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Chapter Eight
The control system for Linear Closed-Loop Systems

In previous chapters, the dynamic behavior of several basic systems was examined. With this background, we can extend the discussion to a complete control system and introduce the fundamental concept of feedback. To work with a familiar system, the treatment will be based on a stirred-tank heater.

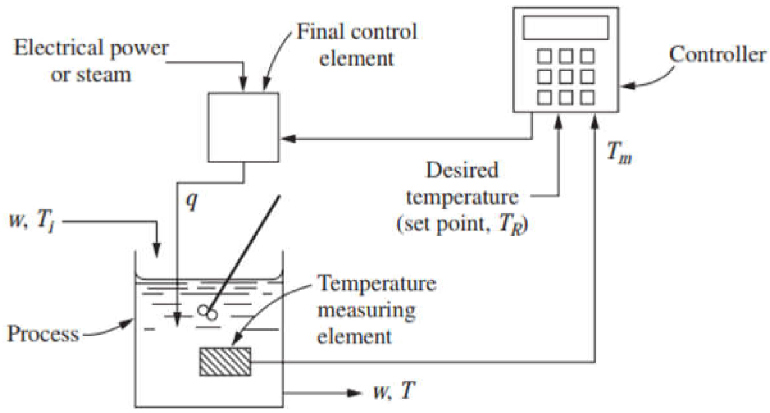


FIGURE 8–1
Control system for a stirred-tank heater.

- A liquid stream at a temperature T_i enters an insulated, well-stirred tank at a constant flow rate w (mass/time). It is desired to maintain (or control) the temperature in the tank at T_R using the controller. If the measured tank temperature T_m differs from the desired temperature T_R, the controller senses the difference or error, $\varepsilon = T_R - T_m$ and changes the heat input in such a way as to reduce the magnitude of ε. If the controller changes the heat input to the tank by an amount that is proportional to ε, we have proportional control.

- In Fig. 8–1, it is indicated that the source of heat input q may be electricity or steam. If an electrical source were used, the final control element might be a variable transformer that is used to adjust current to a resistance heating element; if steam were used, the final control element would be a control valve that adjusts the flow of steam. In either case, the output signal from the controller should adjust q in such a way as to maintain control of the temperature in the tank.

Components of a Control System

The system shown in Fig. 8–1 may be divided into the following components:

1. Process (stirred-tank heater).
2. Measuring element (thermocouple).
3. Controller.
4. Final control element (variable transformer or control valve).

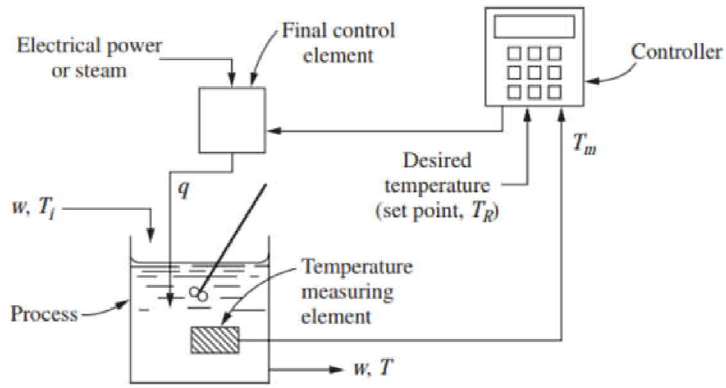
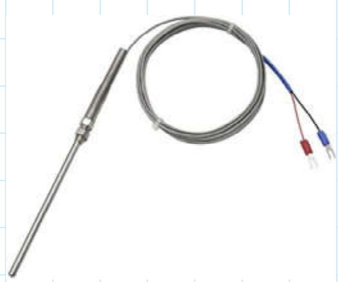


FIGURE 8–1
Control system for a stirred-tank heater.

Stirred-tank heater



Thermocouple

Controller

Variable transformer

Control valve

- In general, these four components will constitute most of the control systems that we consider in this text; however, the reader should realize that more complex control systems exist in which more components are used. For example, some processes require a cascade control system in which two controllers and two measuring elements are used.

Block Diagram

For computational purposes, it is convenient to represent the control system of Fig. 8–1 by means of the block diagram shown in Fig. 8–2. Such a diagram makes it much easier to visualize the relationships among the various signals. New terms, which appear in Fig. 8–2, are set point and load. The set point is a synonym for the desired value of the controlled variable. The load refers to a change in any variable that may cause the controlled variable of the process to change. In this example, the inlet temperature T_i is a load variable. Other possible loads for this system are changes in flow rate and heat loss from the tank. (These loads are not shown on the diagram.)

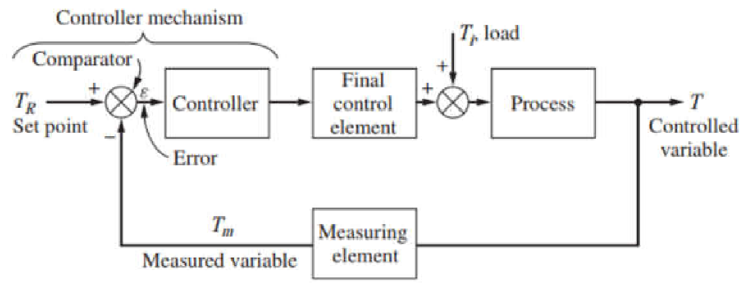


FIGURE 8–2
Block diagram of a simple control system.

The control system shown in Fig. 8–2 is called a closed-loop system or a feedback system because the measured value of the controlled variable is returned or "fed back" to a device called the comparator. In the comparator, the controlled variable is compared with the desired value or set point. If there is any difference between the measured variable and the set point, an error is generated. This error enters a controller, which in turn adjusts the final control element to return the controlled variable to the set point.

Negative Feedback Versus Positive Feedback

Negative Feedback

The feedback principle, which is illustrated by Fig. 8–2, involves the use of the controlled variable T to maintain itself at the desired value T_R. The arrangement of the apparatus of Fig. 8–2 is often described as negative feedback to contrast with another arrangement called positive feedback.
Negative feedback ensures that the difference between T_R and T_m , ($\varepsilon = T_R - T_m$),is used to adjust the control element so that the tendency is to reduce the error.
For example, assume that the system is at steady-state and that $T = T_R = T_m$. If the load T_i should increase, T and T_m would start to increase, which would cause the error ε to become negative. With proportional control, the decrease in error would cause the controller and final control element to decrease the flow of heat to the system, with the result that the flow of heat would eventually be reduced to a value such that T approaches T_R

Positive Feedback

If the signal to the comparator were obtained by adding T_R and T_m, we would have a positive feedback system, which is inherently unstable. To see that this is true, again assume that the system is at steady-state and that $T = T_R = T_m$. If T_i were to increase, T and T_m would increase, which would cause the signal from the comparator (ε in Fig. 8–2) to increase, with the result that the heat to the system would increase. However, this action, which is just the opposite of that needed, would cause T to increase further. It should be clear that this situation would cause T to "run away" and control would not be achieved. For this reason, positive feedback would never be used intentionally in the system of Fig. 8–2 . However, in more complex systems it may arise naturally.

At st. st. $\quad T = T_R = T_m$

$\varepsilon = T_R + T_m$

Servo Problem Versus Regulator Problem

Servo Problem

The control system of Fig. 8–2 can be considered from the point of view of its ability to handle either of two types of situations. In the first situation, which is called the servomechanism-type (or servo) problem, we assume that there is no change in load T_i and that we are interested in changing the bath temperature according to some prescribed function of time. For this problem, the set point T_R would be changed in accordance with the desired variation in bath temperature. If the variation is sufficiently slow, the bath temperature may be expected to follow the variation in T_R very closely.
The servo problem can be viewed as trying to follow a moving target (i.e., the changing set point).

Regulator Problem

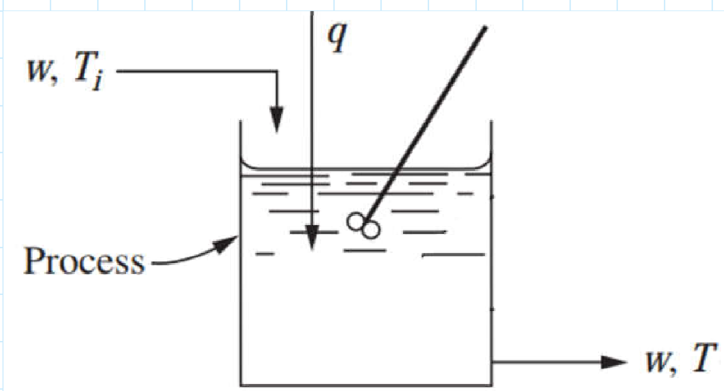
The other situation will be referred to as the regulator problem. In this case, the desired value T_R is to remain fixed, and the purpose of the control system is to maintain the controlled variable at T_R in spite of changes in load T_i. This problem is very common in the chemical industry, and a complicated industrial process will often have many self-contained control systems, each of which maintains a particular process variable at the desired value.

Development of Block Diagram

Each block in Fig. 8–2 represents the functional relationship existing between the input and output of a particular component. In previous chapters, such input-output relations were developed in the form of transfer functions. In block diagram representations of control systems, the variables selected are *deviation variables,* and inside each block is placed the transfer function relating the input-output pair of variables. Finally, the blocks are combined to give the overall block diagram. This is the procedure to be followed in developing Fig. 8–2.

1. Process

Consider first the block for the process. This block will be seen to differ somewhat from those presented in previous chapters in that two input variables are present; however, the procedure for developing the transfer function remains the same.



Energy balance at unsteady state around the tank

$$\left[\begin{array}{c}\text{Rate of enthalpy}\\ \text{into heating}\\ \text{vessel}\end{array}\right] - \left[\begin{array}{c}\text{Rate of enthalpy}\\ \text{leaving heating}\\ \text{vessel}\end{array}\right] + \left[\begin{array}{c}\text{Rate of energy}\\ \text{input into heating}\\ \text{vessel from heater}\end{array}\right] = \left[\begin{array}{c}\text{Rate of accumulation}\\ \text{of enthalpy in}\\ \text{heating vessel}\end{array}\right]$$

$$\left[\begin{array}{c}\text{into heating} \\ \text{vessel}\end{array}\right] - \left[\begin{array}{c}\text{leaving heating} \\ \text{vessel}\end{array}\right] + \left[\begin{array}{c}\text{input into heating} \\ \text{vessel from heater}\end{array}\right] = \left[\begin{array}{c}\text{of enthalpy in} \\ \text{heating vessel}\end{array}\right]$$

$$w C_p (T_i - T_0) - w C_p (T - T_0) + q = \rho C_p V \frac{dT}{dt} \qquad \text{—— (1)}$$

where

T_0 : reference temp. $= 25\ {}^\circ C$

At st. st.

$$\mp w C_p (T_{is} - T_0) \pm w C_p (T_s - T_0) \mp q = \mp \rho C_p V \frac{dT_s}{dt} = 0 \qquad \text{—— (2)}$$

$$w C_p (T_i - T_{is}) - w C_p (T - T_s) + (q - q_s) = \rho C_p V \frac{d(T - T_s)}{dt} \qquad \text{—— (3)}$$

let

$$\left.\begin{aligned} \bar{T}_i &= T_i - T_{is} \\ \bar{T} &= T - T_s \\ Q &= q - q_s \end{aligned}\right\} \text{sub. into Eq. (3)}$$

$$w C_p \bar{T}_i - w C_p \bar{T} + Q = \rho C_p V \frac{d\bar{T}}{dt}$$

$$\left[w C_p (\bar{T}_i - \bar{T}) + Q = \rho C_p V \frac{d\bar{T}}{dt} \right] \div \rho C_p V$$

$$\frac{w \cancel{C_p}}{\rho \cancel{C_p} V} (\bar{T}_i - \bar{T}) + \frac{1}{\rho C_p V} Q = \frac{d\bar{T}}{dt}$$

$$\frac{d\bar{T}}{dt} = \frac{w}{\rho V} (\bar{T}_i - \bar{T}) + \frac{1}{\rho C_p V} Q$$, By taking the Laplace transform

$$s\bar{T}(s) - \overset{0}{\bar{T}(0)} = \frac{w}{\rho V} \left(\bar{T}_i(s) - \bar{T}(s) \right) + \frac{1}{\rho C_p V} Q(s)$$

$$s\bar{T}(s) - \cancel{\bar{T}(0)} = \frac{w}{\rho V}\left(\bar{T}_i(s) - \bar{T}(s)\right) + \frac{1}{\rho C_p V} Q(s)$$

$$s\bar{T}(s) + \frac{w}{\rho V}\bar{T}(s) = \frac{w}{\rho V}\bar{T}_i(s) + \frac{1}{\rho C_p V} Q(s)$$

$$\frac{w}{\rho V}\left(\frac{\rho V}{w} s\bar{T}(s) + \bar{T}(s)\right) = \frac{w}{\rho V}\bar{T}_i(s) + \frac{1}{\rho C_p V} Q(s)$$

$$\frac{\rho V}{w} s\bar{T}(s) + \bar{T}(s) = \frac{\rho V}{w}\left[\frac{w}{\rho V}\bar{T}_i(s) + \frac{1}{\rho C_p V} Q(s)\right]$$

since $\dfrac{\rho V}{w} = \dfrac{\frac{\cancel{kg}}{\cancel{m^3}} \times \cancel{m^3}}{\frac{\cancel{kg}}{s}} = \sec = \tau$

$$\tau s\bar{T}(s) + \bar{T}(s) = \bar{T}_i(s) + \frac{1}{wC_p} Q(s)$$

$$\bar{T}(s)(\tau s + 1) = \bar{T}_i(s) + \frac{1}{wC_p} Q(s)$$

$$\bar{T}(s) = \frac{1}{\tau s + 1}\bar{T}_i(s) + \frac{1/wC_p}{\tau s + 1} Q(s)$$, stirred heater transfer function

where

$\tau = \dfrac{\rho V}{w}$, time constant of the system

$\dfrac{1}{wC_p} = \dfrac{1}{\frac{\cancel{kg}}{min} \frac{kJ}{\cancel{kg}\cdot °C}} = \dfrac{°C}{kJ/min}$, the gain for $Q(t)$

$$\bar{T}(s) = \frac{1}{\tau s + 1}\bar{T}_i(s) + \frac{1/wC_p}{\tau s + 1} Q(s)$$

From the above equation, we can see that two independent quantities, the heater input $Q(s)$ and the inlet temperature $\bar{T}_i(s)$, can cause changes in the outlet temperature.

- If there is a change in $Q(t)$ only, then $\bar{T}_i(t) = 0$ and the transfer function relating $\bar{T}$ to Q is

$$\bar{T}(s) = \cancelto{0}{\frac{1}{\tau s+1}\bar{T}_i(s)} + \frac{1/wC_p}{\tau s+1}Q(s)$$

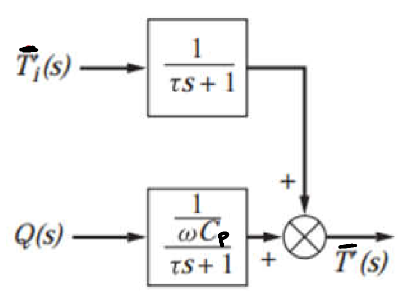
$$\bar{T}(s) = \frac{1/wC_p}{\tau s+1}Q(s)$$

$$\boxed{\frac{\bar{T}(s)}{Q(s)} = \frac{1/wC_p}{\tau s+1}}$$

If there is a change in $\bar{T}_i(t)$ only, then $Q(t) = 0$ and the transfer function relating $\bar{T}$ to $\bar{T}_i$ is

$$\bar{T}(s) = \frac{1}{\tau s+1}\bar{T}_i(s) + \cancelto{0}{\frac{1/wC_p}{\tau s+1}Q(s)}$$

$$\boxed{\frac{\bar{T}(s)}{\bar{T}_i(s)} = \frac{1}{\tau s+1}}$$



(a)

FIGURE 8–3
Block diagram for process.