ORDINARY DIFFERENTIAL EQUATION

1.3. Linear first order differential equations

- An equation of the form $\frac{dy}{dx} + Py = Q$; where "P" and "Q" are functions of "x" only is called a linear differential equation since "y" and its derivatives are of first degree.

- Procedure to solve differential equations of the form

$$\frac{dy}{dx} + Py = Q$$

(i) Rearrange the differential equation in the form of $\frac{dy}{dx} + Py = Q$; Where "P and Q" are functions of "x".

(ii) Determine $\int P\,dx$

(iii) Determine the integrating factor $e^{\int P\,dx}$

(IV) Substitute $e^{\int P\,dx}$ into equation given below

$$y\,e^{\int P\,dx} = \int e^{\int P\,dx}\,Q\,dx$$

(V) Integrate the R.H.S. of the equation above to give the general solution of the differential equation. Given B.Cs., the Particular solution may be determined.

Ex-1. Solve $\frac{1}{x}\frac{dy}{dx} + 4y = 2$; given the boundary Conditions $x = 0$; when $y = 4$

Rearranging gives:- $\frac{dy}{dx} + 4xy = 2x$; which is the form $\frac{dy}{dx} + Py = Q$ where $P = 4x$ and $Q = 2x$.

$$\int P\,dx = \int 4x\,dx = \frac{4x^2}{2} = 2x^2$$

Integrating factor $e^{\int P dx} = e^{2x^2}$

$$y\,e^{\int P dx} = \int e^{\int P dx}\,Q\,dx$$

$$y\,e^{2x^2} = \frac{1}{2}\int e^{2x^2}\,2(2x)\,dx$$

$$y\,e^{2x^2} = \frac{1}{2}e^{2x^2} + c$$ The general Solution.

At $x = 0$; $y = 4 \rightarrow c = \frac{7}{2}$

Hence; The Particular Solution is

$$y\,e^{2x^2} = \frac{1}{2}e^{2x^2} + \frac{7}{2}$$

Ex.2. Solve the differential equation

$\frac{dy}{d\theta} = \sec\theta + y\tan\theta$ given the boundary conditions

$y = 1$ when $\theta = 0$.

Rearranging gives ; $\frac{dy}{d\theta} - (\tan\theta)y = \sec\theta$

Which is of the form: $\frac{dy}{d\theta} + Py = Q$ where

$P = -\tan\theta$ and $Q = \sec\theta$

$\int P\,dx = \int -\tan\theta\,d\theta = -\ln(\sec\theta) = \ln(\sec\theta)^{-1}$

$= \ln(\cos\theta)$

Integrating factor: $e^{\int P\,d\theta} = e^{\ln\cos\theta} = \cos\theta$

$y e^{\int P dx} = \int e^{\int P dx} Q\,dx$

$y\cos\theta = \int \cos\theta \cdot \sec\theta\,d\theta$

$y\cos\theta = \int \cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}}\,d\theta = \int d\theta$

$\therefore y\cos\theta = \theta + c$ which is the general Sol

At $\theta = 0$; $y = 1 \rightarrow c = 1$ $\therefore$ The Particular Sol is :

$y\cos\theta = \theta + 1 \Rightarrow y = (\theta + 1)\sec\theta$.

Ex. 3. Find the general solution of the equation:

$$(x-2)\frac{dy}{dx} + \frac{3(x-1)}{(x+1)} y = 1$$

Given the boundary conditions that $y=5$ when $x=-1$; find the Particular solution of the differential equation.

Rearranging gives;

$$\frac{dy}{dx} + \frac{3(x-1)}{(x+1)(x-2)} y = \frac{1}{x-2}$$

which is the form of

$$\frac{dy}{dx} + P y = Q \quad ; \text{ where } P = \frac{3(x-1)}{(x+1)(x-2)}$$

$$\text{and } Q = \frac{1}{(x-2)}.$$

$$\int P\,dx = \int \frac{3(x-1)}{(x+1)(x-2)}\,dx$$, which is

integrated by Partial Fraction Method

$$\text{let } \frac{3x-3}{(x+1)(x-2)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} = \frac{A(x-2)+B(x+1)}{(x+1)(x-2)}$$

$$3x-3 = A(x-2) + B(x+1)$$

When; $x = -1$

$$3(-1) - 3 = A((-1)-2)$$

$$-3 - 3 = A(-3)$$

$$-6 = A(-3)$$

$$\therefore A = 2$$

When, $x = 2$

$$6 - 3 = B(2+1) \Rightarrow 3 = 3B \Rightarrow B = 1$$

Hence; $$\int \frac{3(x-1)}{(x+1)(x-2)}\,dx = \int \left[\frac{2}{x+1} + \frac{1}{x-2}\right] dx$$

$$= 2\ln(x+1) + \ln(x-2)$$

$$= \ln(x+1)^2 + \ln(x-2)$$

$$= \ln\left[(x+1)^2(x-2)\right]$$

Integrating factor

$$e^{\int P\,dx} = e^{\ln[(x+1)^2(x-2)]} = (x+1)^2(x-2)$$

$$\therefore y e^{\int P\,dx} = \int e^{\int P\,dx} Q\,dx$$

$$y(x+1)^2(x-2) = \int (x+1)^2(x-2)\frac{1}{(x-2)}\,dx$$

$$y(x+1)^2(x-2) = \int (x+1)^2\,dx$$

Hence the general solution;

$$y(x+1)^2(x-2) = \frac{1}{3}(x+1)^3 + c$$

When; $x = -1$; $y = 5 \implies c = 0$

$\therefore$ $y(x+1)^2(x-2) = \frac{1}{3}(x+1)^3$

$$y = \frac{(x+1)^{\cancel{3}}}{3\cancel{(x+1)^2}(x-2)}$$

In this way; the particular solution is

$$y = \frac{(x+1)}{3(x-2)}$$

1.4. Exact First Order differential equation

- A differential expression $M(x,y)\,dx + N(x,y)\,dy$ is an Exact differential in a region R of the xy-plane if it correspounds to the differential of some function $f(x,y)$. A first-order differential equation of the form:

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

- The equation is said to be Exact if:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Ex.1 Solve $\underbrace{(2xy^2-4)}_{M}\,dx + \underbrace{(2x^2y+3)}_{N}\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left[2xy^2-4\right]$$

$$\frac{\partial M}{\partial y} = 4xy$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[2x^2y+3\right]$$

$$\frac{\partial N}{\partial x} = 4xy$$

Since; $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; Then it is Exact Equation

$\frac{\partial f}{\partial x} = M = 2xy^2 - 4$; $\frac{\partial f}{\partial y} = N = 2x^2y + 3$

$\int df = \int (2xy^2 - 4)\, dx$

$f = x^2y^2 - 4x + g(y)$

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left\{ x^2y^2 - 4x + g(y) \right\}$

$\frac{\partial f}{\partial y} = 2x^2y + g'(y)$

But; $\frac{\partial f}{\partial y} = N = 2x^2y + 3$

Then; $\cancel{2x^2y} + 3 = \cancel{2x^2y} + g'(y)$

$\therefore g'(y) = 3$

$g(y) = 3y$

$\therefore f(x,y) = x^2y^2 - 4x + 3y$

$$\boxed{x^2y^2 - 4x + 3y = c}$$

Ex.2. Solve $2xy\,dx + (x^2-1)\,dy = 0$

$M(x,y) = 2xy \quad ; \quad N(x,y) = x^2 - 1$

$M(x,y)\,dx + N(x,y)\,dy = 0$

$$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x}$$

Thus; the equation is Exact.

$$\frac{\partial f}{\partial x} = M(x,y) = 2xy \quad ; \quad \frac{\partial f}{\partial y} = N(x,y) = x^2 - 1$$

$$\int df = \int M\,dx$$

$$f(x,y) = \int 2xy\,dx$$

$$f(x,y) = (x^2 y) + g(y)$$

$$\cancel{\frac{\partial f}{\partial N}} \quad \frac{\partial f}{\partial y} = N = x^2 - 1$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left\{ x^2 y + g(y) \right\}$$

$$\frac{\partial f}{\partial y} = x^2 + g'(y) = x^2 - 1$$

$$\therefore \; g'(y) = -1 \Rightarrow g(y) = -y$$

Hence, $f(x,y) = x^2 y - y$

$$x^2 y - y = c$$

Ex.3 Solve

$$(e^{2y} - y\cos xy)dx + (2xe^{2y} - x\cos xy + 2y)dy = 0$$

The equation is exact because;

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left\{e^{2y} - y\cos xy\right\}$$

$$\frac{\partial M}{\partial y} = 2e^{2y} + xy\sin xy - \cos xy$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left\{2xe^{2y} - x\cos xy + 2y\right\}$$

$$\frac{\partial N}{\partial x} = 2e^{2y} + xy\sin xy - \cos xy$$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; Then the equation is exact

$$M(x,y) = \frac{\partial f}{\partial x} \quad \text{and} \quad N(x,y) = \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial x} = M(x,y) \Rightarrow \int df = \int M(x,y)\,dx$$

$$f(x,y) = \int (e^{2y} - y\cos xy)\,dx$$

$$f(x,y) = xe^{2y} - \sin xy + g(y)$$

$$\frac{\partial f}{\partial y} = 2xe^{2y} - x\cos xy + g'(y)$$

But; $\frac{\partial f}{\partial y} = N(x,y) = -x\cos xy + 2y + 2xe^{2y}$

Then;

$$\cancel{2xe^{2y}} - x\cancel{\cos}\,xy + g'(y) = -x\cancel{\cos}\,xy + 2y + \cancel{2xe^{2y}}$$

$$g'(y) = 2y$$

$$g(y) = y^2$$

Hence; $f(x,y) = xe^{2y} - \sin xy + y^2$

Thus the solution is:

$$xe^{2y} - \sin xy + y^2 = C$$

Special Case: Non-Exact Differential Equations.

- We discuss in the previous section that;

$$M(x,y)\,dx + N(x,y)\,dy = 0$$ will be <u>Exact</u>

If $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

However; If $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$; then we can convert it into Exact D.E by multiplying the giving Non-Exact D.E by Integrating Factor [IF]

- If $\dfrac{M_y - N_x}{N}$ is a function of "x" <u>alone</u>, then the integrating factor is:

$$IF = e^{\int \frac{M_y - N_x}{N}\,dx}$$

If $\dfrac{N_x - M_y}{M}$ is a function of "<u>y</u>" <u>alone</u>, then the integrating factor is:

$$If = e^{\int \frac{N_x - M_y}{M}\,dy}$$

Ex.1 Solve the D.E.:

$$xy\,dx + (2x^2+3y^2-20)\,dy = 0$$

$M = xy \quad ; \quad N = 2x^2+3y^2-20$

$M_y = x \quad ; \quad N_x = 4x$

Since, $M_y \neq N_x$; then the D.E. is not Exact.

$$\frac{M_y - N_x}{N} = \frac{x-4x}{2x^2+3y^2-20}$$

$$\frac{N_x - M_y}{M} = \frac{4x - \cancel{x}}{xy} = \frac{3x}{xy} = \frac{3}{y}$$

Then, the integrating factor:

$$I.F. = e^{\int \frac{3}{y}dy} = e^{3\ln y} = e^{\ln y^3} = y^3$$

Now, by multiplying the given D.E. by I.F.;

$$xy^4\,dx + (2x^2y^3 + 3y^5 - 20y^3)\,dy = 0$$

let us verify if it is Exact ODE:

$M_y = 4xy^3 \quad ; \quad N_x = 4xy^3$ since $M_y = N_x$ then it is now Exact D.E.

Complete the solution

Ex.2. Solve the DE :

$$(3x^2y + 2xy + y^3)\,dx + (x^2+y^2)\,dy = 0$$

$$M_y = \frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left[3x^2y + 2xy + y^3\right]$$

$$M_y = 3x^2 + 2x + 3y^2$$

$$N_x = \frac{\partial}{\partial x}\left[x^2+y^2\right] \rightarrow N_x = 2x$$

Since; $M_y \neq N_x$ → Then the given DE is non exact

$$\frac{M_y - N_x}{N} = \frac{3x^2 + \cancel{2x} + 3y^2 - \cancel{2x}}{x^2+y^2} = \frac{3x^2+3y^2}{x^2+y^2}$$

$$= \frac{3\,\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}} = 3$$

$\therefore$ I.F $= e^{\int 3\,dx} = e^{3x}$

Multiplying D.E. by " e^{3x} "; we get :

$$e^{3x}\left[3x^2y + 2xy + y^3\right]dx + e^{3x}\left[x^2+y^2\right]dy = 0$$

$$M_y = \frac{\partial}{\partial y}\left\{e^{3x}\left[3x^2y + 2xy + y^3\right]\right\}$$

$$M_y = e^{3x}\left[3x^2 + 2x + 3y^2\right]$$

$$N_x = \frac{\partial}{\partial x}\left\{ e^{3x}\left[x^2+y^2\right]\right\}$$

$$= e^{3x}(2x) + (x^2+y^2)(3e^{3x})$$

$$N_x = e^{3x}\left[3x^2 + 2x + 3y^2\right]$$

Since; $M_y = N_x$; Then the D.E. is <u>Exact</u>.

Complete the Solution.

1.5. Bernoulli's Equation

- The differential equation :

$$\frac{dy}{dx} + P(x)\,y = f(x)\,y^{n} \quad ---- (1)$$

where "n" is any real number, is called Bernoulli's Equation and is named after the Swiss Mathematican Jacob Bernoulli [1654–1705]

- Note that for $n=0$ and $n=1$, Eq. (1) is linear. [section 1.3 page 19]
- For $n \neq 0$ and $n \neq 1$, the substitution $u = y^{1-n}$ reduces any equation of form (1) to alinear equation.

<u>Ex.1</u> Solve $x\frac{dy}{dx} + y = x^{2}y^{2}$

We first re-write it as below;

$$\frac{dy}{dx} + \frac{1}{x}\,y = x\,y^{2}$$

$$\frac{dy}{dx} + P(x)\,y = f(x)\,y^{n}$$

Then; $n = 2$; we substitute $\left[u = y^{-1}\right]$ $\therefore\ y = u^{-1}$

$\cancel{\frac{dy}{dx} = y - \frac{1}{x}y + xy^{2}}$

$$\frac{dy}{dx} = \frac{d}{dx}(u^{-1}) = -u^{-2}\frac{du}{dx} \quad \text{[Chain Rule]}$$

$$\therefore \frac{dy}{dx} + \frac{1}{x} y = x y^{2}$$

$$\left[-u^{-2}\frac{du}{dx} + \frac{1}{x}u^{-1} = x u^{-2} \right] * -u^{2}$$

$$\frac{du}{dx} - \frac{1}{x} u = -x$$

which is a linear D.E.

$$\frac{du}{dx} + P u = Q$$

$$\int -\frac{1}{x}\, dx = -\ln x$$

$$e^{[-\ln x]} = e^{\ln x^{-1}} = x^{-1}$$

$$u [x^{-1}] = \int -x^{-1} x \, dx = -\int dx$$

$$\left[u x^{-1} = -x + c \right] * x$$

$$u = -x^{2} + cx \quad \text{but} \quad u = y^{-1} \Rightarrow y = \frac{1}{u}$$

$$\therefore y = \frac{1}{-x^{2} + cx}$$

Tutorial on D.E.

Ex.1 Solve the D.E. given below;

$(2x-1)\,dx + (3y+7)\,dy = 0$

$M(x,y)\,dx + N(x,y)\,dy = 0$

$M = 2x-1 \quad ; \quad N = 3y+7$

$M_y = \frac{\partial M}{\partial y} = 0 \quad ; \quad N_x = \frac{\partial N}{\partial x} = 0$

Since; $M_y = N_x$; Then the given D.E. is Exact.

$\frac{\partial f}{\partial x} = M = 2x-1 \quad ; \quad \frac{\partial f}{\partial y} = N = 3y+7$

$\int df = \int (2x-1)\,dx \Rightarrow f = x^2 - x + g(y)$

$\frac{\partial f}{\partial y} = g'(y) \quad ; \quad$ But $\frac{\partial f}{\partial y} = N = 3y+7$

Then; $3y+7 = g'(y)$

$\therefore \; g(y) = \frac{3}{2}y^2 + 7y$

$\therefore \; f(x,y) = x^2 - x + \frac{3}{2}y^2 + 7y$

$x^2 - x + \frac{3}{2}y^2 + 7y = c$ (The general Solution).

Ex.2 Solve the O.D.E given below:

$$(x - y^3 + y^2 \sin x)\, dx = (3xy^2 + 2y \cos x)\, dy$$

$$M = x - y^3 + y^2 \sin x \quad ; \quad N = -[3xy^2 + 2y \cos x]$$

$$M_y = \frac{\partial}{\partial y}(x - y^3 + y^2 \sin x)$$

$$M_y = -3y^2 + 2y \sin x$$

$$N_x = \frac{-\partial}{\partial x}(3xy^2 + 2y \cos x)$$

$$N_x = -3y^2 + 2y \sin x$$

$\because M_y = N_x \quad \therefore$ The given D.E. is Exact.

$$\frac{\partial f}{\partial x} = M = x - y^3 + y^2 \sin x \quad ; \quad \frac{\partial f}{\partial y} = N = -3xy^2 - 2y \cos x$$

$$\int \partial f = \int (x - y^3 + y^2 \sin x)\, dx$$

$$f = \frac{x^2}{2} - y^3 x - y^2 \cos x + g(y)$$

$$\frac{\partial f}{\partial y} = -3y^2 x - 2y \cos x + g'(y)$$

$$-3x\cancel{y^2} - 2y\cancel{\cos x} = -3\cancel{x}y^2 - 2\cancel{y}\cos x + g'(y)$$

$$\therefore g'(y) = 0 \rightarrow g(y) = c$$

Then; the general solution will be:

$$\frac{x^2}{2} - y^3 x - y^2 \cos x = c$$

Ex. 3 Solve the following D.E:

$$x\frac{dy}{dx} + y = \frac{1}{y^2}$$

$$\frac{dy}{dx} + \frac{1}{x}y = x^{-1}\frac{1}{y^2}$$

$$y' + \frac{1}{x}y = x^{-1}y^{-2}$$ (This is Bernoullii D E).

$$y' + P(x)y = f(x)\,y^n$$

$$u = y^{1-n} \rightarrow u' = (1-n)\,y^{1-n-1}\,y'$$

$$u' = (1-n)\,y^{-n}\,y' \quad ; \quad n = -2$$

$$u' = [1-(-2)]\,y^{+2}\,y'$$

$$u' = 3y^{+2}\,y' \rightarrow y^2 y' = \frac{u'}{3}$$

$$\left\{ y' + \frac{1}{x}y = x^{-1}y^{-2} \right\} y^{+2}$$

$$y^{+2}y' + \frac{1}{x}y\,y^2 = x^{-1}y^{-2}y^2$$

$$\frac{u'}{3} + \frac{1}{x}y^3 = x^{-1}$$ $\therefore$ ~~let~~ $u = y^{1-n} = y^{1-(-2)}$

$\therefore u = y^3$

$$\therefore \frac{u'}{3} + \frac{1}{x}u = x^{-1}$$

$$u' + \frac{3}{x}u = \frac{3}{x}$$

$$u' + P(x)\,u = Q(x)$$ which is a linear D E.

$$P(x) = \frac{3}{x} \Rightarrow \int P\,dx = 3\int \frac{dx}{x} = 3\ln x$$

$$I.F. = e^{\int P dx} = e^{3\ln x} = e^{\cancel{\ln} x^3} = x^3$$

$\therefore$ Integrating factor $= x^3$

$$\therefore \quad u x^3 = \int x^3 \cdot \frac{3}{x}\, dx$$

$$u x^3 = \int 3x^2\, dx$$

$$u x^3 = 3\frac{x^3}{3} + c \Rightarrow u x^3 = x^3 + c$$

or $y^3 = u$ ~~, $y = u^{1/3}$~~

$$\therefore u = 1 + c x^{-3}$$

$$\therefore y^3 = u \Rightarrow y^3 = 1 + c x^{-3}$$

Ex. 4. Solve the following D.E.

$$\frac{dy}{dx} = y(xy^3 - 1)$$

$$\frac{dy}{dx} = xy^4 - y \Rightarrow \frac{dy}{dx} + y = xy^4$$

which is Similar to the: $\frac{dy}{dx} + P(x)y = Q(x)y^n$

$\therefore$ It is Bernoulli's Equation, $n = 4$

$$u = y^{1-n} \rightarrow u' = (1-n)y^{1-n-1}\, y'$$

$$u' = (1-n)y^{-n}y' \Rightarrow u' = (1-4)y^{-4}y'$$

$$\therefore u' = -3y^{-4}y' \Rightarrow y^{-4}y' = -\frac{u'}{3}$$

$$\{y' + y = xy^4\} * y^{-4}$$

$$y^{-4}y' + y\,y^{-4} = x\cancel{y^4}\,\cancel{y^{-4}}$$

$$-\frac{u'}{3} + y^{-3} = x \qquad \therefore u = y^{1-4} \rightarrow u = y^{-3}$$

$$-\frac{u'}{3} + u = x \Rightarrow -\frac{1}{3}\frac{du}{dx} + u = x \quad * -3$$

$$\frac{du}{dx} - 3u = -3x$$

$\frac{du}{dx} + P(x)\,u = Q(x)$ which is linear D.E.

$\int P\,dx = \int -3\,dx = -3x$

Integrating factor $= I.F. = e^{\int P\,dx} = e^{\int -3\,dx}$

$$I.F. = e^{-3x}$$

$$\therefore e^{-3x} u = \int e^{-3x}(-3x)\,dx$$

$$e^{-3x} u = -3\int e^{-3x} \cdot x\,dx$$

D		I
x	$+$	e^{-3x}
1	$-$	$-\frac{1}{3} e^{-3x}$
0		$\frac{1}{9} e^{-3x}$

$$\therefore e^{-3x} u = -3\left\{ -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} \right] + c$$

$$e^{-3x} u = x e^{-3x} + \frac{1}{3} e^{-3x} + c \quad ; \text{Since} ; u = y^{-3}$$

$$y^{-3} = x + \frac{1}{3} + c e^{3x}$$

Homework:

Solve the following differential equations:—

1- $\frac{dy}{dx} = \cos 4x - 2x$

2- $2x \frac{dy}{dx} = 3 - x^3$

3. $\frac{dy}{dx} + x = 3$; given $y = 2$ when $x = 1$.

4. $\frac{dy}{dx} = 2\cos^2 y$

5- $(y^2+2)\frac{dy}{dx} = 5y$; given $y = 1$ when $x = 1$.

6. $\frac{dy}{dx} = 2y \cos x$

7- $(2y-1)\frac{dy}{dx} = (3x^2+1)$; $x = 1$ $y = 2$

8- $\frac{dy}{dx} = e^{2x-y}$; given $x = 0$ when $y = 0$

9- $x^2 = y^2 \frac{dy}{dx}$

10- $x - y + x\frac{dy}{dx} = 0$

11- $(x^2+y^2)\,dy = xy\,dx$; given $x = 1$ when $y = 1$

12. $\frac{x+y}{\cancel{y}-x} = \frac{dy}{dx}$

13. $xy^3\,dy = (x^4+y^4)\,dx$

14. $(9xy-11xy)\dfrac{dy}{dx} = 11y^2-16xy+3x^2$

15- $2xy\dfrac{dy}{dx} = x+3y$; $x=y=1$

16- $\cot x\,\dfrac{dy}{dx} = 1-2y$; $y=1$ when $x=\dfrac{\pi}{4}$

17- $t\dfrac{d\theta}{dt} + \sec t\,[t\sin t + \cos t]\,\theta = \sec t$.

given, $t=\pi$, $\theta=1$

18- $x\dfrac{dy}{dx} = \dfrac{2}{x+2} - y$

19- $\dfrac{dy}{dx} - 2(x+1)^3 = \dfrac{4}{(x+1)}\,y$

20- $\left(1+\ln x+\dfrac{y}{x}\right)dx = (1-\ln x)\,dy$

21- $(x-y^3+y^2\sin x)\,dx = (3xy^2+2y\cos x)\,dx$

22. $(x^3+y^3)\,dx + 3xy^2\,dy = 0$

23- $(x+y)^2\,dx + (2xy+x^2-1)\,dy = 0$; $y(1)=1$

24- $x\dfrac{dy}{dx} - (1+x)y = xy^2$

25- $3(1+t^2)\dfrac{dy}{dt} = 2ty(y^3-1)$.

Assignment

* It is required from each student to solve examples in the subject of the first Ordinary Differential Equations [O.D.E.] considering that the selected examples should cover all of the methods in solving the differential equations (separable, Exact, ...)

* The solved examples should be taken from the textbooks of Mathematics, YouTube videos as well are welcomed.