# Organization of Optimization Problem

## General procedure of optimization:

1. Define a suitable objective for the problem.
2. Examine the restrictions imposed upon the problem.
3. Choose a system or system to study.
4. Examine the structure of the system and the inter-relationship of the system elements.
5. Construct a mathematical model for the system.
6. Examine and define the internal restrictions planned upon the system variables.
7. Carry-out the simulation by expressing the objective function.
8. Verify the proposed model.
9. Determine the optimum solution and discuss the nature of system conditions.
10. Using the information that obtained, repeat the procedure until a satisfactory results are found.

## Formulation of optimization model:

1. Type of variable (Decision variable): these are defined to capture decisions that need to be made. These variables have different types depending on the values they can take.

The basic variable types are:

A- continuous: this can take any real value such as, how much should I change my current investment in stocks?

B- continuous-non-negative: this can take only non negative values $x >= 0$. Such as, how much milk should I drink every day?

C- Binary : this can take values 0 and 1 only, $x \in \{0,1\}$, such as, should I build my new house in Baghdad?

D- Integer: this can take any integer value, $x \in z$, where z is a set of integer. If it is required to be non-negative then $x \geq 0$. such as, how many workers should be hired to meet lunch time demand in a café?

E- finite sets: this can take a small set of values $x \in s$, where s is the set of values x can take. Such as, which road should I take to go to collage today?

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2- Objective function: can be defined as a function of decision variables whose output is a number. There are uncountable possible functions of this kind, these are classified into two groups:

A- linear functions:

$$f(x_1, x_2, x_3) = x_1 + x_2 + 5x_3$$

B- non-linear functions:

Polynomials , like, $f(x, y, z) = x^2 + y^2 + z^2$

Cross terms, like, $f(x, y, z) = xy$

Eponentionals, like, $f(x) = e^x$

Maximum, *like,* $f(x, y, z) = \max\{x, y, z\}$

Absolute, like, $f(x) = | x |$

3- constraints or restrictions or limitations:

These represent limitation on the choice of decision variables, either internals or external imposed by the designer. Can be classified into two major types:

A- linear constraints, which is three types ($\leq, \geq, =$), such as:

$$\left.\begin{array}{r} x_1 + x_2 \geq 5 \\ 0 \leq x_1 \leq 5 \\ x \geq 0 \end{array}\right\} \text{inequality constraints}$$

$$x_1 + x_2 + x_3 = 12 \; \textit{equality constraints}$$

B- non-linear constrains, such as,

$$x^2 + y^2 \geq 3$$
$$x + e^x \geq 0$$
$$x^2 + y^2 + z^2 = 1$$

4- parameters: represent the given data, or any information necessary in specifying the ranking function and rules.

## Two important notes:

1- The constraints are suited to the independent decision variables ($x_i$).

2- most of the chemical engineering problems are constrained because of the physical and economical considerations. So they can be called constrained optimization problems.

If the problem has no limitations or restrictions, it is called unconstrained optimization problems.

## Example#1:

AB steel incorporation produces two kinds of iron $I_1$ and $I_2$, by using three types of raw materials $R_1$, $R_2$, and $R_3$ (scrap iron and two types of ore), as shown below. Formulate this optimization problem to maximize the company profit.

| Raw material | Raw material needed per ton | | Raw material available ton/day |
|---|---|---|---|
| | Iron I1 | Iron I2 | |
| R1 | 2 | 1 | 16 |
| R2 | 1 | 1 | 8 |
| R3 | 0 | 1 | 3.5 |
| Net profit $/ton | 150 | 300 | |

## Solution:

A- Decision variables:

$X_1$= ton of iron $I_1$ produced per day, ton/day.

$X_2$= ton of iron $I_2$ produced per day, ton/day.

B- Objective function:

To maximize the company profit, hence

$$\text{Max} \quad f(x)= 150x1 + 300x2$$

C- constraints:

$2x1+x2 \leq 16$ (raw material R1)

$x1+x2 \leq 8$ (raw material R2)

$x2 \leq 3.5$ (raw material R3)

D- parameters:

All the data available

## Example#2:

Sara wonders how much money she must spend on her food in order to get all the 2000 Kcal (at least energy), protein 55 gm, and calcium 800 mg that she needs every day. She chooses six foods that seem to be cheap sources of the nutrients; her data are collected in table below:

| Food item | Serving size | Energy Kcal | Protein gm | Calcium mg | Price per serving cents | Max. serving per day |
|---|---|---|---|---|---|---|
| Rice | 28 gm | 110 | 4 | 2 | 3 | 4 |
| Chicken | 110 gm | 205 | 32 | 12 | 24 | 3 |
| Eggs | 2 large | 160 | 13 | 54 | 13 | 2 |
| Milk | 237 ml | 160 | 8 | 285 | 9 | 8 |
| Fruits | 170 gm | 420 | 4 | 22 | 20 | 2 |
| Meat | 260 gm | 260 | 14 | 80 | 19 | 2 |

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Formulate the objective function for minimum and most economical cost with important constraints that satisfy the problem.

**Solution:**

**1- The decision variables,** Let:

$X_1$= number of serving of rice.

$X_2$= number of serving of chicken.

$X_3$= number of serving of eggs.

$X_4$= number of serving of milk.

$X_5$= number of serving of fruits.

$X_6$= number of serving of meat.

**2- Objective function:**

In designing the most economical menu, Sara wants to find numbers $x_1 \rightarrow x_6$ that satisfy all the constraints and makes the cost as small as possible. As a mathematician would put it in:

**3- Constraints or limitations:**

Energy $110x_1 + 205x_2 + 160x_3 + 160x_4 + 420x_5 + 260x_6 \geq 2000$

Protien $4x_1 + 32x_2 + 13x_3 + 8x_4 + 4x_5 + 14x_6 \geq 55$

Calcium $2x_1 + 12x_2 + 54x_3 + 285x_4 + 22x_5 + 80x_6 \geq 800$

Also,

$$\begin{array}{ll} \mathbf{0 \leq x_1 \leq 4} & \textit{rice} \\ \mathbf{0 \leq x_2 \leq 3} & \textit{chicken} \\ \mathbf{0 \leq x_3 \leq 2} & \textit{eggs} \\ \mathbf{0 \leq x_4 \leq 8} & \textit{milk} \\ \mathbf{0 \leq x_5 \leq 2} & \textit{fruits} \\ \mathbf{0 \leq x_6 \leq 2} & \textit{meat} \end{array}$$

To solve the problem we have to minimize the cost C.
this kind of problems are called Linear Programming Problems.

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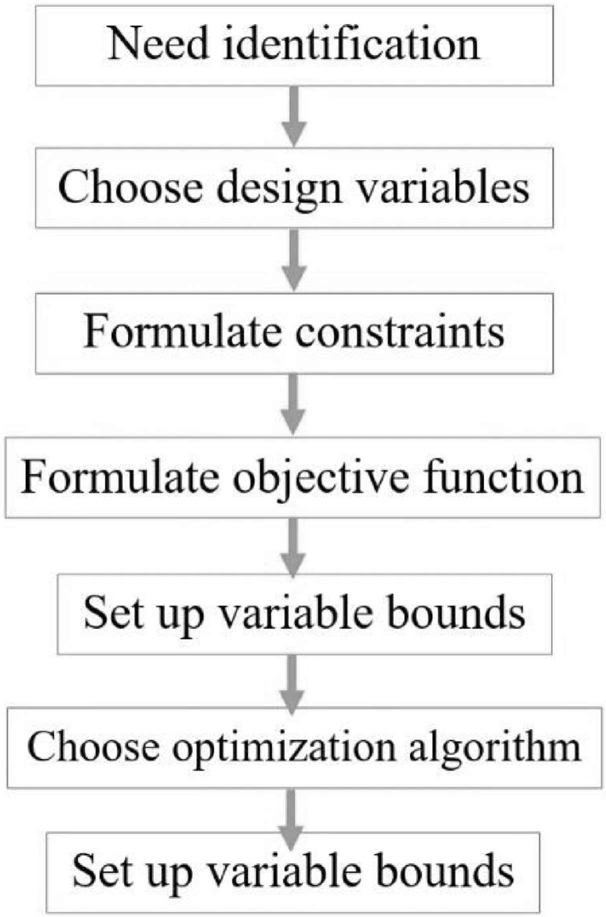
In general, the steps in formulation of an optimization problems are:

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# Assignments

A chemical plant makes three products E, F, and G, and utilize three raw materials A, B and C in limited supply. Each of the three products is produced in a separate process. The available material of A, B and C do not have to be consumed totally. The reactions are:

Process I $A+B \rightarrow E$

Process II $A+B \rightarrow F$

Process III $3A+2B+C \rightarrow G$

Formulate the objective function to maximize the total profit in $/day and satisfy all the constraints for this problem?

The available data are tabulated below:

Table#1

| Raw material | Max. available Ib/day | Cost $/Ib |
|---|---|---|
| A | 40000 | 1.5 |
| B | 30000 | 2.0 |
| C | 25000 | 2.5 |

Table#2

| Products | Ib react/Ib produced | Process Cost $/Ib | Selling price of product $/Ib |
|---|---|---|---|
| E | (2/3)A, (1/3)B | 1.5 | 4.0 |
| F | (2/3)A, (1/3)B | 0.5 | 3.3 |
| G | (1/2)A, (1/6)B, (1/3)C | 1.0 | 3.8 |

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