Al-Mustaqbal University / College of Engineering & Technology
Department: Medical Instrumentation Techniques Engineering
Class: Fourth
Subject: Control Systems / Code: MU0244002
Lecturer: Dr. Hasan Hamad Ali
2nd term – Lecture No. 11
Lecture name: Time Response Analysis

Steady-State Error

Type 0: $G(s) = \dfrac{K(T_a s + 1)(T_b s + 1)\cdots}{s^0(T_1 s + 1)(T_2 s + 1)\cdots} = \dfrac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^0(1 + a_1 s + a_2 s^2 + \cdots)}$

$$e_{ss} = e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}$$

- **Unit-Step Input:** $R(s) = 1/s \qquad r(t) = 1$

$$e_{ss} = \lim_{s\to 0} \frac{s(1/s)}{1 + G(s)} = \frac{1}{1 + G(0)}$$

- **Static Position Error Constant, K_p:** $\quad K_p = G(0)$

$$e_{ss} = \frac{1}{1 + K_p} \quad \textit{Constant Value}$$

Type 0: $G(s) = \dfrac{K(T_a s + 1)(T_b s + 1)\cdots}{s^0(T_1 s + 1)(T_2 s + 1)\cdots} = \dfrac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^0(1 + a_1 s + a_2 s^2 + \cdots)}$

$$e_{ss} = e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}$$

- **Unit-Ramp Input:** $R(s) = 1/s^2 \qquad r(t) = t$

$$e_{ss} = \lim_{s\to 0} \frac{s(1/s^2)}{1 + G(s)} = \lim_{s\to 0} \frac{1}{s + sG(s)} = \frac{1}{\lim_{s\to 0} sG(s)}$$

- **Static Velocity Error Constant, K_v:** $\quad K_v = \lim_{s\to 0} sG(s) = 0$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

Type 0: $G(s) = \dfrac{K(T_a s + 1)(T_b s + 1)\cdots}{s^0(T_1 s + 1)(T_2 s + 1)\cdots} = \dfrac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^0(1 + a_1 s + a_2 s^2 + \cdots)}$

$$e_{ss} = e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}$$

- **Unit-Parabolic Input (Acceleration Input):** $R(s) = 1/s^3 \quad r(t) = \frac{1}{2}t^2$

$$e_{ss} = \lim_{s\to 0} \frac{s(1/s^3)}{1 + G(s)} = \lim_{s\to 0} \frac{1}{s^2 + s^2 G(s)} = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

- **Static Acceleration Error Constant, K_a:** $\quad K_a = \lim_{s\to 0} s^2 G(s) = 0$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

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Type 1: $G(s) = \dfrac{K(T_a s+1)(T_b s+1)\cdots}{s^1(T_1 s+1)(T_2 s+1)\cdots} = \dfrac{K(1+b_1 s+b_2 s^2+\cdots)}{s^1(1+a_1 s+a_2 s^2+\cdots)}$

$$e_{ss} = e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

- **Unit-Step Input:** $R(s) = 1/s \qquad r(t) = 1$

$$e_{ss} = \lim_{s\to 0}\frac{s(1/s)}{1+G(s)} = \frac{1}{1+G(0)}$$

- **Static Position Error Constant, K_p:** $\quad K_p = G(0) = \infty$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

Type 1: $G(s) = \dfrac{K(T_a s+1)(T_b s+1)\cdots}{s^1(T_1 s+1)(T_2 s+1)\cdots} = \dfrac{K(1+b_1 s+b_2 s^2+\cdots)}{s^1(1+a_1 s+a_2 s^2+\cdots)}$

$$e_{ss} = e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

- **Unit-Ramp Input:** $R(s) = 1/s^2 \qquad r(t) = t$

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- **Static Velocity Error Constant, K_v:** $\quad K_v = \lim_{s\to 0} sG(s) = K$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K} \quad \textit{Constant Value}$$

Type 1: $G(s) = \dfrac{K(T_a s+1)(T_b s+1)\cdots}{s^1(T_1 s+1)(T_2 s+1)\cdots} = \dfrac{K(1+b_1 s+b_2 s^2+\cdots)}{s^1(1+a_1 s+a_2 s^2+\cdots)}$

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- **Static Acceleration Error Constant, K_a:** $\quad K_a = \lim_{s\to 0} s^2G(s) = 0$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

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Type 2: $G(s) = \dfrac{K(T_a s + 1)(T_b s + 1)\cdots}{s^2(T_1 s + 1)(T_2 s + 1)\cdots} = \dfrac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^2(1 + a_1 s + a_2 s^2 + \cdots)}$

$$e_{ss} = e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}$$

- **Unit-Step Input:** $R(s) = 1/s \qquad r(t) = 1$

$$e_{ss} = \lim_{s\to 0} \frac{s(1/s)}{1 + G(s)} = \frac{1}{1 + G(0)}$$

- **Static Position Error Constant, K_p:** $\qquad K_p = G(0) = \infty$

$$e_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + \infty} = 0$$

Type 2: $G(s) = \dfrac{K(T_a s + 1)(T_b s + 1)\cdots}{s^2(T_1 s + 1)(T_2 s + 1)\cdots} = \dfrac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^2(1 + a_1 s + a_2 s^2 + \cdots)}$

$$e_{ss} = e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}$$

- **Unit-Ramp Input:** $R(s) = 1/s^2 \qquad r(t) = t$

$$e_{ss} = \lim_{s\to 0} \frac{s(1/s^2)}{1 + G(s)} = \lim_{s\to 0} \frac{1}{s + sG(s)} = \frac{1}{\lim_{s\to 0} sG(s)}$$

- **Static Velocity Error Constant, K_v:** $\qquad K_v = \lim_{s\to 0} sG(s) = \infty$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

Type 2: $G(s) = \dfrac{K(T_a s + 1)(T_b s + 1)\cdots}{s^2(T_1 s + 1)(T_2 s + 1)\cdots} = \dfrac{K(1 + b_1 s + b_2 s^2 + \cdots)}{s^2(1 + a_1 s + a_2 s^2 + \cdots)}$

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$$e_{ss} = \lim_{s\to 0} \frac{s(1/s^3)}{1 + G(s)} = \lim_{s\to 0} \frac{1}{s^2 + s^2 G(s)} = \frac{1}{\lim_{s\to 0} s^2 G(s)}$$

- **Static Acceleration Error Constant, K_a:** $\qquad K_a = \lim_{s\to 0} s^2 G(s) = K$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K}$$

Steady–State Errors for Unity Feedback Systems

$K_p = G(0)$ $\quad R(s) = 1/s$

$K_v = \lim_{s\to 0} sG(s)$ $\quad R(s) = 1/s^2$

$K_a = \lim_{s\to 0} s^2G(s)$

$R(s) = 1/s^3$

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots}{s^q(T_1 s + 1)(T_2 s + 1)\cdots}$$

	Step Input $R(s) = \frac{1}{s}$	Ramp Input $R(s) = \frac{1}{s^2}$	Acceleration Input $R(s) = \frac{1}{s^3}$
Type 0 System	$\frac{1}{1+K}$	∞	∞
Type 1 System	0	$\frac{1}{K}$	∞
Type 2 System	0	0	$\frac{1}{K}$

Example: What is the steady state error due to a unit step-input to a type 1 system?

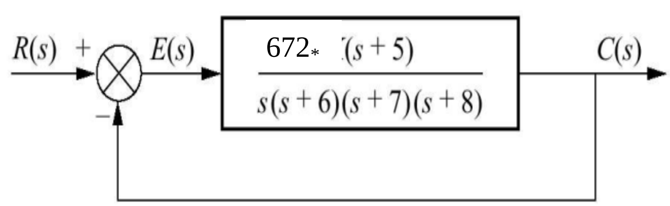
Ans.

$$\frac{1}{1 + K_P}$$

Example: What is the type and order of the system with the open loop transfer function 1/s(1+s)?

Ans. Type 1, second order.

Example: What is the steady state error (ess(∞)) of the following system for a unit ramp input?



Solution:

$$K = \lim_{s\to 0} s.G(s) = \frac{672 * (0+5)}{(0+6)(0+7)(0+8)} = \frac{3360}{336} = 10$$

$$e_{ss} = \frac{1}{K} = \frac{1}{10} = 0.1$$

Example: A unity feedback system with $(s)= \mathbf{48(s+5)/s(s+4)(s+2)(s+3)}$, For a unit ramp input, what is the steady state error?

Solution:

$$K = \lim_{s\to 0} s.G(s) = \frac{48 * (0+5)}{(0+4)(0+2)(0+3)} = \frac{240}{24} = 10$$

$$e_{ss} = \frac{1}{K} = \frac{1}{10} = 0.1$$

Example: What is the step error coefficient of a system $G(s) = \frac{1}{(s+2)(s+3)}$ with unity feedback?

Solution: $K_p = \frac{1}{(0+2)(0+3)} = \frac{1}{6}$