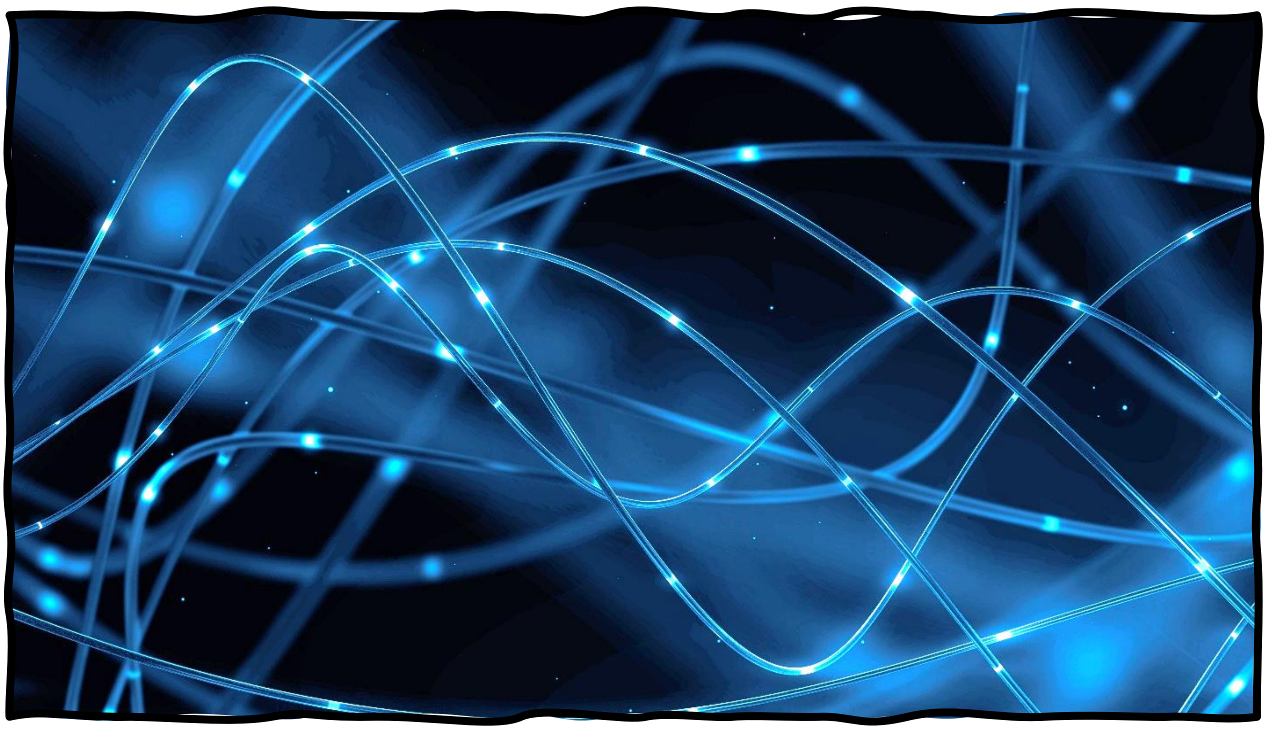
Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture:3

# Lecture 3

# Signal Operations & Fourier Transform



Lecturer: Prof. Dr. Bayan Mahdi Sabbar

M.Sc. Huda Wasfi Hassoon

Email: huda.wasfi@uomus.edu.iq

Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture:3

## 1.1 Signal Operations

In this section, we discuss three useful signal operations: inversion, shifting, and scaling.

- **Time Inversion:** It is simply flipping the signal around the y-axis.

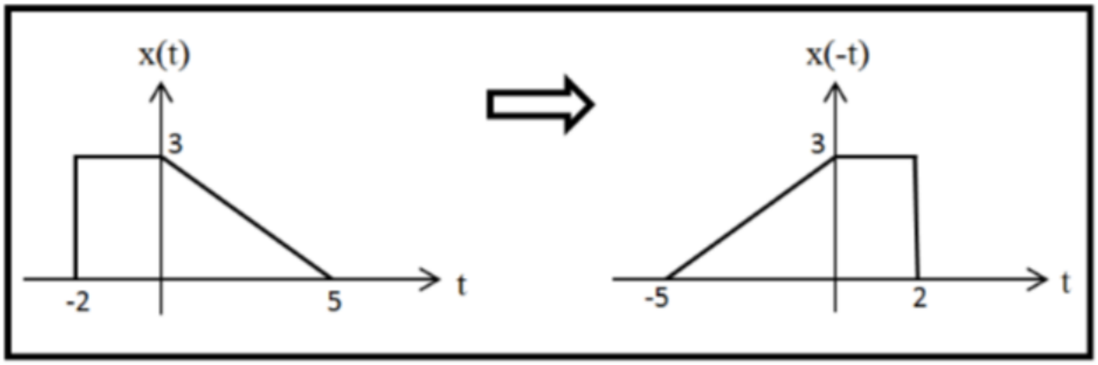



Fig. 1: Time inversion along y-axis.

- **Time Shifting**: involves shifting the signal along the time axis. When a constant is added to the time variable, the signal shifts forward, resulting in an **advanced signal**. Conversely, when time is reduced by a constant, the signal shifts backward, producing a **delayed signal**.

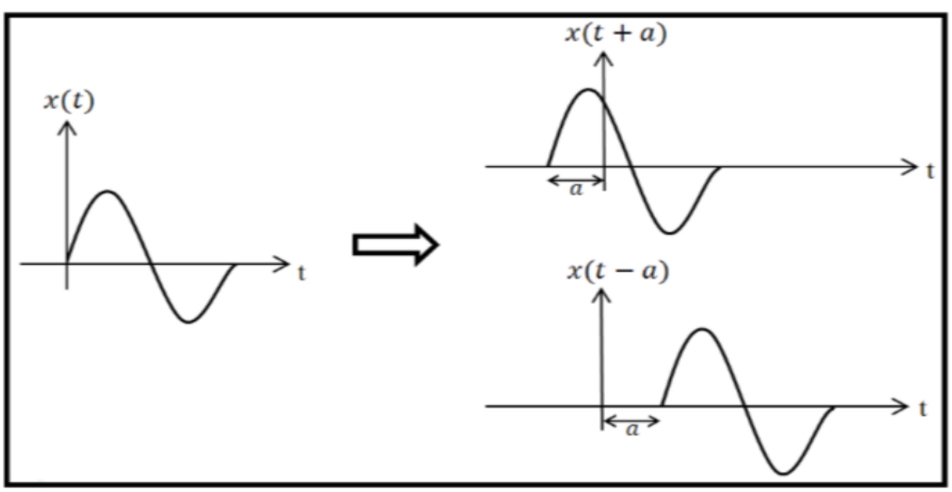



Fig. 2: Examples of Time Shifting.

Email: huda.wasfi@uomus.edu.iq

- Time Scaling: is defined as the process of compression or expansion the time of a signal

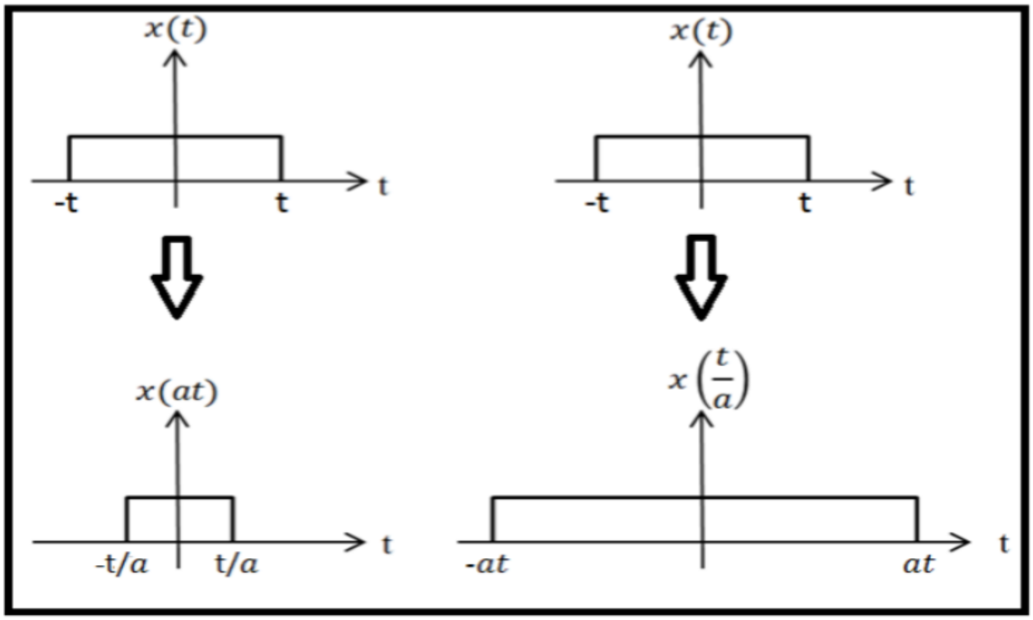
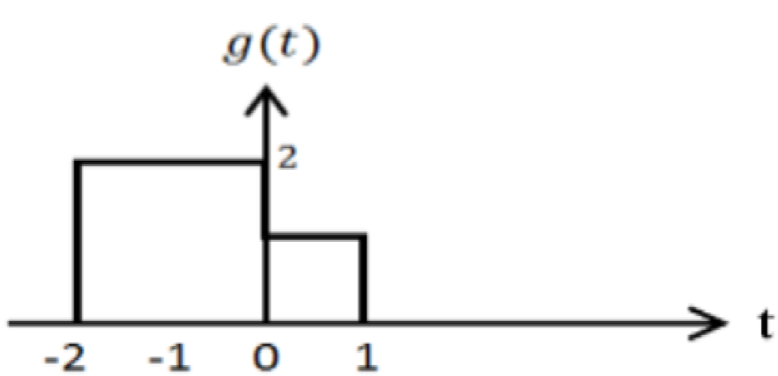



Fig. 3: Examples of Time Scaling.

- If $a > 1$, then we have time compression.
- If $a < 1$, then we have time expansion.

**Example 1:** For the signal shown in the figure below, sketch $g\left(-1 - \frac{t}{2}\right)$.

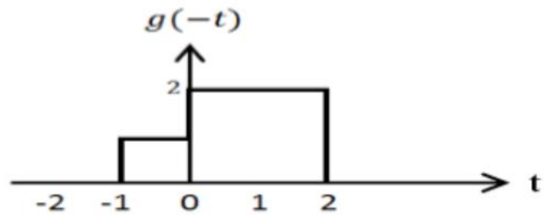



Email: huda.wasfi@uomus.edu.iq

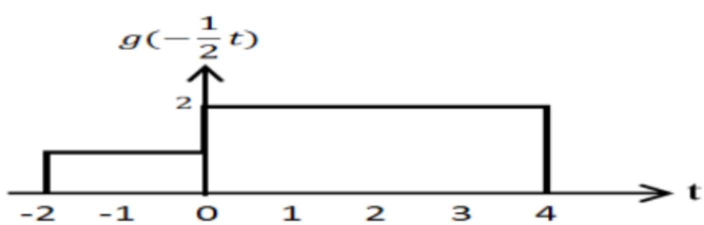
Solution:

$$g\left(-1-\frac{t}{2}\right)=g\left(-\frac{t}{2}-1\right)=g(-\frac{1}{2}(t+2))$$

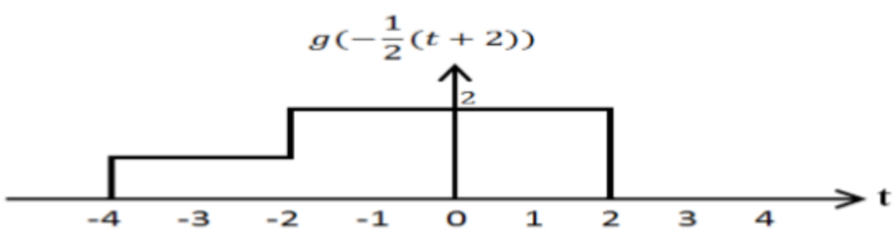
1- Time-inverse




2- Time-scaling by $\frac{1}{2}$




3- shifting to the left by 2




Email: huda.wasfi@uomus.edu.iq

## 1.2 Basic Continous-Time Signals

- Unit Step Function:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

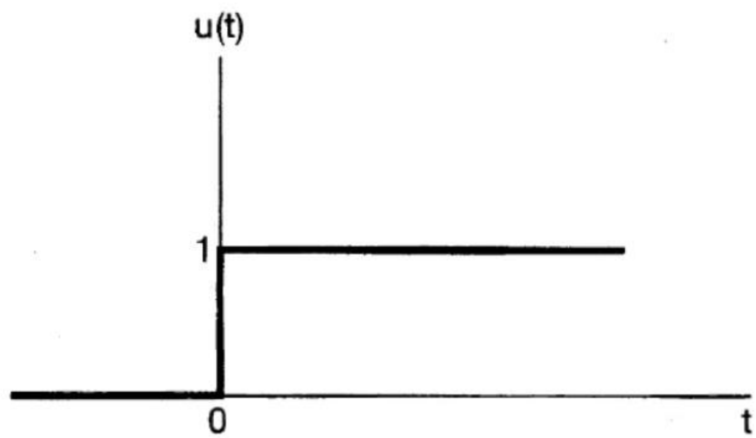



Fig. 4: Unit step function.

- Unit Impulse Function:

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

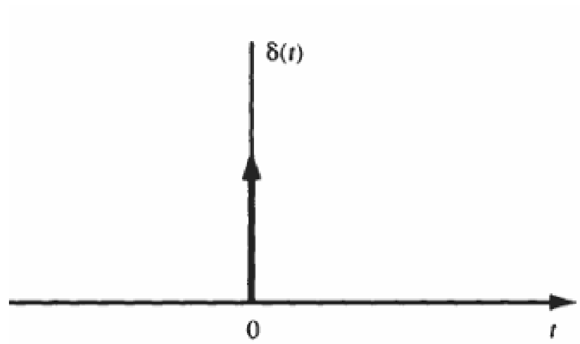



Fig. 5: Unit Impulse function.

Email: huda.wasfi@uomus.edu.iq

Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture:3

Some properties of impulse function:

| |
|---|
| $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$ |
| $\int_{a}^{b} \delta(t)\, dt = 1 \quad , \quad a < 0 < b$ |
| $\int_{a}^{b} \delta(t - t_o)\, dt = 1 \quad , \quad a < t_o < b$ |
| **Symmetry:** $\delta(t) = \delta(-t)$ |
| **Time-scaling:** $\delta(at) = \frac{1}{\lvert a \rvert}\delta(t)$ |
| **Multiplication of a function by an impulse function:** <br>• $f(t)\,\delta(t) = f(0)\,\delta(t)$ <br>• $f(t)\,\delta(t - t_o) = f(t_o)\,\delta(t - t_o)$ |

**Example 2:** Evaluate the following integrals.

a. $\int_{-\infty}^{\infty} \delta(t-2)e^{-t}dt$

b. $\int_{1}^{5} \delta(t)\cos(t)dt$

c. $\int_{-10}^{10} \delta(t+2)\cos(\pi t)dt$

d. $\int_{-\infty}^{\infty}[\delta(t) + u(t) - u(t-2)]dt$

Email: huda.wasfi@uomus.edu.iq

Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture:3

**Solution:**

a) $\int_{-\infty}^{\infty} \delta(t-2)e^{-t}dt = \int_{-\infty}^{\infty} \delta(t-2)e^{-2}dt = 0.1353$

b) $\int_{1}^{5} \delta(t)\cos(t)dt$

$$According\ to\ prperty\ \int_{a}^{b} \delta(t)dt = 1 \quad , \quad a < 0 < b$$

And $a = 1 > 0$ so, $\int_{a}^{b} \delta(t)dt = 0$

$$\int_{a}^{b} \delta(t)f(\mathrm{t})dt = f(0)\int_{a}^{b} \delta(t)dt = f(0) \times 0 = 0 \quad , \quad a < 0 < b$$

$$\int_{1}^{5} \delta(t)\cos(t)\,dt = \cos(0) \times 0 = 0$$

c) $\int_{-10}^{10} \delta(t+2)\cos(\pi t)dt$

$$According\ to\ prperty\ \int_{a}^{b} \delta(t-t_o)dt = 1 \quad , \quad a < t_o < b$$

$t_o = -2$, $a = -10$ and $b = 10$, so

$$\int_{-10}^{10} \delta(t+2)\cos(\pi t)dt = \cos(\pi \times -2) = 1$$

Email: huda.wasfi@uomus.edu.iq

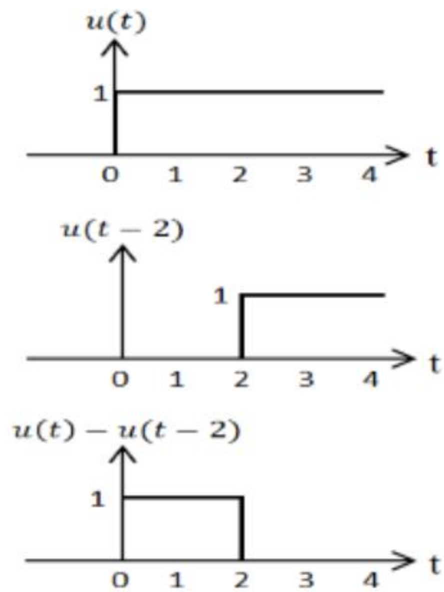
Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture:3

d) $\int_{-\infty}^{\infty}[\delta(t)+u(t)-u(t-2)]\,dt$

$= \int_{-\infty}^{\infty}\delta(t)\,dt + \int_{-\infty}^{\infty}[u(t)-u(t-2)]\,dt$

$= \int_{-\infty}^{\infty}\delta(t)dt + \int_{0}^{2}1\,dt$

$= 1+2 = 3$




## Assignment 1:

Evaluate the following signal:

$$\int_{-\infty}^{\infty} tu(2-t)u(t)dt$$

Email: huda.wasfi@uomus.edu.iq

## 1.3 Fourier Transform

The Fourier transform is used to convert a continuous and non-periodic time-domain signal into the frequency domain. The resulting frequency domain representation from performing the Fourier transform is continuous and non-periodic. The Fourier transform of $x(t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

The inverse Fourier transform of $X(w)$:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

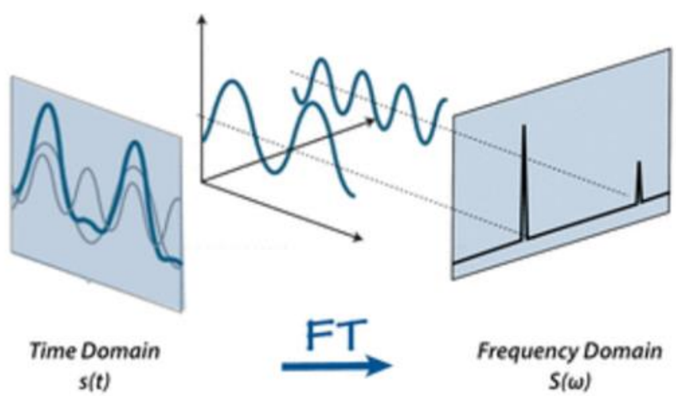



Fig. 6: Fourier Transform.

Email: huda.wasfi@uomus.edu.iq

**Example 3:** Suppose that a signal gets turned on at t=0 and then decays exponentially, So that

$$f(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

For $a > 0$. Find the Fourier transform of this signal.

**Solution:**

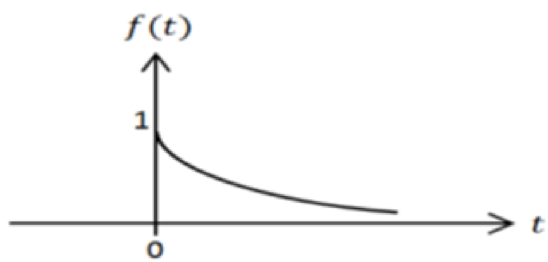
$$F(w) = \int_{-\infty}^{\infty} f(t)e^{-jwt}\ dt$$

$$F(w) = \int_{0}^{\infty} e^{-at}\ e^{-jwt}\ dt$$

$$F(w) = \int_{0}^{\infty} e^{-t(a+jw)}\ dt$$

$$F(w) = \frac{1}{-(a+jw)}[\ \mathrm{e}^{-\infty} - e^{0}]$$

$$F(w) = \frac{1}{(a+jw)}$$



## 1.4 Properties of the Continuous-Time Fourier Transform

There are some properties of the Fourier transform that may be used to simplify the evaluation of the Fourier transform and its inverse. Some of these properties are described below:

| Property | Signal | Fourier Transform |
|---|---|---|
| Linearity | $ax(t) + by(t)$ | $aX(w) + bY(w)$ |
| Time-Shift | $x(t - t_o)$ | $e^{-jwt_o} X(w)$ |
| Time-Reversal | $x(-t)$ | $X(-w)$ |
| Frequency Shifting | $e^{jw_o t} x(t)$ | $X(w - w_o)$ |
| Time Scaling | $x(at)$ | $\frac{1}{\lvert a \rvert} X\left(\frac{w}{a}\right)$ |
| Convolution | $x(t) * y(t)$ | $X(w)\, Y(w)$ |
| Time differentiation | $\frac{dx(t)}{dt}$ | $jwX(w)$ |
| Parseval's relation | $\int_{-\infty}^{\infty} \lvert x(t) \rvert^2 \, dt$ | $\frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(w) \rvert^2 \, dw$ |

## 1.5 Frequency response

The output $y(t)$ of a continuous-time system equals the convolution of the input $x(t)$ with the impulse response $h(t)$; that is

$$y(t) = x(t) \otimes h(t)$$

Applying the convolution property, we obtain

$$Y(\omega) = X(\omega).H(\omega)$$

Where $Y(w)$, $X(w)$ and $H(\omega)$ are the Fourier transforms of $y(t)$, $x(t)$ and $h(t)$, respectively. we have

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

There are some relationships are widely used to transform between the time domain and the frequency domain.

- $\delta(t) \leftrightarrow 1$
- $\delta(t - t_o) \leftrightarrow e^{-jwt_o}$
- $1 \leftrightarrow 2\pi\delta(w)$
- $e^{jw_o t} \leftrightarrow 2\pi\delta(w - w_o)$
- $\cos w_o t \ \leftrightarrow \ \pi\delta(w - w_o) + \pi\delta(w + w_o)$
- $\sin w_o t \ \leftrightarrow -j\pi\delta(w - w_o) + j\pi\delta(w + w_o)$
- $u(t) \leftrightarrow \pi\delta(w) + \frac{1}{jw}$
- $e^{-at}u(t) \leftrightarrow \frac{1}{jw+a}, \quad a > 0$
- $e^{-a|t|} \leftrightarrow \frac{2a}{w^2+a^2}, \quad a > 0$

Email: huda.wasfi@uomus.edu.iq

Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Prof. Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture:3

**Example 4:** Consider the linear time-invariant system characterized by the first-order linear constant coefficient difference equation

$$y'(t) + 2y(t) = x(t) + x'(t)$$

By using Fourier Transform, find the Impulse response, $h(t)$.

**Solution:**

$$jwY(w) + 2Y(w) = X(w) + jwX(w)$$

$$Y(w)[jw + 2] = X(w)[1 + jw]$$

$$\frac{Y(w)}{X(w)} = \frac{jw + 1}{jw + 2}$$

$$H(w) = \frac{jw + 2 - 1}{jw + 2}$$

$$H(w) = 1 - \frac{1}{jw + 2}$$

$$h(t) = \delta(t) - e^{-2t}u(t)$$

# Assignment 2:

Consider a continuous-time system described by:

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Using the Fourier transform to find the output y(t) if the input signal is:

$$x(t) = e^{-t}u(t)$$

Email: huda.wasfi@uomus.edu.iq