

اسم التدريسي: أ.م. د محد على صيهود

المرحلة: Second

السنة الدراسية: 2024-2025

non exact first order differential equation:عنوان المحاضرة



(20)

Non-Exact Differential Equation

المادلات التفاصليه العرام 3-

The equation which is not exact can be made exact by multiplying it by the integrating factor (I.F):

$$M(x,y) dx + N(x,y) dy = 0$$

$$P(x,y) dx + N(x,y) dy = 0$$

$$P(x,y) dx + N(x,y) dy = 0$$

أذا كانت اكعامله الديمًا طلبه غير تامه (Mon - exact) فأننا دفس تبطه عملها تامه بواسط عزبها بعماط التحميل (I.F) وهوصامل دخرب به الحمادله الغير تامه فيصولها اكا معاهله تامه. هنا بكون لدنيا جالستن

(1)
$$f(x) = \frac{1}{N} \left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right]$$

 $I \cdot F = e^{\int f(x) dx}$

(2)
$$f(y) = \frac{1}{M} \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right]$$

If $F = e$



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$$M(x,y) = y$$
 $\Rightarrow \frac{\partial M}{\partial y} = 1$ $\Rightarrow \frac{\partial M}{\partial x} = 3$ $\Rightarrow \frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial x}$

30 Non-Exact.

$$f(x) = \frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$= \frac{1}{3+3x-y} \left(1-3\right) = \frac{-2}{3+3x-y}$$

$$(I \cdot F)_{x} = e^{\int f(x) dx} = e^{\int \frac{-2}{3+3x-y} dx}$$

$$-\frac{2}{3}\ln(3+3x-y) = \ln(3+3x-y)$$
= e = e -\frac{-\frac{7}{3}}{-\frac{7}{3}}

 $= (3+3X-9)^{-73}$

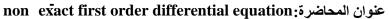
الي من الي مل لانه واله من X و ل



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(55).

$$f(y) = \frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$$

$$= \frac{1}{y} \left(3 - 1 \right) = \frac{2}{y}$$

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$$= \frac{2 \ln y}{2} = \frac{\ln y^2}{2} = \frac{2}{y^2}.$$

& y2 is integrating factor.

$$\Rightarrow y \cdot dx + (3+3x-y) \cdot dy = 0 \quad \boxed{*} y^{2}$$

$$y^{3} \cdot dx + (3y^{2} + 3xy^{2} - y^{3}) \cdot dy = 0$$

$$M(x_1y) = y^3 \implies \frac{\partial M}{\partial y} = 3y^2$$

$$\mathcal{N}(x_{2}y) = 3y^{2} + 3xy^{2} - y^{3} \Rightarrow \frac{\partial \mathcal{N}}{\partial x} = 3y^{2}.$$

$$3 \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3y^2 \implies 50 \text{ Grapt.}$$

To solve the equation

$$\int_{0}^{x} y \, dx + \int_{0}^{y} (3+3x-y) \, dy = 0$$



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$$\int_{a}^{x} y^{3} dx + \int_{b}^{y} (3y^{2} + 3q^{2} - y^{3}) dy = 0$$

$$\left[y^{3} x \right]_{a}^{x} + \left[3y^{3} + 3qy^{2} - y^{3} \right]_{b}^{y} = 0$$

$$\begin{bmatrix}
 y^{3}x - y^{3}q
 \end{bmatrix} + \begin{bmatrix}
 y^{3} + ay^{3} - \frac{y^{4}}{4} - b^{3} - ab^{3} + \frac{b^{4}}{4}
 \end{bmatrix} = c$$

$$y^{3}x - y^{3}q + y^{3} + ay^{3} - \frac{y^{4}}{4} - b^{3} - ab^{3} + \frac{b^{4}}{4} = c$$

$$- \frac{y^{4}}{4} + y^{3}x + y^{3} + (\frac{b^{4}}{4} - b^{3} - ab^{3}) = c$$

$$- \frac{y^{4}}{4} + y^{3}x + y^{3} + k = 0$$

$$K = \frac{b^{4}}{4} - b + ab^{3}$$

Ex(2): Find a general solution for
$$(x^2+y^2+x) dx + xy dy = 0$$

$$M(x,y) = x^2+y^2+x \implies \frac{\partial M}{\partial y} = 2y.$$

$$N(x,y) = xy \implies \frac{\partial N}{\partial x} = y.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \implies \frac{\partial N}{\partial x} = y.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \implies \frac{\partial N}{\partial x} = y.$$



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We must find I.F

$$f(x) = \frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \frac{1}{xy} \left(2y - y \right) = \frac{y}{xy} = \frac{1}{x}.$$

$$(I.F)_{x} = e^{\int \frac{1}{x} dx} = e^{\int \frac{1}{x} dx} = e^{\int \frac{1}{x} dx}$$

$$(x^{2}+y^{2}+x) dx + xy dy = 0$$

$$(x^{3}+xy^{2}+x^{2}) dx + x^{2}y dy = 0$$

$$\frac{\partial M}{\partial y} = 2xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial x} = 2xy$$

$$\int_{a}^{x} (x^{3} + xy^{2} + x^{2}) dx + \int_{b}^{y} x^{2}y dy = 0$$

$$\left[\frac{x^{4}}{4} + y^{2} \frac{x^{2}}{2} + \frac{x^{3}}{3}\right]_{a}^{x} + \left[\frac{a^{2}y^{2}}{2}\right]_{b}^{y} = 0$$

$$\frac{x^{4}}{4} + y^{2} \frac{x^{2}}{2} + \frac{x^{3}}{3} - \left(\frac{a^{4}}{4} + y^{2} \frac{a^{3}}{2} + \frac{a^{3}}{3}\right)$$

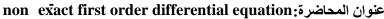
$$+ a^{2} \frac{y^{2}}{2} - a^{2} \frac{b^{2}}{2} = 0$$



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$$\frac{x^{4}}{4} + y^{2} \frac{x^{2}}{2} + \frac{x^{3}}{3} - \frac{a^{4}}{4} - \frac{a^{3}}{3} - a^{2} \frac{b^{3}}{2} = 0$$

$$\left(\frac{x^{4}}{4} + y^{2} \frac{x^{2}}{2} + \frac{x^{3}}{3}\right) - k = 0$$

$$\left[k = \frac{a^{4}}{4} + \frac{a^{3}}{3} + a^{2} \frac{b^{3}}{2}\right],$$

$$(25)$$