جامعة المستقبل
AL MUSTAQBAL UNIVERSITY
كلية العلوم

Medical Nuclear Physics

Presented by

Prof.Dr. Fouad A. Majeed

Medical Physics Department,
College of Science,
Al-Mustaqbal University
Babil, Iraq

Email: fouadattia@gmail.com

Lecture 6: Attenuation of Gamma-Rays

Prof.Dr. Fouad A. Majeed

26/2/2025

Introduction

We covered the interaction of gamma-rays with matter from a descriptive viewpoint in the previous lecture and saw that the Compton and Photoelectric Effects were the major mechanisms.

This lecture considers the subject from an analytical perspective to develop a better understanding of the phenomenon.

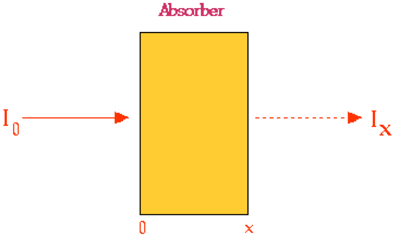
Note: This treatment also applies to X-rays, as gamma-rays and X-rays share similar physical properties.

Our treatment begins with a simple radiation experiment to measure attenuation. The experiment provides insights into developing a general equation for any attenuation scenario.

Attenuation Experiment

Experimental Setup:

- A narrow gamma-ray beam is fired at an absorbing material.
- Measure the intensity of radiation passing through the material.
- Variables: Energy of gamma-rays, absorber type, thickness, and density.



Definitions:

- **Incident Intensity** (I_0): Radiation intensity before absorption.
- **Transmitted Intensity** (I_x): Radiation intensity after

Attenuation Analysis

Relationship Between Intensities:

$$I_x < I_0 \quad (1)$$

Defining the Difference:

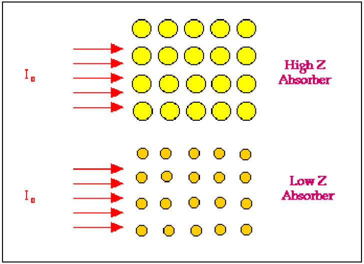
$$\Delta I = I_0 - I_x \quad (2)$$

Effect of Atomic Number

- Attenuation (ΔI) is proportional to Z^3:

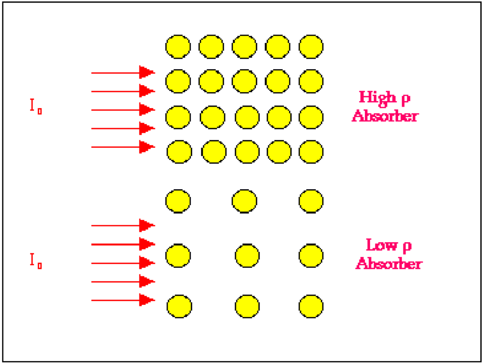
$$\Delta I \propto Z^3 \tag{3}$$

- Example: Doubling Z increases attenuation by a factor of 8.
- High-Z materials like lead (Pb) are used for radiation shielding.



Effect of Density

- Higher density absorbers have more atoms per unit volume.
- Increased probability of radiation interacting with matter.
- Lower density materials allow more radiation to pass through.



Effect of Thickness & Effect of Gamma-Ray Energy

Effect of Thickness

- The thicker the absorber, the greater the attenuation.
- More material means more interactions reducing intensity.

Effect of Gamma-Ray Energy

- Higher-energy gamma rays experience less attenuation.
- Think of it as a spaceship traveling at higher speed through a meteor cloud – less chance of interaction.

Mathematical Model

- Consider a uniform absorber (same material and density).
- The change in intensity depends on the thickness:

$$-dI \propto I \cdot dx \tag{4}$$

- Introducing the **Linear Attenuation Coefficient (μ)**:

$$-dI = \mu I dx \tag{5}$$

- Dividing both sides by I:

$$-\frac{dI}{I} = \mu dx \tag{6}$$

$$-\int_{I_0}^{I_x} \frac{dI}{I} = \mu \int_0^x dx \tag{7}$$

$$\ln\left(\frac{I_x}{I_0}\right) = -\mu x \tag{8}$$

$$\frac{I_x}{I_0} = e^{-\mu x} \tag{9}$$

$$I_x = I_0 e^{-\mu x} \tag{10}$$

$$I = I_0 e^{-\mu x} \tag{11}$$

I_0: Initial intensity of gamma rays.

I: Intensity after passing through thickness x.

μ: Linear Attenuation Coefficient.
This equation describes the exponential decrease in intensity with thickness.

Spaceship Analogy & Applications in Radiation Protection

Spaceship Analogy

- Think of atomic number as the size of meteors in a meteor cloud.
- Density represents how many meteors are in the cloud.
- Thickness represents the depth of the cloud.
- Higher atomic number, density, and thickness increase chances of collision (attenuation).

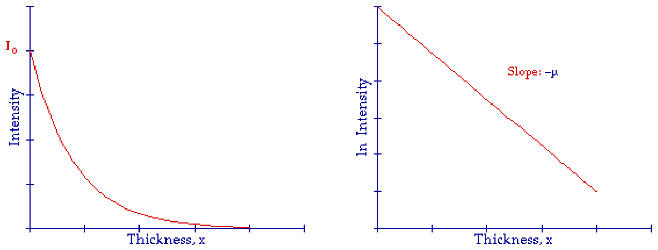
Applications in Radiation Protection

- Lead shielding in medical imaging and nuclear reactors.
- Dense materials used in protective gear.
- Understanding attenuation helps in designing safe environments.

Exponential Attenuation Model

- Radiation intensity decreases exponentially with absorber thickness.
- Governed by the **Linear Attenuation Coefficient (μ)**.

Graphical Representation of Attenuation



- Left: Intensity vs. Thickness (Exponential decay)
- Right: ln(Intensity) vs. Thickness (Linear plot with slope $-\mu$)

Influence of Linear Attenuation Coefficient

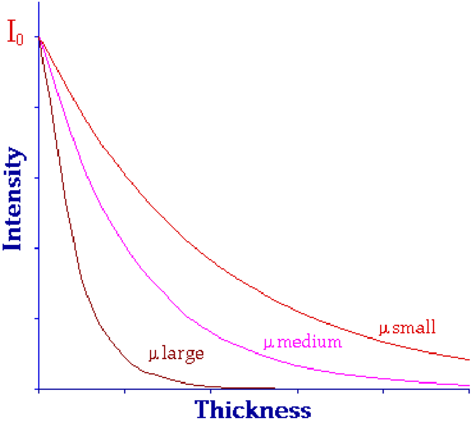
- The rate of decay depends on the value of μ.
- Higher μ leads to **faster attenuation**.
- Lower μ results in **slower attenuation**.

Effect of Linear Attenuation Coefficient

- The Linear Attenuation Coefficient (μ) determines how quickly intensity decreases with thickness.
- Large μ values cause rapid attenuation.
- Small μ values allow gamma rays to penetrate more easily.

Exponential Attenuation with Different μ

- Different values of μ lead to different attenuation behaviors.
- Large μ results in steep decline, small μ results in gradual decline.



Linear Attenuation Coefficients for Various Materials

Absorber	100 keV	200 keV	500 keV
Air	0.000195	0.000159	0.000112
Water	0.167	0.136	0.097
Carbon	0.335	0.274	0.196
Aluminium	0.435	0.324	0.227
Iron	2.72	1.09	0.655
Copper	3.8	1.309	0.73
Lead	59.7	10.15	1.64

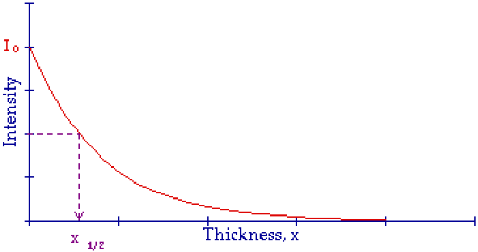
- Materials with higher μ values are more effective at blocking gamma rays.
- Lead has the highest attenuation coefficient, making it ideal for radiation shielding.

Half Value Layer (HVL)

- The Half Value Layer (HVL) is the thickness of an absorber required to reduce radiation intensity by half.
- It provides a useful measure for comparing materials in terms of radiation shielding.
- The HVL can be derived from the exponential attenuation equation:

$$I_x = \frac{I_0}{2} \quad (12)$$

Graphical Representation of HVL



- The intensity drops to half at thickness $x = HVL$.
- The HVL increases with higher gamma-ray energy.

Half Value Layer for Various Absorbers

Absorber	100 keV	200 keV	500 keV
Air	3,555	4,359	6,189
Water	4.15	5.1	7.15
Carbon	2.07	2.53	3.54

- Higher energy gamma rays require thicker absorbers to achieve HVL.
- Air has a much larger HVL compared to dense materials like water and carbon.

Half Value Layers for More Absorbers

Absorber	100 keV	200 keV	500 keV
Aluminium	1.59	2.14	3.05
Iron	0.26	0.64	1.06
Copper	0.18	0.53	0.95
Lead	0.012	0.068	0.42
Aluminium	1.59	2.14	3.05
Iron	0.26	0.64	1.06
Copper	0.18	0.53	0.95
Lead	0.012	0.068	0.42

- Higher atomic number materials have smaller Half Value Layers (HVL).
- HVL increases with increasing gamma-ray energy.

Relationship Between μ and HVL

- The Half Value Layer (HVL) and the Linear Attenuation Coefficient (μ) are reciprocally related.
- Definition of HVL:

$$I_x = \frac{I_0}{2} \tag{13}$$

- When $x = x_{1/2}$, inserting into the exponential attenuation equation:

$$I_x = I_0 \exp(-\mu x) \tag{14}$$

- Solving for HVL:

$$x_{1/2} = \frac{\ln 2}{\mu} \tag{15}$$

- This shows that a larger μ leads to a smaller HVL.

Derivation of μ and HVL Relationship

$$\frac{I}{2} = \exp(-\mu x_{1/2}) \tag{16}$$

$$2^{-1} = \exp(-\mu x_{1/2}) \tag{17}$$

$$\ln 2^{-1} = -\mu x_{1/2} \tag{18}$$

$$\ln 2 = \mu x_{1/2} \tag{19}$$

$$0.693 = \mu x_{1/2} \tag{20}$$

$$\mu = \frac{0.693}{x_{1/2}} \tag{21}$$

$$x_{1/2} = \frac{0.693}{\mu} \tag{22}$$

- These equations express the reciprocal relationship between μ and HVL.
- Useful for solving numerical problems related to attenuation.

Mass Attenuation Coefficient

- The **Mass Attenuation Coefficient** incorporates the density (ρ) of the absorber.
- Defined as:

$$\text{Mass Attenuation Coefficient} = \frac{\mu}{\rho} \tag{23}$$

- Measurement unit: cm^2/g.
- Useful for comparing attenuation across materials with different densities.

Question 1

Problem: How much aluminium is required to reduce the intensity of a 200 keV gamma-ray beam to 10% of its initial intensity? Assume Half Value Layer (HVL) for 200 keV gamma-rays in Al is 2.14 cm.

Step 1: Express Given Data

$$I_x = \frac{I_0}{10}, \quad x = ? \tag{24}$$

Step 2: Compute the Linear Attenuation Coefficient (μ)

$$\mu = \frac{0.693}{x_{1/2}} = \frac{0.693}{2.14} = 0.324 \text{ cm}^{-1} \tag{25}$$

Step 3: Apply Exponential Attenuation Formula

$$I_x = I_0 \exp(-\mu x) \tag{26}$$

$$\frac{I_0}{10} = I_0 \exp(-0.324x) \tag{27}$$

Solution to Question 1

Step 4: Solve for x

$$\frac{1}{10} = \exp(-0.324x) \tag{28}$$

$$x = \frac{\ln 10}{0.324} = \frac{2.3}{0.324} = 7.1 \text{ cm} \tag{29}$$

- Therefore, approximately **7 cm** of aluminium is required.
- Aluminium is not ideal for shielding due to its lower atomic number.
- Compare with lead (Pb), which only requires **2.2 mm** for the same attenuation.

Question 2

Problem: A 10^5 MBq source of ^{137}Cs is to be contained in a Pb box so that the exposure rate at 1 m away is less than 0.5 mR/hour. If the Half Value Layer (HVL) for ^{137}Cs gamma-rays in Pb is 0.6 cm, what thickness of Pb is required?

- The Specific Gamma Ray Constant for ^{137}Cs is:

$$3.3 \text{ R hr}^{-1} \text{ mCi}^{-1} \text{ at 1 cm from the source.} \tag{30}$$

Understanding the Question

- This type of problem is common when dealing with radioactive sources.
- The goal is to ensure radiation exposure is kept within safe limits.
- We use the **Half Value Layer (HVL)** to determine required shielding thickness.

Applying the Inverse Square Law

- The given Specific Gamma Ray Constant:

$$3300 \text{ mR hr}^{-1} \text{ mCi}^{-1} \text{ at 1 cm} \tag{31}$$

- At 1 meter using the Inverse Square Law:

$$\frac{3300}{(100)^2} \text{ mR hr}^{-1} \text{ mCi}^{-1} \tag{32}$$

Conversion to Becquerel

- Expressing per becquerel using:

$$1 \text{ mCi} = 3.7 \times 10^7 \text{ Bq} \tag{33}$$

- Resulting exposure per Bq:

$$\frac{3300}{10^4(3.7 \times 10^7)} \text{ mR hr}^{-1} \text{ Bq}^{-1} \tag{34}$$

Exposure Rate for 10^5 MBq Source

$$\frac{3300 \times 10^5 \times 10^6}{10^4(3.7 \times 10^7)} \text{ mR hr}^{-1} \tag{35}$$

- The exposure rate at 1 meter from the source is **891.9 mR hr^{-1}**.
- We need to reduce it to less than **0.5 mR hr^{-1}** using Pb shielding.

Required Lead Thickness Calculation

- Using the exponential attenuation equation with the given Half Value Layer (HVL) for Pb:
- Required thickness of Pb calculated as approximately **6.5 cm**.

Questions 1-2

Question 1: What is the primary mechanism responsible for the attenuation of gamma rays in matter?

(A) Reflection

(B) Refraction

(C) Absorption and scattering

(D) Diffraction

Question 2: The attenuation of gamma rays in a material is governed by which law?

(A) Ohm's Law

(B) Inverse Square Law

(C) Exponential Attenuation Law

(D) Snell's Law

Questions 3-4

Question 3: What is the term for the thickness of a material required to reduce gamma-ray intensity by half?

(A) Mass Attenuation Coefficient

(B) Half Value Layer (HVL)

(C) Linear Absorption Coefficient

(D) Mean Free Path

Question 4: Which material is most commonly used for gamma-ray shielding?

(A) Wood

(B) Aluminum

(C) Lead

(D) Plastic

Questions 5-6

Question 5: The linear attenuation coefficient (μ) depends on:

(A) Energy of the gamma-ray

(B) Density of the material

(C) Atomic number of the material

(D) All of the above

Question 6: If the Half Value Layer (HVL) of a material is small, it means:

(A) The material has a low attenuation coefficient

(B) The material has a high attenuation coefficient

(C) The material allows more radiation to pass

(D) The material is ineffective at shielding

Questions 7-8

Question 7: The mathematical form of the Exponential Attenuation Law is:

(A) $I = I_0 e^{-\mu x}$

(B) $I = I_0 + \mu x$

(C) $I = I_0/(1 + \mu x)$

(D) $I = I_0 x^{-\mu}$

Question 8: Higher-energy gamma rays experience:

(A) More attenuation

(B) Less attenuation

(C) No attenuation

(D) Instant absorption

Questions 9-10

Question 9: The Half Value Layer (HVL) is related to the linear attenuation coefficient (μ) by:

(A) $HVL = 0.693/\mu$

(B) $HVL = \mu/0.693$

(C) $HVL = \mu \times 0.693$

(D) $HVL = 1/\mu$

Question 10: What happens to the intensity of gamma rays as absorber thickness increases?

(A) Increases exponentially

(B) Remains constant

(C) Decreases exponentially

(D) Increases linearly

Questions 11-12

Question 11: What is the SI unit of the linear attenuation coefficient (μ)?

(A) cm

(B) cm^{-1}

(C) g/cm^2

(D) kg/m^3

Question 12: Which factor increases the probability of gamma-ray interaction with matter?

(A) Lower atomic number

(B) Higher atomic number

(C) Decreasing material thickness

(D) Lower density

Questions 13-14

Question 13: The mass attenuation coefficient is defined as:

(A) μ/ρ

(B) ρ/μ

(C) $\mu \times \rho$

(D) ρ^2/μ

Question 14: What happens to the Half Value Layer (HVL) if the material's density increases?

(A) It increases

(B) It decreases

(C) It remains unchanged

(D) It first increases, then decreases

Questions 15-16

Question 15: The exponential attenuation law is useful for predicting:

(A) Radiation dose levels

(B) The number of photons absorbed

(C) The intensity of transmitted radiation

(D) All of the above

Question 16: Which of the following is NOT a method of gamma-ray interaction with matter?

(A) Compton scattering

(B) Pair production

(C) Photoelectric effect

(D) Refraction

Questions 17-18

Question 17: The Half Value Layer (HVL) can be calculated using:

(A) $HVL = 0.693/\mu$

(B) $HVL = \mu/0.693$

(C) $HVL = \mu \times 0.693$

(D) $HVL = 1/\mu$

Question 18: If a material has a high mass attenuation coefficient, it means:

(A) It is good at shielding radiation

(B) It has a low density

(C) It transmits radiation effectively

(D) It is a poor attenuator

Questions 19-20

Question 19: The effectiveness of a material in attenuating gamma rays is most strongly influenced by:

(A) Its density

(B) Its atomic number

(C) Its thickness

(D) All of the above

Question 20: Which factor does NOT affect the attenuation of gamma rays?

(A) Thickness of the absorber

(B) Energy of the gamma ray

(C) Material's atomic number

(D) Distance from the source