Heat transfer by flow in tubes

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Laminar flow in tube

- If $Re_d = \frac{\rho d u}{\mu} = \frac{du}{\nu} < 2300$
- The flow is laminar flow
- For constant temperature $Nu = 3.66$
- For constant heat flux Nu=4.36
- For average Nusslet number for in side tube, the following correlation, have been developed
- $Nu=3.66+\frac{0.0668(D/L)Re.Pr}{1+0.04[(D/L)Re.Pr]^{2/3}}$

Laminar flow in tube

- This correlation is used for laminar flow with constant wall temperature, fully developed flow,
- The properties are evaluated at bulk temperature.
- 0.5<Pr< 100 and Re< 2300 and also
- $Nu = 1.86\left[\frac{D}{L}Re.Pr\right]^{1/3}\left[\frac{\mu}{\mu_s}\right]^{0.14}$

Laminar flow in tubes

- This equation is valid for
- For short tube $\frac{L}{D} > 2$, Re< 2100
- 0.48<Pr<16400 , 0.0044<$\left(\frac{\mu}{\mu_s}\right)$<9.75
- All properties are evaluated at bulk temperature except μ_s it is evaluated at surface temperature.

Ex.1 Water at $20^{o}C$ with a flow rate of 0.015kg/s enters a 2.5 cm ID tube which is maintained at uniform temperature of $90^{o}C$. Assuming hydrodynamic and thermally fully developed flow. Determine the heat transfer coefficient and the tube length required to heat the water to $70^{o}C$.

Solution: water at $T_{b1} = 20^{o}C$ and $T_{b2} = 70^{o}C$

Tube is at $T_s = 90^{o}C$ mass flow rate $\dot{m} = 0.015kg/s$, D=2.5cm=0.025m

$$T_b = \frac{T_{b1}+T_{b2}}{2} = \frac{20+70}{2} = 45^{o}C$$

Properties of water at $45^{o}C$ are: ρ=992.3kg/m^3, k=0.638W/m.^{o} C, Cp=4180J/$kg.^{o}$ C, ν=0.613× $10^{-6} m^2/s$

Analysis: at the beginning we will find the type of flow.

$$\dot{m} = \rho A u = \rho \frac{\pi}{4} D^2 u$$

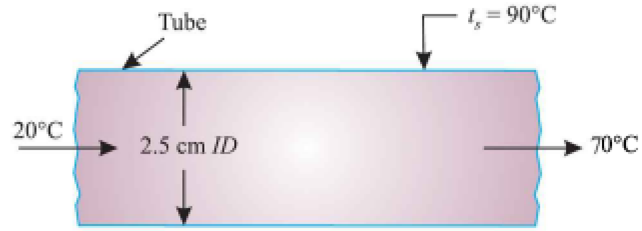
$$0.015=992.3\times \frac{\pi}{4}(0.025)^2 u$$

u=0.0308m/s

$$Re_d = \frac{Du}{\nu} = \frac{0.025\times 0.0308}{0.613\times 10^{-6}} = 1256.12$$

The flow is laminar because Re< 2300

Then Nu=0.366



$$h = \frac{Nu.k}{D} = \frac{3.66\times 0.638}{0.025} = 93.4 W/m^2.^{o}C$$

The heat transfer to water is

$$\dot{Q} = \dot{m} Cp (T_{b2} - T_{b1}) = 0.015 \times 4180(70-20) = 3135W$$

To find the length of the tube

$$\dot{Q} = A_s h \Delta T_m$$

$$\Delta T_m = \frac{(T_s - T_{b1}) - (T_s - T_{b2})}{ln\frac{(T_s - T_{b1})}{(T_s - T_{b2})}} = \frac{(90-20)-(90-70)}{ln\frac{(90-20)}{(90-70)}} = \frac{70-20}{ln\frac{70}{20}} = 39.9^{o}C$$

$$\dot{Q} = \pi D L h \Delta T_m \rightarrow 3135 = \pi \times 0.025 \times L \times 93.4 \times 39.9 \rightarrow L = 10.71m$$

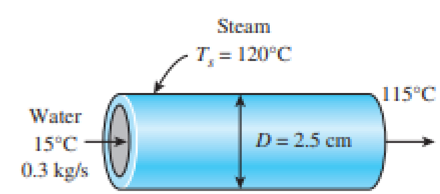
Ex. 2 Water enters a 2.5cminternal-diameter thin copper tube of a heat exchanger at 15^{o}C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120^{o}C. If the average heat transfer coefficient is 800W/ $m^2.^o\,C$, determine the length of the tube required in order to heat the water to 115^{o}C.

Solution: D=0.025m, $\dot{m} = 0.3kg/s$

$T_{b1} = 15^o C, T_{b2} = 115^o C$

$T_s = 120^o C$, h=800W/$m^2.^o\,C$

Cp=4187J/kg.$^o\,C$



To find the length of the tube

$$\dot{Q} = \dot{m}Cp(T_{b1} - T_{b2}) = 0.3 \times 4187(115 - 15)$$

$$\dot{Q} = 125610W$$

$$\Delta T_m = \frac{(T_s - T_{b1}) - (T_s - T_{b2})}{ln\frac{(T_s - T_{b1})}{(T_s - T_{b2})}} = \frac{(120 - 15) - (120 - 15)}{ln\frac{(120 - 15)}{(120 - 115)}} =$$

$$\frac{105 - 5}{ln\frac{105}{5}} = 32.85^{o}\text{C}$$

Heat transfer between water and tube surface is

$$\dot{Q} = A_s h \Delta T_m = \pi D L h \Delta T_m$$

$125610 = \pi \times 0.025 \times L \times 800 \times 32.85 \rightarrow L = 60.86m$

Turbulent flow in tube

- For turbulent flow in tube $Re_D > 2300$

Following correlations are used

1- $\overline{Nu} = 0.023(Re_d)^{0.8}(Pr)^n$

n=0.4 for heating

n=0.3 for cooling

It is valid for $\frac{L}{D} \geq 60$, $10^4 < Re < 12 \times 10^4$

$0.7 \leq Pr \leq 160$, all properties are evaluated at mean bulk temperature

Turbulent Flow

2. $Nu = 0.023(Re)^{0.8}(Pr)^{1/3}$

   $Nu = 0.023\ (Re)^{-0.2}(Pr)^{2/3}$

It is valid for $\frac{L}{D} \geq 60,\ Re \geq 10^4$,

$0.7 < Pr < 160$, The properties are evaluated at bulk temperature.

3. $\overline{Nu} = 0.025(Re)^{0.8}(Pr)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$

It is valid for $\frac{L}{D} > 60$, $Re \geq 10^4$, $0.7 \leq Pr \leq 16700$

Properties are at mean bulk temperature except μ_s is at surface temperature.

Hydraulic Diameter

The hydraulic diameter is $D_H = \frac{4A}{P}$

A= cross-sectional area, P=Peremeter

1. For circular tube $D_H = \frac{4A}{P} = \frac{4\frac{\pi}{4}D^2}{\pi D} = D$
2. For square tube with side length=B

$$D_H = \frac{4A}{P} = \frac{4B^2}{4B} = B$$

3. In the annular for concentric circular tubes

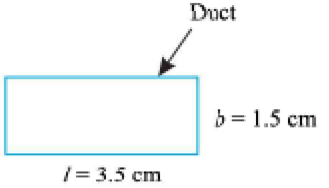
$$D_H = \frac{4A}{P} = \frac{4\left(\frac{\pi}{4}{D_o}^2 - \frac{\pi}{4}{D_i}^2\right)}{\pi D_o + \pi D_i} =$$

$$\frac{{D_o}^2 - {D_i}^2}{D_o + D_i} = \frac{(D_o + D_i)(D_o - D_i)}{D_o + D_i} = D_o - D_i$$

Ex.3 Water is heated while flowing through a 1.5cmx3.5cm rectangular tube at a velocity of 1.2m/s. The entering water temperature is $40^o C$ and tube wall is maintained at $85^o C$. Determine the length of the tube required to the temperature of water to raise by $35^o C$.

Solution: tube l=0.035m, b=0.015m

u=1.2m/s, $T_{b1} = 40^o C, T_S = 85^o$C



$\Delta T_b = 35^oC$, then $T_{b2} = T_{b1} + \Delta T_b = 40 + 35 = 75^oC$

$$T_b = \frac{T_{b1}+T_{b2}}{2} = \frac{40+75}{2} = 57.5^oC$$

Properties of water at 57.5^oC, ρ=985.5kg/m^3, k=0.653W/m.$^o C$, Cp=4190J/kg.$^o C$, ν=0.517x$10^{-6} m^2/s$.

$$\dot{m} = \rho A_c u = \rho(l \times b)u = 985.5 \times (0.015 \times 0.035)1.2 = 0.621 kg/s$$

$$\dot{Q} = \dot{m}Cp(T_{b2} - T_{b1}) = 0.621 \times 4190 \times 35 = 91069.65W$$

$The\ hydraulic\ diameter\ D_H = \frac{4lb}{2(l+b)} = \frac{2(0.035\times0.015)}{0.035+0.015} = 0.021m$

$R_e = \frac{D_H u}{\nu} = \frac{0.021\times1.2}{0.517\times10^{-6}} = 48742.75$

Re> 2300 It is turbulent flow

$Pr = \frac{\mu.Cp}{k} = \frac{\rho\nu Cp}{k} = \frac{985.5\times0.517\times10^{-6}\times4190}{0.653} =$ 3.27

$Nu = 0.023(Re)^{0.8}(Pr)^n$ n=0.4 for heating

$Nu = 0.023(48742.75)^{0.8}(3.27)^{0.4} = 207.89$

$$h = \frac{Nu.k}{D_H} = \frac{207.89 \times 0.653}{0.021} = 6464.4 W/m^2.^o C$$

$$\Delta T_m = \frac{(T_s - T_{b1}) - (T_s - T_{b2})}{ln\frac{(T_s - T_{b1})}{(T_s - T_{b2})}} = \frac{(85-40)-(85-75)}{ln\frac{(85-40)}{(85-75)}} =$$

$$\frac{45-10}{ln\frac{45}{10}} = 23.27^o C.$$

$$\dot{Q} = PLh\Delta T_m$$

$$91069.65 = 2(0.035 + 0.015)L \times 6464.4 \times 23.27$$

L=6.054m