Indefinite integrals

The set of all anti derivatives of a function is called indefinite integral of the function. Assume u and v denote differentiable functions of x, and a, n, and c are constants, then the integration formulas are:-

1) $\int du = u(x) + c$

2) $\int a \cdot u(x)dx = a\int u(x)dx$

3) $\int \left(u(x) \mp v(x)\right)dx = \int u(x)\,dx \mp \int v(x)\,dx$

4) $\int u^n du = \dfrac{u^{n+1}}{n+1} + c \quad when \quad n \neq -1 \quad \& \quad \int u^{-1}du = \int \dfrac{1}{u}du = \ln u + c$

5) $\int a^u du = \dfrac{a^u}{\ln a} + c \quad \Rightarrow \quad \int e^u du = e^u + c$

EXAMPLE 1 Using the Power Rule

$$\int \sqrt{1+y^2}\cdot 2y\,dy = \int \sqrt{u}\cdot\left(\frac{du}{dy}\right)dy \qquad \text{Let } u = 1 + y^2,\ du/dy = 2y$$

$$= \int u^{1/2}\,du$$

$$= \frac{u^{(1/2)+1}}{(1/2)+1} + C \qquad \text{Integrate, using Eq. (1) with } n = 1/2.$$

$$= \frac{2}{3}u^{3/2} + C \qquad \text{Simpler form}$$

$$= \frac{2}{3}(1+y^2)^{3/2} + C \qquad \text{Replace } u \text{ by } 1 + y^2.$$

EXAMPLE 2 Adjusting the Integrand by a Constant

$$\int \sqrt{4t-1}\,dt = \int \frac{1}{4}\cdot\sqrt{4t-1}\cdot 4\,dt$$

$$= \frac{1}{4}\int \sqrt{u}\cdot\left(\frac{du}{dt}\right)dt \qquad \text{Let } u = 4t-1,\ du/dt = 4.$$

$$= \frac{1}{4}\int u^{1/2}\,du \qquad \text{With the } 1/4 \text{ out front, the integral is now in standard form.}$$

$$= \frac{1}{4}\cdot\frac{u^{3/2}}{3/2} + C \qquad \text{Integrate, using Eq. (1) with } n = 1/2.$$

$$= \frac{1}{6}u^{3/2} + C \qquad \text{Simpler form}$$

$$= \frac{1}{6}(4t-1)^{3/2} + C \qquad \text{Replace } u \text{ by } 4t-1.$$

EXAMPLE-3 – Evaluate the following integrals:

1) $\int 3x^2\,dx$

2) $\int\left(\frac{1}{x^2}+x\right)dx$

3) $\int x\sqrt{x^2+1}\,dx$

4) $\int\left(2t+t^{-1}\right)^2\,dt$

5) $\int\sqrt{(z^2-z^{-2})^2+4}\,dz$

6) $\int\frac{x+3}{\sqrt{x^2+6x}}\,dx$

7) $\int\frac{x+2}{x^2}\,dx$

8) $\int\frac{e^x}{1+3e^x}\,dx$

9) $\int 3x^3\cdot e^{-2x^4}\,dx$

10) $\int 2^{-4x}\,dx$

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Sol. –

$$1)\ \int 3x^2\,dx = 3\int x^2 dx = 3\frac{x^3}{3} + c = x^3 + c$$

$$2)\ \left(x^{-2} + x\right)dx = \int x^{-2}\,dx + \int x\,dx = \frac{x^{-1}}{-1} + \frac{x^2}{2} + c = -\frac{1}{x} + \frac{x^2}{2} + c$$

$$3)\ \int x\sqrt{x^2+1}\,dx = \frac{1}{2}\int 2x(x^2+1)^{1/2}dx = \frac{1}{2}\frac{(x^2+1)^{3/2}}{3/2} + c = \frac{1}{3}\sqrt{(x^2+1)3} + c$$

$$4)\ \int \left(2t + t^{-1}\right)^2 dt = \int \left(4t^2 + 4 + t^{-2}\right)dt = 4\frac{t^3}{3} + 4t + \frac{t^{-1}}{-1} + c = \frac{4}{3}t^3 + 4t - \frac{1}{t} + c$$

$$5)\ \int \sqrt{(z^2 - z^{-2})^2 + 4}\,dz = \int \sqrt{z^4 - 2 + z^{-4} + 4}\,dz = \int \sqrt{z^4 + 2 + z^{-4}}\,dz$$

$$= \int \sqrt{(z^2 + z^{-2})^2}\,dz = \int (z^2 + z^{-2})\,dz = \frac{z^3}{3} + \frac{z^{-1}}{-1} + c = \frac{1}{3}z^3 - \frac{1}{z} + c$$

$$6)\ \int \frac{x+3}{\sqrt{x^2+6x}}\,dx = \frac{1}{2}\int (2x+6)\cdot(x^2+6x)^{-1/2}dx$$

$$= \frac{1}{2}\cdot\frac{(x^2+6x)^{1/2}}{1/2} + c = \sqrt{x^2+6x} + c$$

$$7)\ \int \frac{x+2}{x^2}\,dx = \int\left(\frac{x}{x^2} + \frac{2}{x^2}\right)dx = \int\left(x^{-1} + 2x^{-2}\right)dx = \ln x + \frac{2x^{-1}}{-1} + c = \ln x - \frac{2}{x} + c$$

$$8)\ \int \frac{e^x}{1+3e^x}\,dx = \frac{1}{3}\int 3e^x(1+3e^x)^{-1}dx = \frac{1}{3}\ln(1+3e^x) + c$$

$$9)\ \int 3x^3\cdot e^{-2x^4}\,dx = -\frac{3}{8}\int -8x^3\cdot e^{-2x^4}\,dx = -\frac{3}{8}\cdot e^{-2x^4} + c$$

$$10)\ \int 2^{-4x}dx = -\frac{1}{4}\int 2^{-4x}\cdot(-4dx) = -\frac{1}{4}\cdot 2^{-4x}\cdot\frac{1}{\ln 2} + c$$

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