Rankine's Coefficient on Inclined Backfill

Granular Backfill with Vertical Back Face

If the backfill of a frictionless retaining wall is a granular soil (c=0) and rises at an angle α with respect to the horizontal (see Figure 7.10), the active earth-pressure coefficient may be expressed in the form

$$a = \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \emptyset}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \emptyset}} \qquad \textbf{(1)}$$

$$K_P = \cos \alpha \frac{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \emptyset}}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \emptyset}}$$
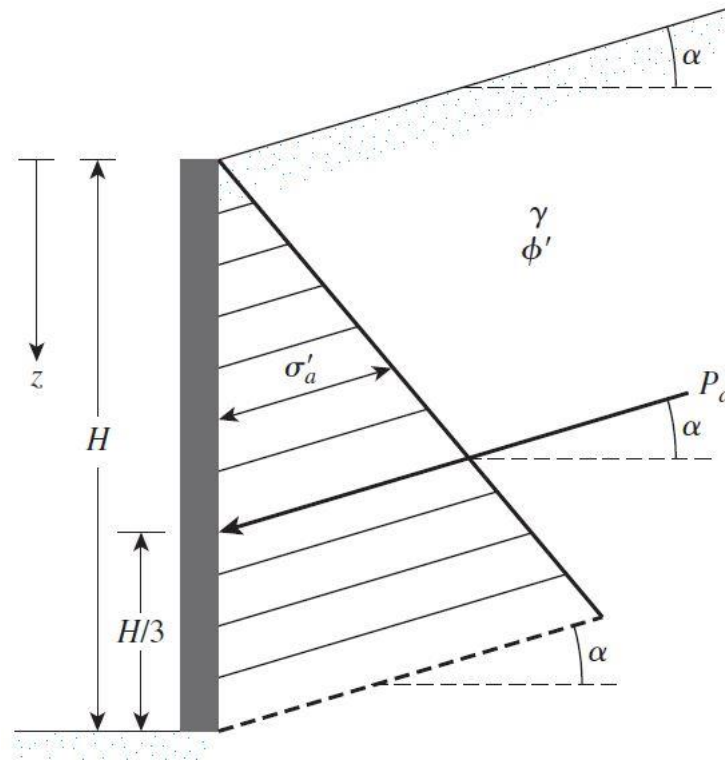

Fig.7-10

Lets $\alpha = 0$, the Eq.1 will reduce to

$$k_a = \frac{1 - sin\emptyset}{1 + sin\emptyset} \qquad (2)$$

Ex: let $\alpha = 10 \; and \; \emptyset = 30$, from Eq.1

$$k_a = \cos(10) \frac{\cos 10 - \sqrt{\cos^2 10 - \cos^2 30}}{\cos 10 + \sqrt{\cos^2 10 - \cos^2 30}} = 0.3495$$

Or use the following Table 17

Table 7.1 Values of K_a [Eq. (7.19)]

α (deg) ↓	28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.2174
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.2175
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.2177
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.2181
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.2186
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.2192
6	0.3676	0.3531	0.3389	0.3253	0.3120	0.2992	0.2868	0.2747	0.2631	0.2518	0.2409	0.2303	0.2200
7	0.3701	0.3553	0.3410	0.3272	0.3138	0.3008	0.2883	0.2761	0.2644	0.2530	0.2420	0.2313	0.2209
8	0.3730	0.3580	0.3435	0.3294	0.3159	0.3027	0.2900	0.2778	0.2659	0.2544	0.2432	0.2325	0.2220
9	0.3764	0.3611	0.3463	0.3320	0.3182	0.3049	0.2921	0.2796	0.2676	0.2560	0.2447	0.2338	0.2233
10	0.3802	0.3646	0.3495	0.3350	0.3210	0.3074	0.2944	0.2818	0.2696	0.2578	0.2464	0.2354	0.2247
11	0.3846	0.3686	0.3532	0.3383	0.3241	0.3103	0.2970	0.2841	0.2718	0.2598	0.2482	0.2371	0.2263
12	0.3896	0.3731	0.3573	0.3421	0.3275	0.3134	0.2999	0.2868	0.2742	0.2621	0.2503	0.2390	0.2281
13	0.3952	0.3782	0.3620	0.3464	0.3314	0.3170	0.3031	0.2898	0.2770	0.2646	0.2527	0.2412	0.2301
14	0.4015	0.3839	0.3671	0.3511	0.3357	0.3209	0.3068	0.2931	0.2800	0.2674	0.2552	0.2435	0.2322
15	0.4086	0.3903	0.3729	0.3564	0.3405	0.3253	0.3108	0.2968	0.2834	0.2705	0.2581	0.2461	0.2346
16	0.4165	0.3975	0.3794	0.3622	0.3458	0.3302	0.3152	0.3008	0.2871	0.2739	0.2612	0.2490	0.2373
17	0.4255	0.4056	0.3867	0.3688	0.3518	0.3356	0.3201	0.3053	0.2911	0.2776	0.2646	0.2521	0.2401
18	0.4357	0.4146	0.3948	0.3761	0.3584	0.3415	0.3255	0.3102	0.2956	0.2817	0.2683	0.2555	0.2433
19	0.4473	0.4249	0.4039	0.3842	0.3657	0.3481	0.3315	0.3156	0.3006	0.2862	0.2724	0.2593	0.2467
20	0.4605	0.4365	0.4142	0.3934	0.3739	0.3555	0.3381	0.3216	0.3060	0.2911	0.2769	0.2634	0.2504
21	0.4758	0.4498	0.4259	0.4037	0.3830	0.3637	0.3455	0.3283	0.3120	0.2965	0.2818	0.2678	0.2545
22	0.4936	0.4651	0.4392	0.4154	0.3934	0.3729	0.3537	0.3356	0.3186	0.3025	0.2872	0.2727	0.2590
23	0.5147	0.4829	0.4545	0.4287	0.4050	0.3832	0.3628	0.3438	0.3259	0.3091	0.2932	0.2781	0.2638
24	0.5404	0.5041	0.4724	0.4440	0.4183	0.3948	0.3731	0.3529	0.3341	0.3164	0.2997	0.2840	0.2692
25	0.5727	0.5299	0.4936	0.4619	0.4336	0.4081	0.3847	0.3631	0.3431	0.3245	0.3070	0.2905	0.2750

Vertical Back face with $C - \emptyset$ Soil Backfill

For a retaining wall with a vertical back and inclined backfill of $C - \emptyset$ soil

$$K_a' = \frac{1}{\cos^2 \phi'} \left\{ \frac{2\cos^2 \alpha + 2\left(\dfrac{c'}{\gamma z}\right) \cos \phi' \sin \phi'}{-\sqrt{\left[4\cos^2 \alpha (\cos^2 \alpha - \cos^2 \phi') + 4\left(\dfrac{c'}{\gamma z}\right)^2 \cos^2 \phi' + 8\left(\dfrac{c'}{\gamma z}\right) \cos^2 \alpha \sin \phi' \cos \phi' \right]}} \right\} - 1$$

Some values of ka are given in Table 7.2. For a problem of this type, the depth of tensile crack is given as

$$z_c = \frac{2c'}{\gamma} \sqrt{\frac{1 + \sin \phi'}{1 - \sin \phi'}}$$

Table 7.2 Values of K_a'

ϕ' (deg)	α (deg)	$\frac{c'}{\gamma z}$			
		0.025	0.05	0.1	0.5
15	0	0.550	0.512	0.435	−0.179
	5	0.566	0.525	0.445	−0.184
	10	0.621	0.571	0.477	−0.186
	15	0.776	0.683	0.546	−0.196
20	0	0.455	0.420	0.350	−0.210
	5	0.465	0.429	0.357	−0.212
	10	0.497	0.456	0.377	−0.218
	15	0.567	0.514	0.417	−0.229
25	0	0.374	0.342	0.278	−0.231
	5	0.381	0.348	0.283	−0.233
	10	0.402	0.366	0.296	−0.239
	15	0.443	0.401	0.321	−0.250
30	0	0.305	0.276	0.218	−0.244
	5	0.309	0.280	0.221	−0.246
	10	0.323	0.292	0.230	−0.252
	15	0.350	0.315	0.246	−0.263

Example 7.5

For the retaining wall shown in Figure 7.10, $H = 7.5$ m, $\gamma = 18$ kN/m³, $\phi' = 20°$, $c' = 13.5$ kN/m², and $\alpha = 10°$. Calculate the Rankine active force, P_a, per unit length of the wall and the location of the resultant force after the occurrence of the tensile crack.

Solution

From Eq. (7.24).

$$z_r = \frac{2c'}{\gamma}\sqrt{\frac{1 + \sin\phi'}{1 - \sin\phi'}} = \frac{(2)(13.5)}{18}\sqrt{\frac{1 + \sin 20}{1 - \sin 20}} = 2.14 \text{ m}$$

At $z = 7.5$ m,

$$\frac{c'}{\gamma z} = \frac{13.5}{(18)(7.5)} = 0.1$$

From Table 7.2, for $\phi' = 20°$, $c'/\gamma z = 0.1$, and $\alpha = 10°$, the value of K_a' is 0.377, so at $z = 7.5$ m,

$$\sigma_a' = \gamma z K_a' \cos \alpha = (18)(7.5)(0.377)(\cos 10) = 50.1 \text{ kN/m}^2$$

After the occurrence of the tensile crack, the pressure distribution on the wall will be as shown in Figure 7.11, so

$$P_a = \left(\frac{1}{2}\right)(50.1)(7.5 - 2.14) = \textbf{134.3 kN/m}$$

and

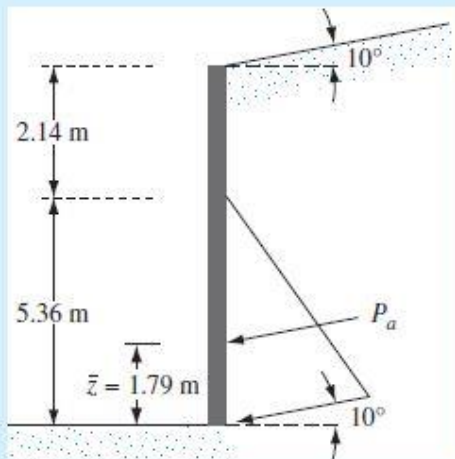$$\bar{z} = \frac{7.5 - 2.14}{3} = \textbf{1.79 m} \qquad \blacksquare$$

Figure 7.11 Calculation of Rankine active force, $c' - \phi'$ soil