Window Method

In this section, the window method (Fourier transform design with window functions) is developed to remedy the undesirable Gibbs oscillations in the passband and stopband of the designed FIR filter. Recalling that Gibbs oscillations originate from the abrupt truncation of the infinite-length coefficient sequence. Then it is natural to seek a window function, which is symmetrical and can gradually weight the designed FIR coefficients down to zeros at both ends for the range of $-N \le n \le N$. Applying the window sequence to the filter coefficients gives:

$$h_{\mathrm{w}}[n] = h[n].w[n]$$

where $w[n]$ designates the window function. Common window functions used in the FIR filter design are as follows:

Rectangular window	$w_{rec}[n]=1$	$-N \le n \le N$
Triangular (Bartlett) window	$w_{tri}[n] = 1 - \frac{n}{N}$	$-N \le n \le N$
Hanning window	$w_{han}[n] = 0.5 + 0.5\cos(\frac{n\pi}{N})$	$-N \le n \le N$
Hamming window	$w_{ham}[n] = 0.54 + 0.46\cos(\frac{n\pi}{N}$	$-N \le n \le N$
Blackman window	$w_{black}[n] = 0.42 + 0.5cos(\frac{n\pi}{N}) + 0.08c$	$-N \le n \le N$

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The design procedure of the FIR filter via windowing is summarized as follows:

1. Obtain the FIR filter coefficients h(n) via the Fourier transforms method (Table 1.1).
2. Multiply the generated FIR filter coefficients by the selected window sequence $h_W[n] = h[n]w[n], \quad n = -N, \dots, 0, 1, \dots, N$ where w(n) is chosen to be one of the window functions listed in the above equations.
3. Delay the windowed impulse sequence $h_w[n]$ by N samples to get the windowed FIR filter coefficients:

$$b_n = h_w[n - N], \qquad n = 0, 1, \dots, 2N$$

Example: Design a 3-tap FIR lowpass filter with a cutoff frequency of 800 Hz and a sampling rate of 8,000 Hz using the Hamming window function

Solution:

FIR coefficients obtained previously:

$$h[0] = \frac{0.2\pi}{\pi} = 0.2$$

$$h[-1] = h[1] = 0.1871$$

Applying the Hamming window function, we have

$$w_{ham}[0] = 0.54 + 0.46\cos\left(\frac{0\pi}{1}\right) = 1$$

$$w_{ham}[1] = 0.54 + 0.46\cos\left(\frac{1 \times \pi}{1}\right) = 0.08$$

Using the symmetry of the window function gives:

$$w_{ham}[-1] = w_{ham}[1] = 0.08$$

The window impulse response is calculated as

$$h_w[0] = h[0]w_{ham}[0] = 0.2 \times 1 = 0.2$$

$$h_w[1] = h[1]w_{ham}[1] = 0.1871 \times 0.08 = 0.01497$$

$$h_w[-1] = h[-1]w_{ham}[-1] = 0.1871 \times 0.08 = 0.01497$$

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