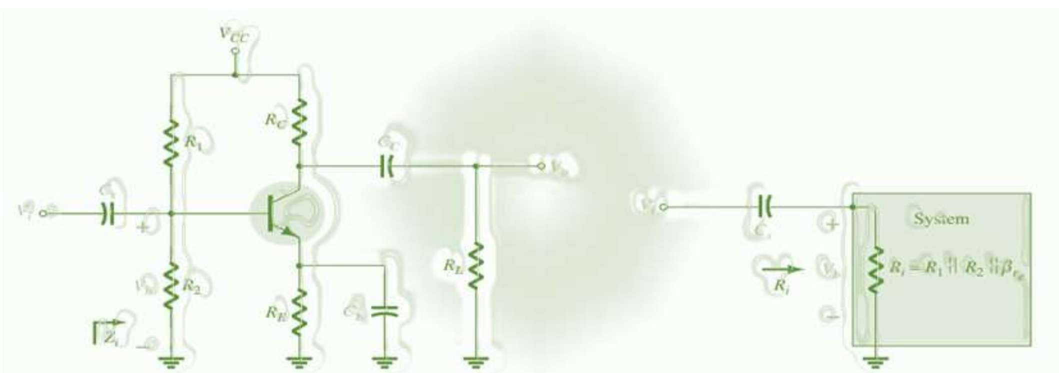
Electronic Circuit

Lecture 3 (4^{th} & 5^{th} Week)

BJT & FET Frequency Response



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1.1. Introduction to BJT Amplifiers

The frequency response of a Bipolar Junction Transistor (BJT) describes how its performance changes with varying signal frequency. The response varies across low, and high frequencies.

- **Low Frequencies:** The performance is affected by coupling and bypass capacitors, leading to a drop in response at very low frequencies.
- **High Frequencies:** The effects of parasitic capacitances (such as base-collector capacitance C_{bc} and base-emitter capacitance C_{be} become significant, reducing gain and altering circuit response.

1.2 Low Frequency Response BJT Amplifier with R_L

The analysis of this section will employ the loaded R_L voltage-divider BJT bias configuration introduced earlier lecture. For the network of Fig. 1, the capacitors C_S, C_C, and C_E will determine the low-frequency response. We will now examine the impact of each independently in the order listed.

C_S Because is normally connected between the applied source and the active device, the general form of the R_C configuration is established by the network of Fig. 2, matching that with $R_i = R_1 \| R_2 \| \beta r_e$.

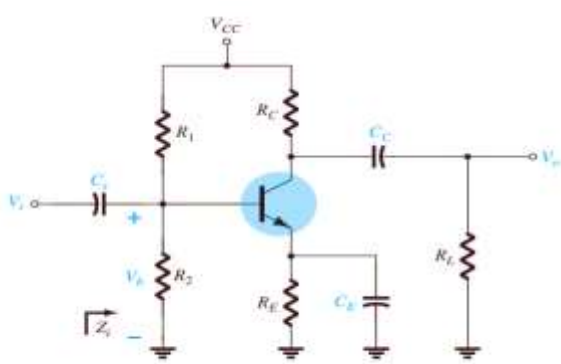

Fig.1. Loaded BJT amplifier with capacitors that affect the low- frequency response.

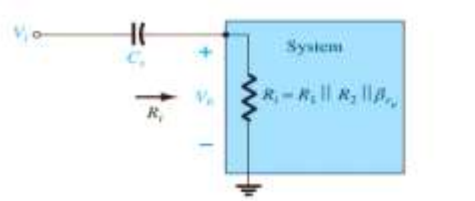

Fig.2. Determining the effect of C s on the low-frequency response.

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Applying the voltage-divider rule:

$$\mathbf{V}_b = \frac{R_i \mathbf{V}_i}{R_i - jX_{C_s}}$$

The cutoff frequency defined by **Cs** can be determined by manipulating the above equation into a standard form or simply. For future RC networks, will simply be applied.

$$f_{L_s} = \frac{1}{2\pi R_i C_s}$$

For the network of Fig.1, when we analyze the effects of Cs we must make the as Sumption that CE and Cc are performing their designed function or the analysis becomes too unwieldy, that is, that the magnitudes of the reactance of CE and Cc permit employing a short-circuit equivalent in comparison to the magnitude of the other series impedances.

Cc Because the coupling capacitor is normally connected between the output of the active device and the applied load, the RC configuration that determines the low-cutoff frequency due to Cc appears in Fig.3. The total series resistance is now $R_o + R_L$, and the cutoff frequency due to Cc is determined by:

$$f_{L_C} = \frac{1}{2\pi (R_o + R_L) C_C}$$

Ignoring the effects of Cs and CE, we find that the output voltage Vo. For the network of Fig.3, the ac equivalent network for the output section with Vi =0 V appears in Fig.3 The resulting value for Ro in Eq. below is then simply:

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$$R_o = R_C \| r_o$$

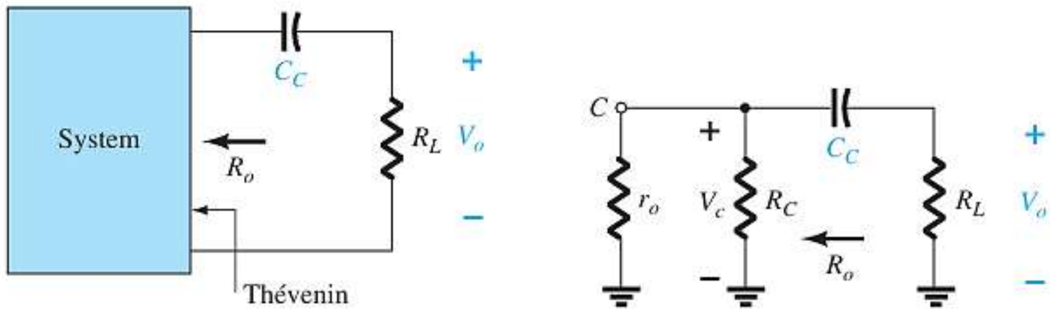

Fig.3. Determining the effect of Cc on the low-frequency response. Localized ac equivalent for Cc with Vi=0 V.

CE To determine f_{LE} , the network "seen" by CE must be determined as shown in Fig.4. Once the level of Re is established, the cutoff frequency due to CE can be determined using the following equation:

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

For the network, the ac equivalent as "seen" by CE appears in Fig.4. The value of Re is therefore determined by:

$$R_e = R_E \| \left(\frac{R_1 \| R_2}{\beta} + r_e \right)$$

The effect of CE on the gain is best described in a quantitative manner by recalling that the gain for the configuration of Fig.4. is given by

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$$A_v = \frac{-R_C}{r_e + R_E}$$

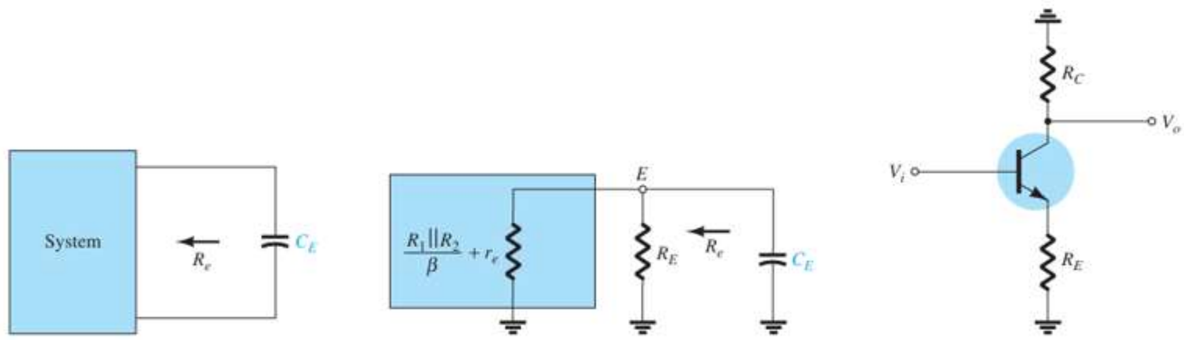

Fig.4. (a)Determining the effect of C_E on the low-frequency response. (b) Localized ac equivalent of C_E. (c) Network employed to describe the effect of C_E on the amplifier gain.

The maximum gain is obviously available where R_E is 0. At low frequencies, with the bypass capacitor C_E in its "open-circuit" equivalent state, all of R_E appears in the gain equation above, resulting in the minimum gain. As the frequency increases, the reactance of the capacitor C_E will decrease, reducing the parallel impedance of R_E and C_E until the resistor R_E is effectively "shorted out" by C_E. The result is a maximum or midband gain determined by $A_v = -R_C/r_e$. At f_{LE} the gain will be 3 dB below the midband value determined with R_E "shorted out."

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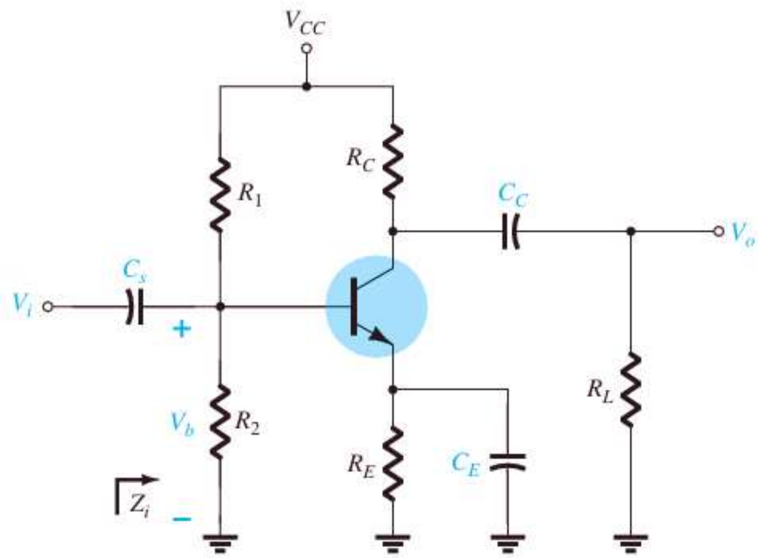
Example1: Determine the cutoff frequencies for the network of Fig.1. using the following parameters:

C_S = 10 mF, C_E = 20 mF, C_C = 1 mF
R1 = 40 kΩ, R2 = 10 k Ω, R_E = 2 kΩ,
R_C = 4 kΩ, R_L = 2.2 kΩ, β = 100 , V_{CC} = 20 V



Sol:

To determine r_e for dc conditions, we first apply the test equation:

$$\beta R_E = (100)(2\,k\Omega) = 200\,k\Omega \gg 10R_2 = 100\,k\Omega$$

Since satisfied the dc base voltage is determined by

$$V_B \cong \frac{R_2 V_{CC}}{R_2 + R_1} = \frac{10\,k\Omega(20\,V)}{10\,k\Omega + 40\,k\Omega} = \frac{200\,V}{50} = 4\,V$$

with $$I_E = \frac{V_E}{R_E} = \frac{4\,V - 0.7\,V}{2\,k\Omega} = \frac{3.3\,V}{2\,k\Omega} = 1.65\,mA$$

so that $$r_e = \frac{26\,mV}{1.65\,mA} \cong \mathbf{15.76\,\Omega}$$

and $$\beta r_e = 100(15.76\,\Omega) = 1576\,\Omega = \mathbf{1.576\,k\Omega}$$

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Midband Gain $$A_v = \frac{V_o}{V_i} = \frac{-R_C \| R_L}{r_e} = -\frac{(4\ \text{k}\Omega)\|(2.2\ \text{k}\Omega)}{15.76\ \Omega} \cong -90$$

C_s
$$R_i = R_1 \| R_2 \| \beta r_e = 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega \cong 1.32\ \text{k}\Omega$$
$$f_{L_S} = \frac{1}{2\pi R_i C_s} = \frac{1}{(6.28)(1.32\ \text{k}\Omega)(10\ \mu\text{F})}$$
$$f_{L_S} \cong \mathbf{12.06\ Hz}$$

C_C
$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C} \quad \text{with} \quad R_o = R_C \| r_o \cong R_C$$
$$= \frac{1}{(6.28)(4\ \text{k}\Omega + 2.2\ \text{k}\Omega)(1\ \mu\text{F})}$$
$$\cong \mathbf{25.68\ Hz}$$

C_E
$$R_e = R_E \| \left(\frac{R_1 \| R_2}{\beta} + r_e\right)$$
$$= 2\ \text{k}\Omega \| \left(\frac{40\ \text{k}\Omega \| 10\ \text{k}\Omega}{100} + 15.76\ \Omega\right)$$
$$= 2\ \text{k}\Omega \| \left(\frac{8\ \text{k}\Omega}{100} + 15.76\ \Omega\right)$$
$$= 2\ \text{k}\Omega \| (80\ \Omega + 15.76\ \Omega)$$
$$= 2\ \text{k}\Omega \| 95.76\ \Omega$$
$$= 91.38\ \Omega$$
$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(91.38\ \Omega)(20\ \mu\text{F})} = \frac{10^6}{11{,}477.73} \cong \mathbf{87.13\ Hz}$$

Since $f_{L_E} \gg f_{L_C}$ or f_{L_S} the bypass capacitor C_E is determining the lower cutoff frequency of the amplifier.

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1.3 Impact of Rs on the BJT Low Frequency response

In this section we will investigate the impact of the source resistance on the various cutoff frequencies. In Fig. 5 a signal source and associated resistance have been added to the configuration of Fig.1. The gain will now be between the output voltage Vo and the signal source Vs.

Cs The equivalent circuit at the input is now as shown in Fig.5, with Ri continuing to be equal to $R_1||R_2||\ \beta r_e$.

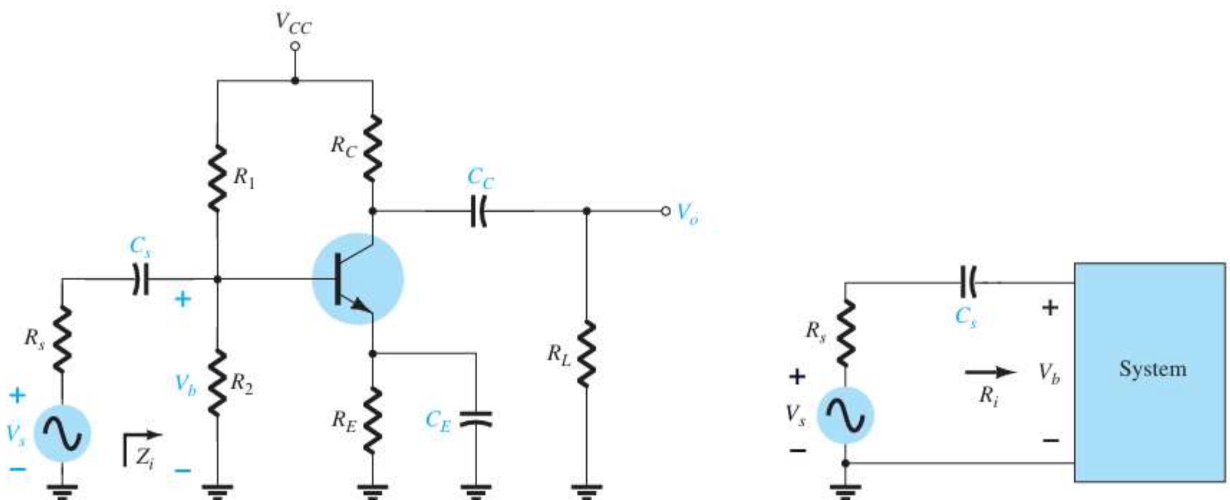

Fig.5. Determining the effect of Rs on the low-frequency response of a BJT amplifier. Determining the effect of Cs on the low frequency response.

Using the results of the last section it would appear we could simply find the total sum of the series resistors. Doing so would result in the following equation for the cutoff frequency:

$$f_{L_s} = \frac{1}{2\pi(R_i + R_s)C_s}$$

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However, it would be best to validate our assumption by first applying the voltage-divider rule in the following manner:

$$\mathbf{V}_b = \frac{R_i \mathbf{V}_s}{R_s + R_i - jX_{C_s}}$$

For the midband frequencies, the input network will appear as shown:

$$A_{v_{\text{mid}}} = \frac{\mathbf{V}_b}{\mathbf{V}_s} = \frac{R_i}{R_i + R_s}$$

Noting the similarities with Eq. above the cutoff frequency is defined by f_{Ls} above and:

$$f_{L_s} = \frac{1}{2\pi(R_s + R_i)C_s}$$

Cc Reviewing the analysis for the coupling capacitor Cc, we find that the derivation of the equation for the cutoff frequency remains the same. That is,

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

C_E Again, following the analysis for the same capacitor, we find that Rs will affect the resistance level substituted into the cutoff equation so that

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

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Example 2: Repeat the analysis of Example 1 but with a source resistance Rs of 1 k**Ω**. The gain of interest will now be Vo /Vs rather than Vo / Vi. Compare results.

Sol/ The dc conditions remain the same:

$$r_e = 15.76\ \Omega \text{ and } \beta r_e = 1.576\ \text{k}\Omega$$

Midband Gain $A_v = \dfrac{V_o}{V_i} = \dfrac{-R_C \| R_L}{r_e} \cong -90$ as before

The input impedance is given by

$$Z_i = R_i = R_1 \| R_2 \| \beta r_e$$
$$= 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega$$
$$\cong 1.32\ \text{k}\Omega$$

and from Fig. 9.35,

$$V_b = \frac{R_i V_s}{R_i + R_s}$$

or
$$\frac{V_b}{V_s} = \frac{R_i}{R_i + R_s} = \frac{1.32\ \text{k}\Omega}{1.32\ \text{k}\Omega + 1\ \text{k}\Omega} = 0.569$$

so that
$$A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_b}{V_s} = (-90)(0.569)$$
$$= \mathbf{-51.21}$$

C_s

$$R_i = R_1 \| R_2 \| \beta r_e = 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega \cong 1.32\ \text{k}\Omega$$
$$f_{L_S} = \frac{1}{2\pi(R_s + R_i)C_s} = \frac{1}{(6.28)(1\ \text{k}\Omega + 1.32\ \text{k}\Omega)(10\ \mu\text{F})}$$
$$f_{L_S} \cong 6.86\ \text{Hz vs. } 12.06\ \text{Hz without } R_s$$

C_C

$$f_{L_C} = \frac{1}{2\pi(R_C + R_L)C_C}$$
$$= \frac{1}{(6.28)(4\ \text{k}\Omega + 2.2\ \text{k}\Omega)(1\ \mu\text{F})}$$
$$\cong \mathbf{25.68\ Hz} \text{ as before}$$

C_E

$$R_s' = R_s \| R_1 \| R_2 = 1\ \text{k}\Omega \| 40\ \text{k}\Omega \| 10\ \text{k}\Omega \cong 0.889\ \text{k}\Omega$$
$$R_e = R_E \left\| \left(\frac{R_s'}{\beta} + r_e\right) = 2\ \text{k}\Omega \right\| \left(\frac{0.889\ \text{k}\Omega}{100} + 15.76\ \Omega\right)$$
$$= 2\ \text{k}\Omega \| (8.89\ \Omega + 15.76\ \Omega) = 2\ \text{k}\Omega \| 24.65\ \Omega \cong 24.35\ \Omega$$
$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{(6.28)(24.35\ \Omega)(20\ \mu\text{F})} = \frac{10^6}{3058.36}$$
$$\cong \mathbf{327\ Hz} \text{ vs. } 87.13\ \text{Hz without } R_s.$$

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1.4. High Frequency response BJT Amplifier

At the high-frequency end, there are two factors that define the -3-dB cutoff point: the network capacitance (parasitic and introduced) and the frequency dependence of h_{fe} (β).

In the high-frequency region, the RC network of concern has the configuration appearing in Fig.6. At increasing frequencies, the reactance XC will decrease in magnitude, resulting in a shorting effect across the output and a decrease in gain. The derivation leading to the corner frequency for this RC configuration follows along similar lines to that encountered for the low-frequency region. The most significant difference is in the following general form of Av:

$$A_v = \frac{1}{1 + j(f/f_H)}$$

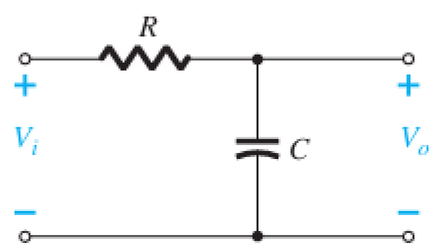

Fig.6. RC combination that will define a high-cutoff frequency.

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In Fig. 8, the various parasitic capacitances (C_{be}, C_{bc}, C_{ce}) of the transistor are included with the wiring capacitances (C_{Wi}, C_{Wo}) introduced during construction.

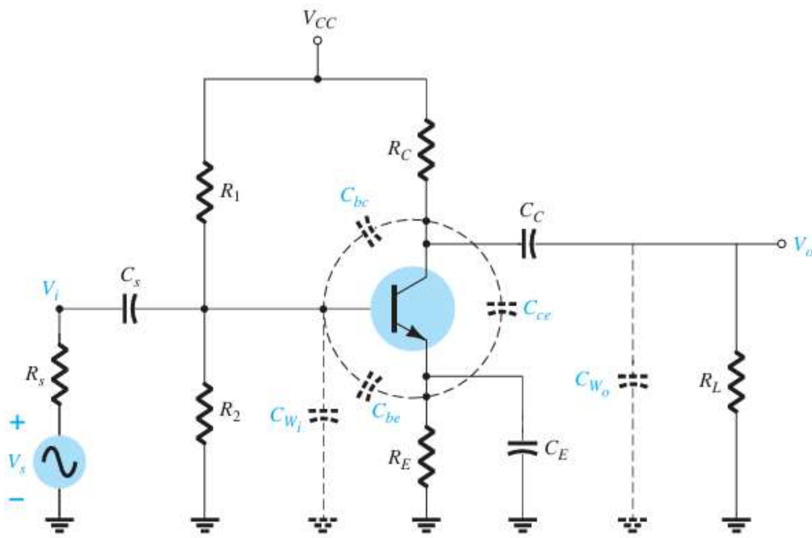


Fig.8. the capacitors that affect the high-frequency response.

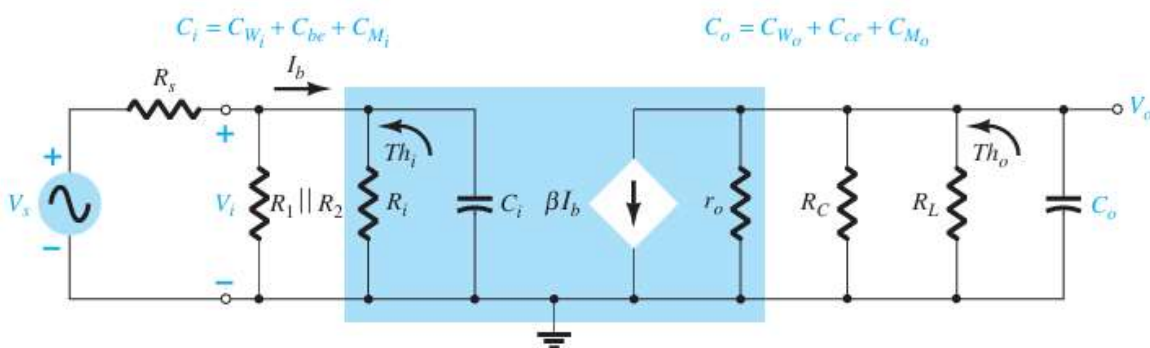


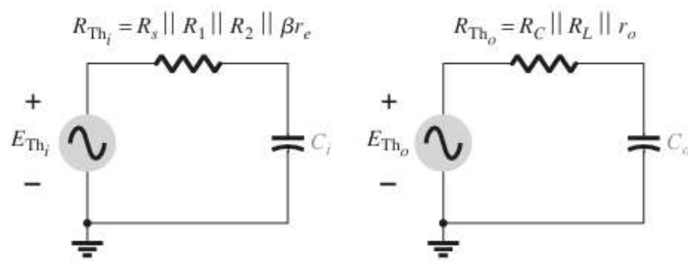
Fig.9. High-frequency ac equivalent model for the network of Fig.8

Determining the Thevenin equivalent circuit for the input and output networks of Fig.8 results in the configurations of Fig.9. For the input network, the 3-dB frequency is defined by:

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

with $R_{Th_i} = R_s \| R_1 \| R_2 \| \beta r_e$

and $C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc}$



At very high frequencies, the effect of Ci is to reduce the total impedance of the parallel combination of R1, R2, βre, and Ci in Fig.9 . The result is a reduced level of voltage across Ci, a reduction in Ib , and a gain for the system. For the output network,

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}$$

with $R_{Th_o} = R_C \| R_L \| r_o$

and $C_o = C_{W_o} + C_{ce} + C_{M_o}$

or $C_o = C_{W_o} + C_{ce} + (1 - 1/A_v)C_{bc}$

For A_v large (typical): $1 \gg 1/A_v$

and $C_o \cong C_{W_o} + C_{ce} + C_{bc}$

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Example 3: Use the network of Fig.1. with the same parameters as in Example.1, that is,

Rs = 1 kΩ, R1 = 40 kΩ , R2 = 10 kΩ ,

RE = 2 kΩ , RC = 4 kΩ , RL = 2.2 kΩ

Cs = 10 mF, Cc = 1 mF, C_E = 20 mF

hfe = 100, ro =∞ , VCC = 20 V

with the addition of $C\pi(C_{be})$ = 36 pF, $Cu(C_{bc})$ = 4 pF, Cce = 1 pF, C_{Wi} = 6 pF, C_{Wo} = 8 pF

a. Determine f_{Hi} and f_{Ho}.

a.

$\beta r_e = 1.576\ \text{k}\Omega,\quad A_{v_{mid}}(\text{amplifier—not including effects of } R_s) = -90$

and $R_{Th_i} = R_s \| R_1 \| R_2 \| \beta r_e = 1\ \text{k}\Omega \| 40\ \text{k}\Omega \| 10\ \text{k}\Omega \| 1.576\ \text{k}\Omega \cong 0.57\ \text{k}\Omega$

with
$$C_i = C_{W_i} + C_{be} + (1 - A_v)C_{bc} = 6\ \text{pF} + 36\ \text{pF} + [1 - (-90)]4\ \text{pF} = 406\ \text{pF}$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi(0.57\ \text{k}\Omega)(406\ \text{pF})} = \mathbf{687.73\ kHz}$$

$$R_{Th_o} = R_C \| R_L = 4\ \text{k}\Omega \| 2.2\ \text{k}\Omega = 1.419\ \text{k}\Omega$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = 8\ \text{pF} + 1\ \text{pF} + \left(1 - \frac{1}{-90}\right)4\ \text{pF} = 13.04\ \text{pF}$$

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o} = \frac{1}{2\pi(1.419\ \text{k}\Omega)(13.04\ \text{pF})} = \mathbf{8.6\ MHz}$$

1.5. Low Frequency Response FET Amplifier

The analysis of the FET amplifier in the low-frequency region will be quite similar to that of the BJT amplifier. There are again three capacitors of primary concern as appearing in the network of Fig.15. C_G , C_C , and C_S .

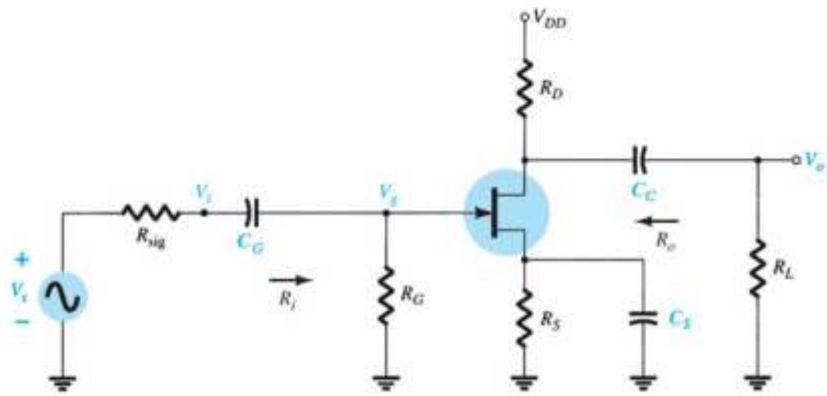

Fig.15. Capacitive elements that affect the low-frequency response of a JFET amplifier.

C_G For the coupling capacitor between the source and the active device, the ac equivalent network is as shown in Fig. 16. The cutoff frequency determined by C_G is:

$$f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G}$$

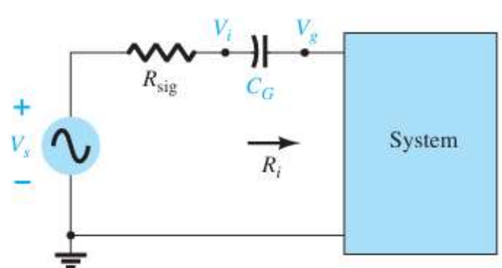

Fig.16. Determining the effect of C_G on the low-frequency response.

$$R_i = R_G$$

Typically, RG»Rsig, and the lower cutoff frequency is determined primarily by RG and CG. The fact that RG is so large permits a relatively low level of CG while maintaining a low cutoff frequency level for f_{LG}.

Cc For the coupling capacitor between the active device and the load the network of Fig.17.results. The resulting cutoff frequency is:

$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

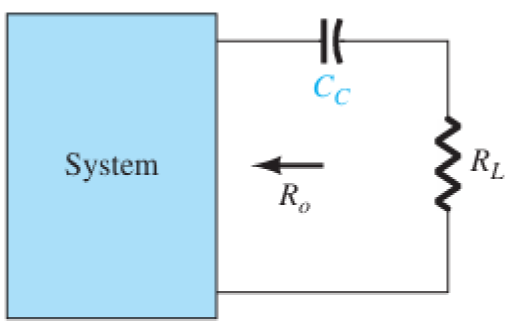

Fig.17. RL Determining the effect of Cc on the low-frequency response.

Cs For the source capacitor Cs, the resistance level of importance. The cutoff frequency is defined by:

$$f_{L_S} = \frac{1}{2\pi R_{eq} C_S}$$

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the resulting value of Req is

$$R_{eq} = \frac{R_S}{1 + R_S(1 + g_m r_d)/(r_d + R_D \| R_L)}$$

which for $r_d \cong \infty\,\Omega$ becomes

$$R_{eq} = R_S \| \frac{1}{g_m} \qquad r_d \cong \infty\,\Omega$$

Example 4:

Determine the lower cutoff frequency for the network of Fig. 15 using the following parameters:

$C_G = 0.01\ \mu F$, $C_C = 0.5\ \mu F$, $C_S = 2\ \mu F$
$R_{sig} = 10\ k\Omega$, $R_G = 1\ M\Omega$, $R_D = 4.7\ k\Omega$, $R_S = 1\ k\Omega$, $R_L = 2.2\ k\Omega$
$I_{DSS} = 8\ mA$, $V_P = -4\ V$, $r_d = \infty\ \Omega$, $V_{DD} = 20\ V$

Sol

DC analysis: Plotting the transfer curve of $I_D = I_{DSS}(1 - V_{GS}/V_P)^2$ and superimposing the curve defined by $V_{GS} = -I_D R_S$ results in an intersection at $V_{GS_Q} = -2\ V$ and $I_{D_Q} = 2\ mA$. In addition,

$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(8\ mA)}{4\ V} = 4\ mS$$

$$g_m = g_{m0}\left(1 - \frac{V_{GS_Q}}{V_P}\right) = 4\ mS\left(1 - \frac{-2\ V}{-4\ V}\right) = 2\ mS$$

C_G $$f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G} = \frac{1}{2\pi(10\ k\Omega + 1\ M\Omega)(0.01\ \mu F)} \cong \mathbf{15.8\ Hz}$$

C_C $$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C} = \frac{1}{2\pi(4.7\ k\Omega + 2.2\ k\Omega)(0.5\ \mu F)} \cong \mathbf{46.13\ Hz}$$

C_S $$R_{eq} = R_S \| \frac{1}{g_m} = 1\ k\Omega \| \frac{1}{2\ mS} = 1\ k\Omega \| 0.5\ k\Omega = 333.33\ \Omega$$

Eq. (9.40): $$f_{L_S} = \frac{1}{2\pi R_{eq} C_S} = \frac{1}{2\pi(333.33\ \Omega)(2\ \mu F)} = \mathbf{238.73\ Hz}$$

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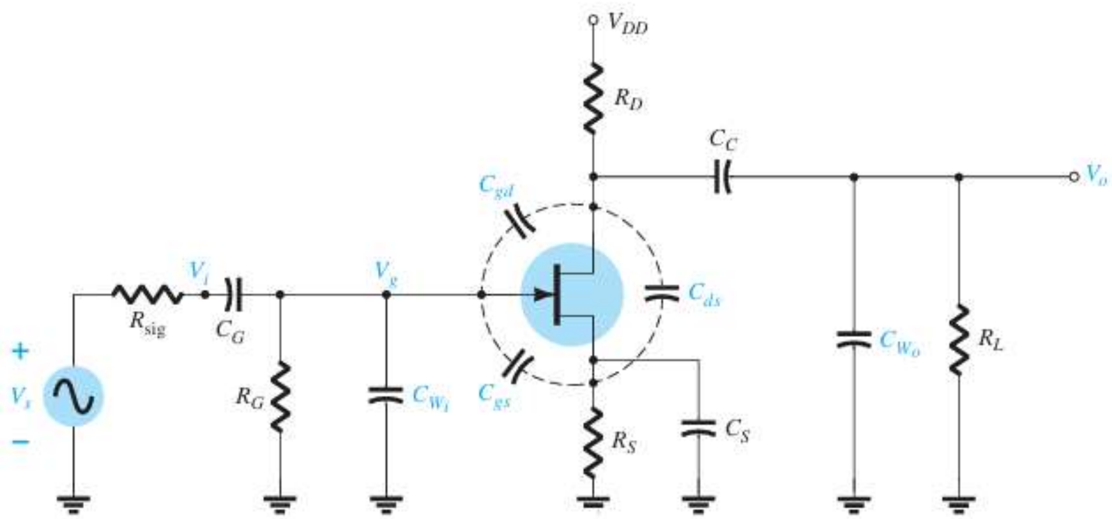
1.5. High Frequency Response FET Amplifier

The analysis of the high-frequency response of the FET amplifier will proceed in a very similar manner to that encountered for the BJT amplifier. As shown in Fig. 18, there are interelectrode and wiring capacitances that will determine the high-frequency characteristics of the amplifier. The capacitors C gs and C gd typically vary from 1 pF to 10 pF, whereas the capacitance C ds is usually quite a bit smaller, ranging from 0.1 pF to 1 pF.



Capacitive elements that affect the high-frequency response of a JFET amplifier.

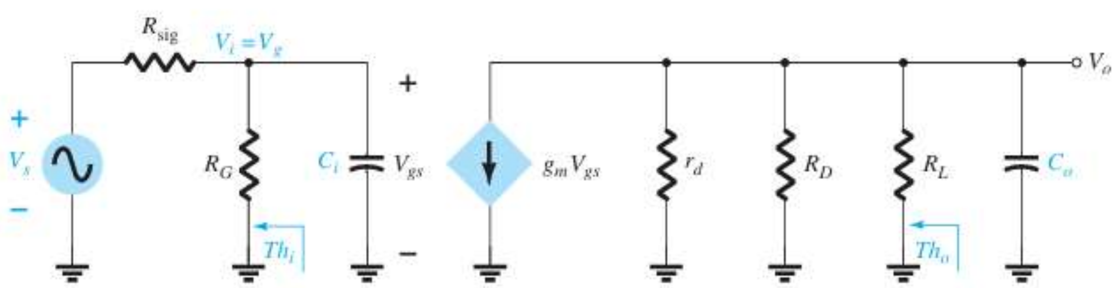


Fig.18. High-frequency ac equivalent circuit

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The cutoff frequencies defined by the input and output circuits can be obtained by first finding the Thevenin equivalent circuits for each section as shown in Fig. 18. For the input circuit,

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

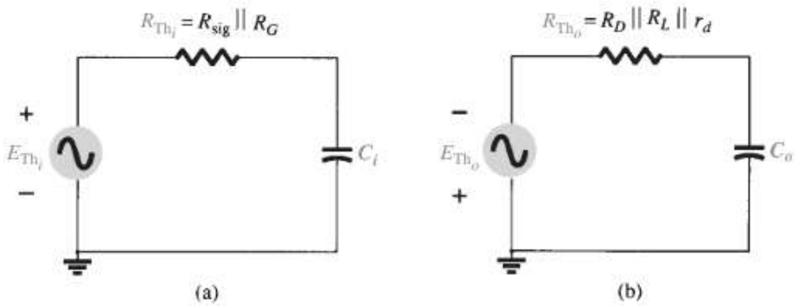

Fig.18. The Thevenin equivalent circuits for: (a) the input circuit and (b) the output circuit.

and
$$R_{Th_i} = R_{sig} \| R_G$$

with
$$C_i = C_{W_i} + C_{gs} + C_{M_i}$$

and
$$C_{M_i} = (1 - A_v)C_{gd}$$

for the output circuit,

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}$$

with
$$R_{Th_o} = R_D \| R_L \| r_d$$

and
$$C_o = C_{W_o} + C_{ds} + C_{M_o}$$

and
$$C_{M_o} = \left(1 - \frac{1}{A_v}\right)C_{gd}$$

Homework

Determine the high-cutoff frequencies for the network of Fig.18. using the same parameters as Example 4:

$$C_G = 0.01\,\mu\text{F}, \quad C_C = 0.5\,\mu\text{F}, \quad C_S = 2\,\mu\text{F}$$

$$R_{sig} = 10\,\text{k}\Omega, \quad R_G = 1\,\text{M}\Omega, \quad R_D = 4.7\,\text{k}\Omega, \quad R_S = 1\,\text{k}\Omega, \quad R_L = 2.2\,\text{k}\Omega$$

$$I_{DSS} = 8\,\text{mA}, \quad V_P = -4\,\text{V}, \quad r_d = \infty\,\Omega, \quad V_{DD} = 20\,\text{V}$$

with the addition of

$$C_{gd} = 2\,\text{pF}, \quad C_{gs} = 4\,\text{pF}, \quad C_{ds} = 0.5\,\text{pF}, \quad C_{W_i} = 5\,\text{pF}, \quad C_{W_o} = 6\,\text{pF}$$