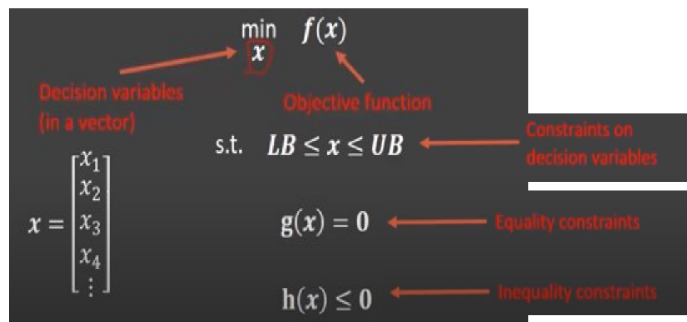




Optimisation Terminologies

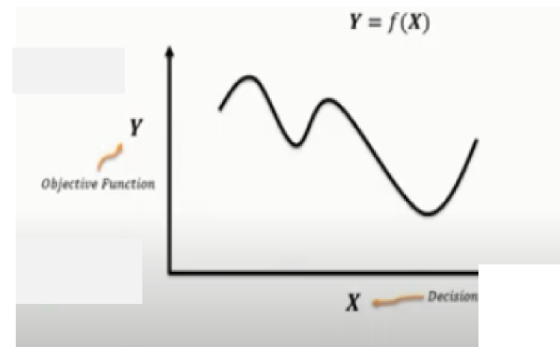
Objective function : the function that it is desired to maximize or minimize



Decision variables :

a- are the inputs to your problem that your optimizer can change to try to improve the objective function

b- all decision variables should be independent of each other.



$$\downarrow$$
$$\underset{x}{\text{minimize}} f(x)$$

$$\text{Objective} = a + b$$

$$\text{Objective} = a \times b$$

$$\text{Objective} = 3a + 4ab$$

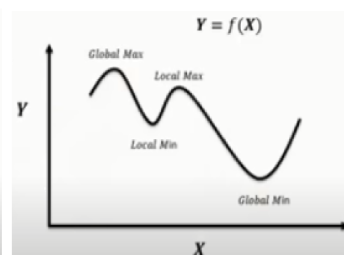
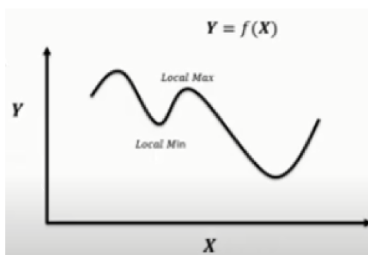
$$\underset{x}{\text{minimize}} f(x_1, x_2, x_3, x_4, x_5 \dots)$$

- Objective Function
 - The value you are trying to optimize
 - Minimized or maximized
- Decision Variables
 - The values the optimizer can change
 - Also called design or manipulated variables

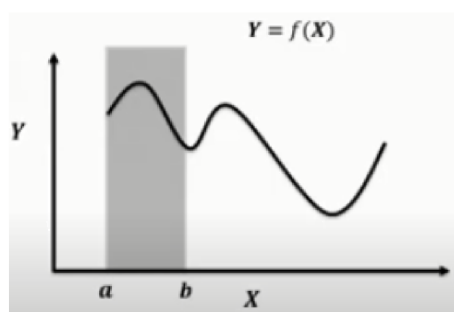
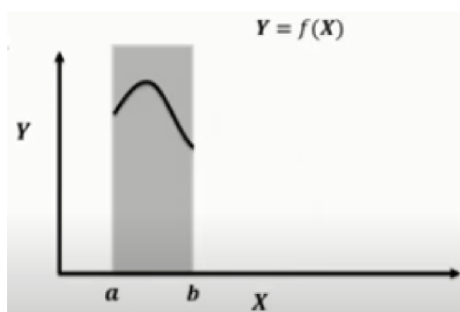
value in the example the decision variables would be the **length of the two sides**
(variable may be called **design variable s** or **manipulated variables**).

Some other important definitions:

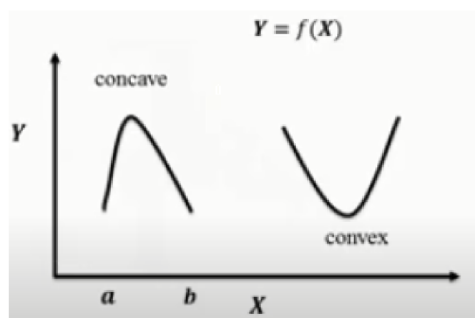
- **Optimum value:** it is a technical term including quantities measurements and mathematical analysis to determine the best setting (maximum or minimum) of a dependent variables.
- **Optimization procedure:** it is the process of determining the optimum value (maximum or minimum) of some criterion function.
- **Optimization problem:** is the specification of the variables that need to be optimized.
- **Local minimum (maximum)** a point where the function value is smaller (greater) than or equal to the value at nearby points.
- **Global minimum (maximum)** a point where the function value is smaller (greater) than or equal to the value at all other feasible points.



❖ **Search space :** a function $f(X)$ defined at a closed interval $[a, b]$.

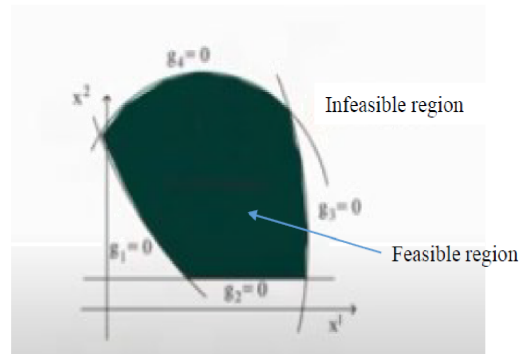


❖ **Unimodal function :** a function has only one peak (maximum, concave) or valley (minimum, concave) in a given interval



Constraint: The constraints represent some functional relationships among the design variables and other design parameters satisfying certain physical phenomenon and certain resource limitations. Figure below shows the feasible region in a two dimensional space.

The set of values of X that satisfy the equation $g_i(x) = 0$ forms a hyper surface in the design space and is called a constraint surface.



Classification of the optimization problems

- In an optimization problem, the types of mathematical relationships between the objective and constraints and decision variables determine how hard it is to solve, the solution methods or algorithms that can be used for optimisation and the confidence you can have that the solution is truly optimal.

Classification based on the number of decision variables

- ❖ single variable
- ❖ Multi variables

• Classification based on the existence of constraints

- ❖ Unconstraint optimisation
- ❖ constraint optimisation

• Classification based on the existence of constraints

- ❖ Linear optimisation
- ❖ Nonlinear optimisation

• Classification based on the number objective function

- ❖ single variable
- ❖ Multi variables

• Classification based on the deterministic nature of the variables

- Static
- ❖ Dynamic



- Classical optimization techniques
 - ❖ single variable optimisation
 - ❖ Multi variables optimisation
 - Multi variables optimisation with **no constrained**
 - Multi variables optimisation with **equality constraints**
 - Multi variables optimisation with **inequality constraints**
- Numerical optimization techniques
 - ❖ elimination method
 - ❖ Golden section method
- Interpolation methods
 - ❖ Newton method
- Advanced optimisation techniques
 - ❖ Heuristic



How to solve an optimisation problem

- Examine the structure of the system and the inter-relationship of the system elements.
- Develop a model of your system
- ❖ consider the model complexity
- ❖ consider the model form (linear, quadratic, nonlinear, mixed-integer,...), these are listed in order of increasing complexity, general speaking.
- Determine your objective function (what are you trying to maximise or minimise)
- ❖ This is usually a combination of additives terms, all put on the same basis (profit= sum of revenue –sum of costs)
- ❖ By conversion, most solvers will require your problem to be a minimisation problem. For example: if you want to
- ❖ maximise profit, you actually need to formulate you problem so that the profit is minimised.
- Examine the restrictions imposed upon the problem.
- Determine your constrains (a flow rate ≥ 0 , temperature \leq some explosive limit, ..)
- Put the math representing your model into the form required by the solver
- ❖ Many solvers require putting the problem into matrix form.
- ❖ Determine the optimum solution and discuss the nature of system conditions.

Setting up the problem in the right format is most of the battle.

Example#4 (Critical insulation thickness)

$$R_{Total} = \frac{\ln\left(\frac{r}{r_i}\right)}{2\pi kL} + \frac{1}{2\pi rL}$$

Conduction
resistance

Convection
resistance

- The convection resistance decreases due to increasing outer surface, but the conduction resistance increases with the addition of insulation (thickness).
- So, to reduce the energy loss we have to increase the insulation.

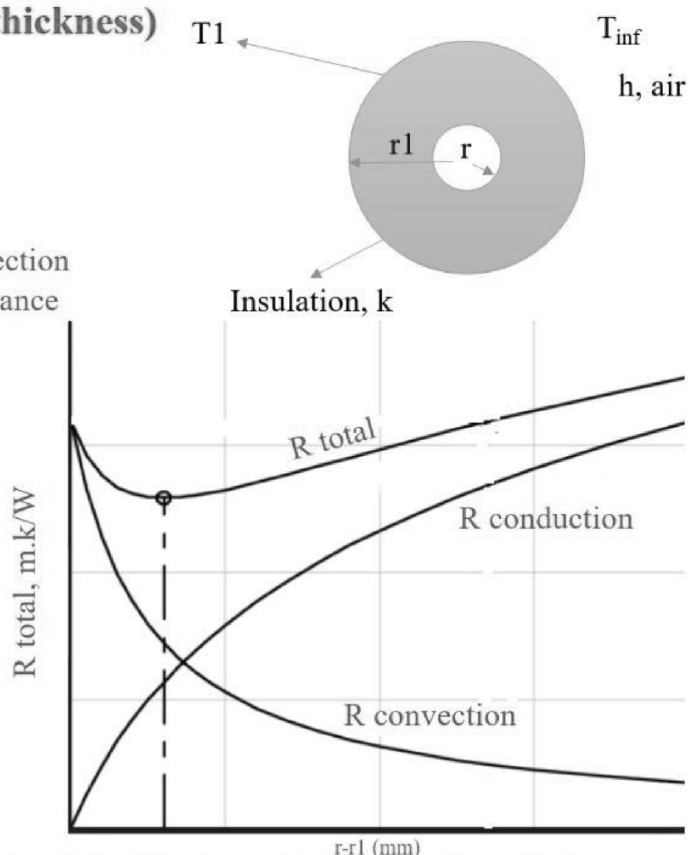


Figure 4. The relation between the insulation thickness on total resistance (x^* = optimum thickness)