Partial Differential Equations (PDEs):

In problems where one variable, say u, depends on more than one independent variable, say that x and t, then any derivatives of u will be (partial derivatives) such as $\frac{\partial u}{\partial x}$ or $\frac{\partial^2 u}{\partial t^2}$ and any differential equation arising will be known as a partial differential equation.

① * Method of Direct Integration:

Ex(1) Solve; $\frac{\partial^2 u}{\partial x^2} = x^2 + y$

sol

① Integrate with respect to (x)

$$\int \frac{\partial^2 u}{\partial x^2} = \int x^2 + y \, dx$$

$$\frac{\partial u}{\partial x} = \frac{x^3}{3} + yx + f(y)$$

② Integrate with respect to (x)

$$\int \frac{\partial u}{\partial x} = \int \frac{x^3}{3} + yx + f(y) \, dx$$

$$\Rightarrow u = \frac{x^4}{12} + y\frac{x^2}{2} + x f(y) + g(y)$$

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Example (2) solve $\dfrac{\partial^2 Z}{\partial x \partial y} = x^2 y$ that Subject to the conditions $Z(x,0) = x^2$ & $Z(1,y) = \cos y$

Sol:

1- Integrate with respect to x.

$$\frac{\partial Z}{\partial y} = \int x^2 y \, dx \Rightarrow \frac{\partial Z}{\partial y} = \frac{1}{3} x^3 y + f(y)$$

2- Integrate with respect to y.

$$Z = \int \frac{1}{3} x^3 y + f(y) \, dy$$

$$\therefore Z = \frac{1}{6} x^3 y^2 + F(y) + g(x) \quad \text{——— } (1)$$

(1) Boundry condition $\quad Z(x,0) = x^2$

$Z = x^2 \quad ; \quad y = 0$

$$\Rightarrow x^2 = \frac{1}{6} x^3 (0)^2 + F(0) + g(x)$$

$$x^2 = F(0) + g(x) \Rightarrow g(x) = x^2 - F(0) \quad \Big] \text{ sub. in eq } (1)$$

$$Z = \frac{1}{6} x^3 y^2 + F(y) + x^2 - F(0) \quad \text{——— } (2)$$

Example (4) Solve $\frac{\partial^3 Z}{\partial x^2 \partial y} = 18xy^2 + \sin(2x-y)$

Sol: Integrate w.r. to x

$$\frac{\partial^2 Z}{\partial x \partial y} = 18\frac{x^2}{2}y^2 - \frac{\cos(2x-y)}{2} + f(y)$$

Integrate w.r. to x

$$\frac{\partial Z}{\partial y} = 9\frac{x^3}{3}y^2 - \frac{1}{2}\frac{\sin(2x-y)}{2} + F(y)$$

Integrate w.r. to y

$$Z = 3x^3\frac{y^3}{3} - \frac{1}{4}\frac{[-\cos(2x-y)]}{-1} + F(y) + g(x)$$

$$\therefore Z = x^3y^3 - \frac{1}{4}\cos(2x-y) + F(y) + g(x)$$

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