Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture:4

Lecture 4

Fourier Transform

Lecturer: Prof. Dr. Bayan Mahdi Sabbar

M.Sc. Huda Wasfi Hassoon

Email: huda.wasfi@uomus.edu.iq

1.1 Fourier Transform

The Fourier transform is used to convert a continuous and non-periodic time-domain signal into the frequency domain. The resulting frequency domain representation from performing the Fourier transform is continuous and non-periodic. The Fourier transform of $x(t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

The inverse Fourier transform of $X(w)$:

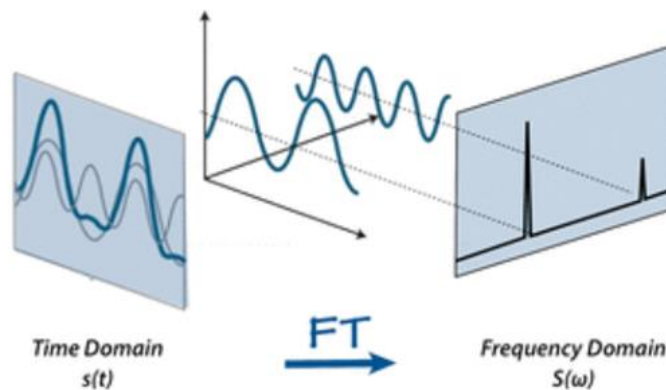$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Fig. 1: Fourier Transform.

Example 1: Suppose that a signal gets turned on at t=0 and then decays exponentially, So that

$$f(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

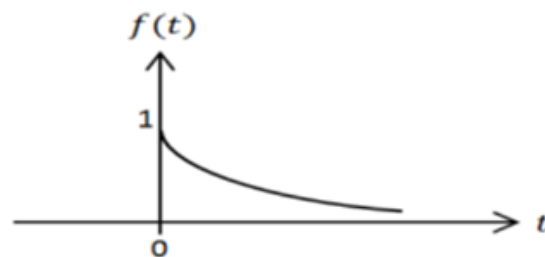For $a > 0$. Find the Fourier transform of this signal.

Solution:

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-jwt} \, dt$$

$$F(w) = \int_{0}^{\infty} e^{-at} e^{-jwt} \, dt$$

$$F(w) = \int_{0}^{\infty} e^{-t(a+jw)} \, dt$$

$$F(w) = \frac{1}{-(a+jw)} [\, e^{-\infty} - e^{0} \,]$$

$$F(w) = \frac{1}{(a+jw)}$$

$f(t)$

1.2 Properties of the Continuous-Time Fourier Transform

There are some properties of the Fourier transform that may be used to simplify the evaluation of the Fourier transform and its inverse. Some of these properties are described below:

Property	Signal	Fourier Transform
Linearity	$ax(t) + by(t)$	$aX(w) + bY(w)$
Time-Shift	$x(t - t_o)$	$e^{-jwt_o} X(w)$
Time-Reversal	$x(-t)$	$X(-w)$
Frequency Shifting	$e^{jw_o t} x(t)$	$X(w - w_o)$
Time Scaling	$x(at)$	$\dfrac{1}{\|a\|} X\left(\dfrac{w}{a}\right)$
Convolution	$x(t) * y(t)$	$X(w)\, Y(w)$
Time differentiation	$\dfrac{dx(t)}{dt}$	$jwX(w)$
Parseval's relation	$\displaystyle\int_{-\infty}^{\infty} \|x(t)\|^2 \, dt$	$\dfrac{1}{2\pi} \displaystyle\int_{-\infty}^{\infty} \|X(w)\|^2 \, dw$

1.3 Frequency response

The output $y(t)$ of a continuous-time system equals the convolution of the input $x(t)$ with the impulse response $h(t)$; that is

$$y(t) = x(t) \otimes h(t)$$

Applying the convolution property, we obtain

$$Y(\omega) = X(\omega).H(\omega)$$

Where $Y(w), X(w)$ and $H(\omega)$ are the Fourier transforms of $y(t), x(t)$ and $h(t)$, respectively. we have

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

There are some relationships are widely used to transform between the time domain and the frequency domain.

- $\delta(t) \leftrightarrow 1$

- $\delta(t - t_o) \leftrightarrow e^{-jwt_o}$

- $1 \leftrightarrow 2\pi\delta(w)$

- $e^{jw_o t} \leftrightarrow 2\pi\delta(w - w_o)$

- $\cos w_o t \leftrightarrow \pi\delta(w - w_o) + \pi\delta(w + w_o)$

- $\sin w_o t \leftrightarrow -j\pi\delta(w - w_o) + j\pi\delta(w + w_o)$

- $u(t) \leftrightarrow \pi\delta(w) + \frac{1}{jw}$

- $e^{-at}u(t) \leftrightarrow \frac{1}{jw+a}, \quad a > 0$

- $e^{-a|t|} \leftrightarrow \frac{2a}{w^2+a^2}, \quad a > 0$

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Example 2: Consider the linear time-invariant system characterized by the first-order linear constant coefficient difference equation

$$y'(t) + 2y(t) = x(t) + x'(t)$$

By using Fourier Transform, find the Impulse response, $h(t)$.

Solution:

$$jwY(w) + 2Y(w) = X(w) + jwX(w)$$

$$Y(w)[jw + 2] = X(w)[1 + jw]$$

$$\frac{Y(w)}{X(w)} = \frac{jw + 1}{jw + 2}$$

$$H(w) = \frac{jw + 2 - 1}{jw + 2}$$

$$H(w) = 1 - \frac{1}{jw + 2}$$

$$h(t) = \delta(t) - e^{-2t}u(t)$$

Assignment 1:

Consider a continuous-time system described by:

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Using the Fourier transform to find the output y(t) if the input signal is:

$$x(t) = e^{-t}u(t)$$

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