Compressors

8.1.6 Compressors

Compressors are open systems utilized to raise the pressure of gas steams. The exit pressure is higher than the inlet pressure. Work is required for the compressor to operate. The following example illustrates the use of a compressor to pressurize a vapor steam.

Example 8.10 Compressor

Problem

The feed to a compressor is superheated steam at 300°C and 20 bar absolute pressure. It enters the compressor at a velocity of 20 m/s. The pipe inlet inside diameter is 0.10 m. The discharging pipe, after the compressor, has a smaller inside diameter and the discharge velocity is 170 m/s. The exit of the compressor is superheated steam at 350°C and 60 bar absolute. Heat loss from the compressor to the surroundings is 5 kW. Determine the compressor horsepower.

Solution

Known quantities: Inlet and exit steam temperature and pressure, inlet and exit velocities, heat loss from the compressor, inlet pipe diameter.

Find: Compressor horsepower.

Assumption: The system is located on a horizontal plane and no change in elevation between inlet and exit of the compressor; hence change in potential energy is negligible.

Analysis: The compressor is an open system. The steady-state energy balance can be used to describe the compressor system:

$$\Delta\dot{H} + \Delta KE + \Delta PE = \dot{Q} - W_s$$

The general energy balance reduces to

$$\Delta H + \Delta KE = \dot{Q} - W_s$$

Determination of the specific enthalpy and specific volume from the superheated steam table (Appendix A.3):

Inlet stream at $P_1 = 20$ *bar*, $T_1 = 300$ ℃, $h_1 = 3025$ *kJ/kg*, $v_1 = 0.125\ m^3/kg$

Exit stream at $P_2 = 600$ *bar*, $T2 = 350$ ℃, $h_2 = 3046$ *kJ/kg*, $v_2 = 0.0422\ m^3/kg$

Mass flow rate of the inlet steam is equal to density multiplied by volumetric flow rate; the steam density is the inverse of steam specific volume:

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$$\dot{m} = \rho \times \dot{V} = \rho \times (\upsilon \times A) = \rho \times \left(\upsilon \times \frac{\pi D^2}{4}\right) = \frac{1}{v} \times \left(\upsilon \times \frac{\pi D^2}{4}\right)$$

Substitute the values of density, velocity, and diameter:

$$\dot{m} = \frac{1}{0.125\ \text{m}^3/\text{kg}} \times \left(\frac{20\ \text{m}}{\text{s}} \times \frac{\pi (0.1m)^2}{4}\right) = 1.25\ \text{kg/s}$$

The change in enthalpy transport rate $\dot{H}$ is given by

$$\Delta\dot{H} = \dot{m}(h_2 - h_1) = 1.25\ \frac{\text{kg}}{\text{s}}(3046 - 3025)\frac{\text{kJ}}{\text{kg}} = 26.5\ \frac{\text{kJ}}{\text{s}} = 26.25\ \text{kW}$$

The change in kinetic energy is

$$\Delta KE = \frac{1}{2}\dot{m}\left(v_2^2 - v_1^2\right)$$

Substitute the values of mass and inlet and exit velocity:

$$\Delta KE = \frac{1}{2} \times 1.25\ \frac{\text{kg}}{\text{s}}\left\{\left(\frac{170\ \text{m}}{\text{s}}\right)^2 - \left(\frac{20\ \text{m}}{\text{s}}\right)^2\right\}$$

$$\times \frac{\text{N}}{\text{kg m/s}^2}\Bigg|\frac{\text{J}}{\text{N}\cdot\text{m}}\Bigg|\frac{\text{kJ}}{1000\ \text{J}} = 18\ \text{kJ/s}$$

The change in the kinetic energy

$$\Delta\ KE = 18kW$$

The heat loss from the system to the surroundings is 5 kW. Since heat is transferred from the system to the surroundings, Q = −5 kW.

The general energy balance equation reduces to

$$\Delta\dot{H} + \Delta\dot{E}_k = \dot{Q} - \dot{W}_s$$

Substituting the values of change in enthalpy, kinetic energy, and heat loss,

$$26.25\ \text{kW} + 18\ \text{kW} = -5\ \text{kW} - \dot{W}_s$$

Rearranging and solving for the shaft work,

$$\dot{W}_s = -49.25\ \text{kW}$$

$$\text{Power} = 49.25\ \text{kW}\left(\frac{1.341\ \text{hp}}{1\ \text{kW}}\right) = 66.04\ \text{hp}$$

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The sign of the shaft work is negative since work is done on the system by compressor blades. To convert the shaft work to horsepower, use the proper conversion factor.

8.2 Mechanical Energy Balance

The mechanical energy balance is most useful for processes in which changes in the potential and kinetic energies are of primary interest, rather than changes in internal energy or heat associated with the process. Thus, the mechanical energy balance is mainly used for purely mechanical flow problems-that is, problems in which heat transfer, chemical reactions, or phase changes are not present. First, we assume the steady-state condition so that all terms on the left-hand side become zero. Second, we assume that the system has only a single inlet and a single outlet. Moreover, steady state implies that the inlet mass flow rate must equal the outlet mass flow rate, in order to avoid accumulation of material in the system. Let us start with the general energy balance equation:

Energy transferred = Energy out − Energy in

$$\dot{Q}-\dot{W}_s=\left(\dot{U}_{out}+K\dot{E}_{,out}+P\dot{E}_{,out}+P_{out}\dot{V}_{out}\right)-\left(\dot{U}_{in}+K\dot{E}_{,in}+P\dot{E}_{,in}+P_{in}\dot{V}_{in}\right) \quad (8.17)$$

Rearrange the earlier equation by taking the mass flow rate ($\dot{m}$) as a common factor. In this case, the internal energy and volumetric flow rate will become specific internal energy and specific volumetric flow rate, respectively:

$$\dot{Q}-W_s=\dot{m}\left(u_{out}+\frac{v_{out}^2}{2}+gz_{out}+P_{out}v_{out}-u_{in}-\frac{v_{in}^2}{2}-gz_{in}-P_{in}v_{in}\right) \quad (8.18)$$

In this equation, subscript "in" refers to the inlet section, and subscript "out" to the outlet port. Now, we divide the entire equation by $\dot{m}$, and express the specific volume (volume/mass) as $v = 1/\rho$, where ρ is the density (mass/volume) of the flowing material. Assuming incompressible flow rate, so that the density is constant, $v_{in} = v_{out} = 1/\rho$. Also, we define $\Delta u = u_{out} - u_{in}$ and $\Delta P = P_{out} - P_{in}$. With these changes, the general energy balance equation becomes

$$\frac{-W_s}{\dot{m}}=\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2}+g\Delta z+\Delta u-\frac{\dot{Q}}{\dot{m}} \quad (8.19)$$

The term ($\Delta u - \dot{Q}/\dot{m}$) in the absence of chemical reactions, phase changes, or other sources of large amounts of heat transfer will generally represent heat generated due to the viscous friction in the fluid. In such situations, this term is called the friction loss and we will write it as F. With this last change, the general energy balance represents the usual form of the mechanical energy balance:

$$\frac{-\dot{W}_s}{\dot{m}}=\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2}+g\Delta z+F \quad (8.20)$$

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where $\dot{W}/\dot{m}$ is the shaft work performed by the system on the surroundings, per unit mass of material passing through the system. The following example illustrates the use of the mechanical energy balance equation.

Example 8.11 Mechanical energy balance equation

Problem

A water supply tank is capable of delivering 0.3 m^3/s of water for fire-fighting purposes in a chemical plant. The water supply is to come from a lake, the elevation of the surface of the lake is 800 m and the elevation of the factory is 852 m from sea level. The water discharge pipe is located at a depth of 100 m from the surface of the lake. The frictional losses in the water line to the plant are given by the relation (0.01 m/s^2) L, where L is the length of the pipe line. The water line to the supply tank has an inner diameter of 0.15 m and a length of 8000 m. How much energy must a pump deliver to the water?

Solution

Known quantities: Discharge line volumetric flow rate, initial and final elevation, friction losses, length and diameter of the pipe.

Find: Pump horsepower.

Assumption: Pressure drop is neglected because the pressure at both ends of line is atmospheric.

Analysis: Use the mechanical energy balance equation:

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z + F = \frac{-\dot{W}_s}{\dot{m}}$$

The pressure at both ends of the line is atmospheric, so *ΔP = 0.* The velocity at the inlet of the lake is zero but the velocity out of the discharge end of the pipe is

$$v_2 = \dot{V} \times \frac{1}{\frac{\pi D^2}{4}} = \left(0.3\,\frac{m^3}{s}\right) \times \frac{1}{\frac{\pi (0.15 m)^2}{4}} = 17\ m/s$$

The mass flow rate

$$\dot{m} = \dot{V} \times \rho = \frac{0.3\,m^3}{s} \times \frac{1000\,kg}{m^3} = 300\ kg/s$$

The elevation change is from 800 m (800 - 100 to the lake) to 852 to the factory, or the difference is equivalent to 152 m. So, the mechanical energy balance becomes

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$$\frac{0}{\rho}+\frac{\left(17\ \frac{\text{m}}{\text{s}}\right)^2-0}{2}+9.81\ \frac{\text{m}}{\text{s}^2}(152\ \text{m})+0.01\ \frac{\text{m}}{\text{s}^2}(8000\ \text{m})=\frac{-\dot{W}_s}{300\ \text{kg/s}}$$

Solving for shaft work,

$$-\dot{W}_s=514{,}686\ \text{W}\times\frac{\text{hp}}{746}=690\ \text{hp}$$

The minus sign indicates that the energy is going into the system.

Example 8.12 Fire Extinguishment Process

Problem

A large tank filled with water and open to atmosphere is used for fire extinguishment in an ethylene production plant. The water is taken from the tank, passed through a pump, and then delivered to hoses. It is desired to deliver 1890 L of water per minute at a pressure of 15 bar (gauge). If there is a negligible elevation change between the water level in the tank and the discharge of the pump, no changes in the diameter of the pipes and hoses, and if the pump has an efficiency of 70.0%, how much work must be supplied to the pump in order to meet the pressure and discharge rate specifications?

Solution

Known quantities: Discharge line volumetric flow rate, initial and final elevation, friction losses, length and diameter of the pipe.

Find: Pump horsepower.

Assumption: Pressure drop is neglected because the pressure at both ends of the line is atmospheric.

Analysis: Use the mechanical energy balance to solve this problem:

$$\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2}+g\Delta z+F=\frac{-\dot{W}_s}{\dot{m}}$$

Because there is no change in elevation or velocity (no change in pipe/ hose diameter) and no frictional losses are given, the earlier equation reduces to

$$\frac{\Delta P}{\rho}=\frac{-\dot{W}_s}{\dot{m}}$$

The water mass flow rate is

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$$\dot{m} = \dot{V} \times \rho = 1890 \frac{\text{L}}{\text{min}} \frac{1\,\text{min}}{60\,\text{s}} \times \frac{1\,\text{kg}}{\text{L}} = 31.5\,\text{kg/s}$$

The discharge pressure is given as 15 bar (gauge). This means that the absolute pressure is this pressure plus the ambient pressure. Substitute the known values to get

$$\frac{\left[(15\,\text{bar} + P_{\text{ambient}}) - P_{\text{ambient}}\right] \left.\frac{10^5\,\text{Pa}}{\text{bar}}\right| \frac{1\,\text{N/m}^2}{\text{Pa}}}{1000\,\frac{\text{kg}}{\text{m}^3}} = \frac{-\dot{W}_s}{31.5\,\frac{\text{kg}}{\text{s}}}$$

Simplifying

$$-\dot{W}_s = 47{,}250 \left.\frac{\text{N}\cdot\text{m}}{\text{s}}\right| \left.\frac{\text{J}}{\text{N}\cdot\text{m}}\right| \frac{\text{kJ}}{1000\,\text{J}} = 47.25\,\frac{\text{kJ}}{\text{s}} = 47.25\,\text{kW}$$

The theoretical shaft work in horsepower is

$$-\dot{W}_s = 47.25\,\text{kW} \left| \frac{1\,\text{hp}}{0.754\,\text{kW}} \right. = 62.7\,\text{hp}$$

The pump has an efficiency of 70.0%; accordingly, the actual work ($\dot{W}$s. a) that must be supplied to the pump in order to meet the pressure and discharge rate specifications is

$$-\dot{W}_{s,a} = \frac{62.7\,\text{hp}}{0.7} = 89.5\,\text{hp}$$

The sign of the work is negative, which means that the work is done on the system. The actual work that must be supplied to the pump in order to meet the pressure and discharge rate specifications is higher than the theoretical work.

8.3 Bernoulli's Equation

In many instances, the amount of energy lost to viscous dissipation in the fluid is small compared to magnitudes of the other terms in the general energy balance equation. In such a case, $F = 0$. Moreover, many common flows such as fluid flow through a pipe do not have any appreciable shaft work associated with them; accordingly, $\dot{W} = 0$. For such frictionless flows with no shaft work, the mechanical energy balance simplifies to Bernoulli's equation:

$$\frac{\Delta P}{\rho} + \frac{\Delta \upsilon^2}{2} + g\Delta z = 0$$

Bernoulli's equation has a wide range of applications, despite its simplified assumptions. The following example illustrates the use of Bernoulli's equation.

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Example 8.13 Bernoulli's Equation

Problem

The pressure difference between the underside of the wing and the top of the wing that is necessary to lift the weight of an aircraft is 0.08 atm. At an elevation of approximately 10,000 m, the aircraft velocity is 275 m/s and the density of air is 0.45 kg/m^3. Assume that the velocity of the air on the underside of the wing is the plane velocity of 275 m/s. What is the velocity of the air on the topside of the wing, which is necessary to generate the pressure difference needed to lift the plane?

Solution

Known values: Pressure drop around the wing, velocity of air on the underside of the wing.

Find: Velocity of air on the topside of the wing

Analysis: Use Bernoulli's equation around the wing (1: topside of the wing, 2: underside of the wing). Use Bernoulli's Equation to relate the pressure difference to a velocity difference so that

$$\frac{P_2 - P_1}{\rho} + \frac{\upsilon_2^2 - \upsilon_1^2}{2} + g(z_2 - z_1) = 0$$

Neglect the effect of wing thickness on change in potential energy. The equation is reduced to

$$\frac{P_2 - P_1}{\rho} + \frac{\upsilon_2^2 - \upsilon_1^2}{2} + 0 = 0$$

Substituting the values of pressure drop, air density, and velocity under the wing,

$$\frac{0.08\ \text{atm} \left| \dfrac{101,325\ \text{Pa}}{1\ \text{atm}} \right.}{0.45\ \text{kg/m}^3} + \frac{275^2 - \upsilon_1^2}{2} = 0$$

Solving for velocity on the topside of the wing,

$$\upsilon_1 = 334\ \frac{\text{m}}{\text{s}}$$

The velocity on the topside of the wing is higher than that on the under-side of the wing.

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