Email: abbas.jawad@uomus.edu.iq

Controller

Controller: A device that outputs a signal to the process or final control element based on the magnitude of the error signal.

The control hardware required to control the temperature of a stream leaving a heat exchanger is shown in Fig. 9–2. This hardware consists of the following components listed here along with their respective conversions:

1. Transducer (temperature-to-current)
2. Computer/ Controller (current-to-current)
3. Converter (current-to-pressure)
4. Control valve (pressure-to-flow rate)

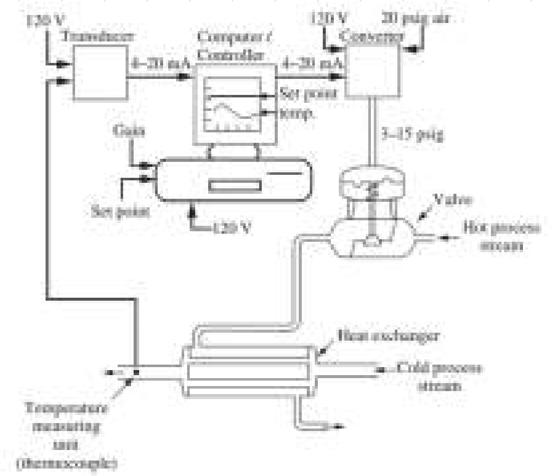


Figure 9.2: Schematic diagram of control system

- Figure 9–2 shows that a thermocouple is used to measure the temperature; the signal from the thermocouple is sent to a transducer, which produces a current output in the range of 4 to 20 mA, which is a linear function of the input. The output of the transducer enters the controller where it is compared to the set point to produce an error signal. The computer/controller converts the error to an output signal in the range of 4 to 20 mA in accordance with the computer control algorithm. The only control algorithm we have considered so far has been proportional. Later in this chapter, other control algorithms will be described. The output of the computer/controller enters the converter, which produces an output in the range of 3 to 15 psig, as a linear function of the input. Finally, the air pressure output of the converter is sent to the top of the control valve, which adjusts the flow of steam to the heat exchanger. We assume that the valve is linear and is the air-to-open type. The external power (120 V) needed for each component is also shown in Fig. 9–2. Electricity is needed for the transducer, computer/controller, and converter. A source of 20 psig air is also needed for the converter.

  - To see how the components interact with one another, consider the process to be operating at steady state with the outlet temperature equal to the set point. If the temperature of the cold process stream decreases, the following events occur: After some delay the thermocouple detects a decrease in the outlet temperature and produces a proportional change in the signal to the controller. As soon as the controller detects the drop in temperature, relative to the set point, the controller output increases according to proportional action. The increase in signal to the converter causes the output pressure from the converter to increase and to open the valve wider to admit a greater flow of the hot process stream. The increased flow of hot stream will eventually increase the output temperature and move it toward the set point. From this qualitative description, we see that the flow of signals from one component to the next is such that the outlet temperature of the heat exchanger should return toward the set point.

An equivalent P&ID (piping and instrumentation diagram) for this control system is shown in Fig. 9–3

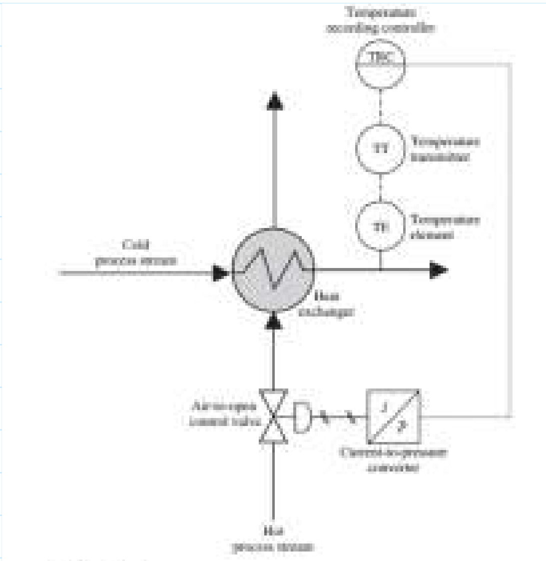


FIGURE 9-3
Piping and instrumentation diagram for control system of Fig. 9-2.

For convenience in describing various control laws (or algorithms) in the next part of this chapter, the transducer, controller, and converter will be lumped into one block, as shown in Fig. 9–4.

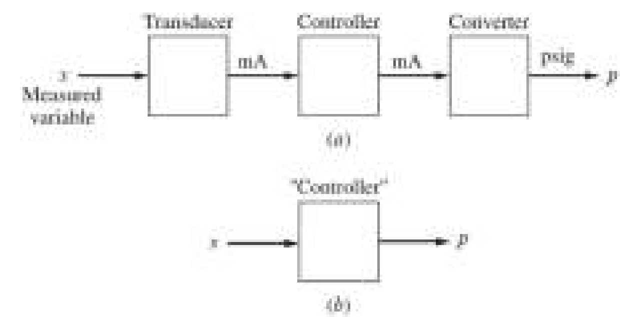


FIGURE 9–4
Equivalent block for transducer, controller, and converter.

1. Proportional controller (P controller)

Proportional control. The simplest type of controller is the proportional controller. Our goal is to reduce the error between the process output and the set point. The proportional controller, as we will see, can reduce the error, but cannot eliminate it. If we can accept some residual error, proportional control may be the proper choice for the situation.
The proportional controller has only one adjustable parameter, the controller gain. The proportional controller produces an output signal (pressure in the case of a pneumatic controller, current, or voltage for an electronic controller) that is proportional to the error ε. This action may be expressed as

$$p = K_c\varepsilon + p_s \qquad (1)$$

where
p = output signal from controller, psig or mA
Kc= proportional gain, or sensitivity
ε=error= (set point)-(measured variable)
ps=a constant, the steady-state output from the controller

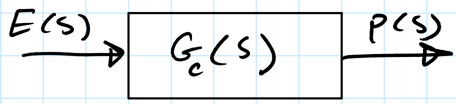
In a controller having adjustable gain, the value of the gain K_c can be varied by entering it into the controller, usually by means of a keypad (or a knob on older equipment).
The value of p_s is the value of the output signal when ε is zero, and in most controllers p_s can be adjusted to

obtain the required output signal when the control system is at steady-state and $\varepsilon = 0$.
To obtain the transfer function of Eq. 1, we first introduce the deviation variable

$$P = p - p_s$$

At time t = 0, we assume the error es to be zero. Then ε is already a deviation variable, E. Equation 1 becomes

$$p - p_s = K_c E(t)$$



$$P = K_c E(t)$$

By taking the Laplace transform for the above equation, we obtain

$$P(s) = K_c E(s) \quad \text{—— } 2$$

$$\text{Proportional controller transfer function} = G_s(s) = \frac{P(s)}{E(s)} = K_c$$

- The actual behavior of a proportional controller is depicted in Fig. 9–6. The controller output will saturate (level out) at p_{max} = 15 psig or 20 mA at the upper end and at p_{min} = 3 psig or 4 mA at the lower end of the output. The ideal transfer function Eq. (2) does not predict this saturation phenomenon.

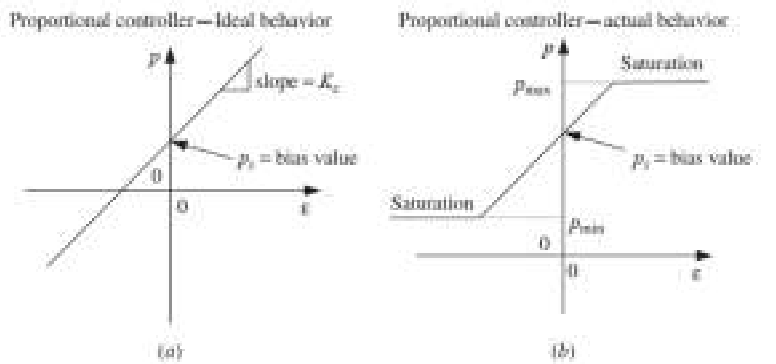


FIGURE 9–6
Proportional controller output as a function of error input to the controller. *(a)* Ideal behavior; *(b)* actual behavior.

Proportional band, P.B%, Band width

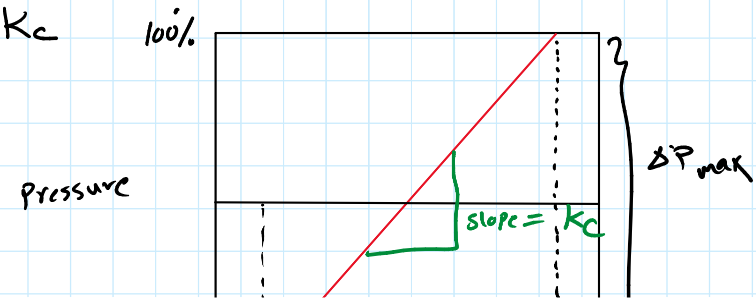
Proportional band is used in some controllers instead of gain. This tends to be somewhat confusing because gain-type control is referred to as proportional control. Proportional band is the amount of change in error that will cause the output to go from full on to full off. The amount of change in error is calculated was a percentage of full-scale error.

Proportional band P.B (band width) is defined as the error ΔE (expressed as a percentage of the total range of measured error ΔE_{max}) required to move the final control element from minimum value to maximum value Δp_{max}

Note: from B.P%, you can get Kc

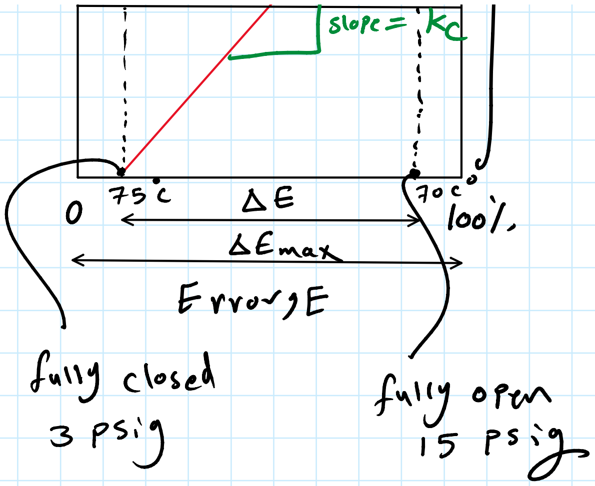
$$P.B\% = \frac{\Delta E}{\Delta E_{max}} \times 100 \quad \text{—— } 1$$

since



since

Transfer function of proportional controller $G_c(s)$ is equal



$$G_c(s) = K_c = \frac{\Delta P_{max}}{\Delta E} \quad \text{—— (2)}$$

so $\Delta E = \frac{\Delta P_{max}}{K_c}$ sub. into Eq. (1)

$$P.B\% = \frac{\frac{\Delta P_{max}}{K_c}}{\Delta E_{max.}} * 100$$

$$P.B\% = \frac{100}{K_c} * \frac{\Delta P_{max}}{\Delta E_{max}} \quad \text{—— (3)}$$

If ΔP_{max} + ΔE_{max} are taken as percentage of total change Therefore, the Eq. (3) becomes

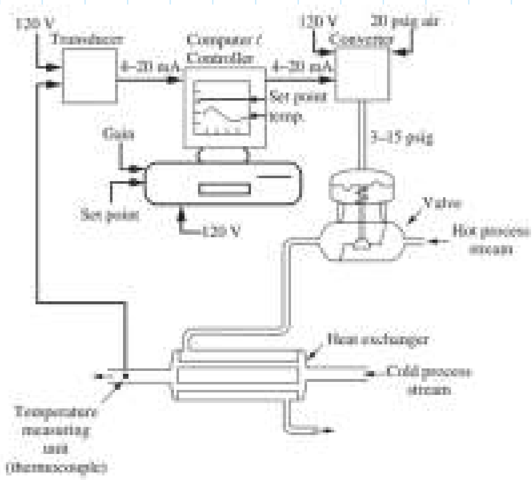
$$P.B\% = \frac{100}{K_c} * \frac{\cancel{100\%}}{\cancel{100\%}} \Rightarrow P.B = \frac{100}{K_c}$$

this the range of comeany

Example 1: A pneumatic proportional controller is used in the process shown in the below figure to control the cold stream outlet temperature within the range of 60 to 100 °F. The controller gain is adjusted so that the output pressure goes from 3 psig (valve fully closed) to 15 psig (valve fully open) as the measured temperature goes from 71 to 75 °F with the set point held constant. Find the controller gain K_c and proportional band, P.B%

this is adjusted by operator

Solution:



$$\text{Gain} = K_c = \frac{\Delta P_{max}}{\Delta E} = \frac{P_{max} - P_{min}}{E_2 - E_1}$$

$$K_c = \frac{P_{max} - P_{min}}{T_{m_2} - T_R - (T_{m_1} - T_R)}$$

$$K_c = \frac{P_{max} - P_{min}}{T_{m_2} - \cancel{T_R} - T_{m_1} + \cancel{T_R}}$$

$$K_c = \frac{P_{max} - P_{min}}{T_{m_2} - T_{m_1}}$$

$$K_c = \frac{15\ psig - 3\ psig}{75\,^\circ F - 71\,^\circ F} = 3\ psig/^\circ F$$

$$B.P\% = \frac{\Delta E}{\Delta E_{max}} \times 100\% = \frac{75-71}{100-60} \times 100 = 10\%$$

another way to find gain Kc

$$P.B\% = \frac{100}{K_c} \times \frac{\Delta P_{max}}{\Delta E_{max}} \Rightarrow 10 = \frac{100}{K_c} \times \frac{15-3}{100-60} \Rightarrow K_c = 3 \ psig/{}^{\circ}F$$

Now assume that the gain of the controller is changed to 0.4 psi/°F. Find the error in temperature that will cause the control valve to go from fully closed to fully open.

Since $K_c = \frac{\Delta P_{max.}}{\Delta E}$

$$\Delta E = \frac{\Delta P_{max.}}{K_c} = \frac{(15-3)\ psig}{0.4\ \frac{psig}{{}^{\circ}F}} = 30\ {}^{\circ}F$$

At this level of gain, the valve will be fully open if the error signal reaches 30 °F. The gain Kc has the units of psi per unit of measured variable. [Regarding the units on controller gain, if the actual controller of Fig. 9–4 is considered, both the input and the output units are in milliamperes. In this case the gain will be dimensionless (i.e., mA/mA).]

Note: the larger we make the proportionality constant for the proportional controller (called the controller gain), the smaller the steady-state error will become.

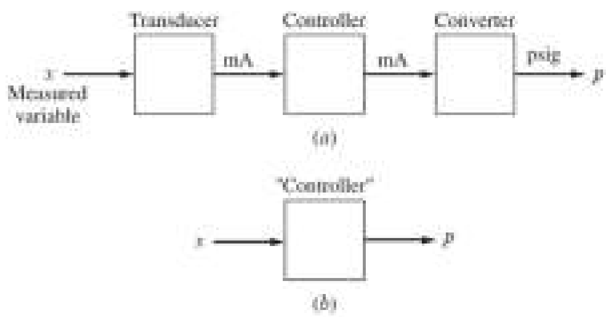
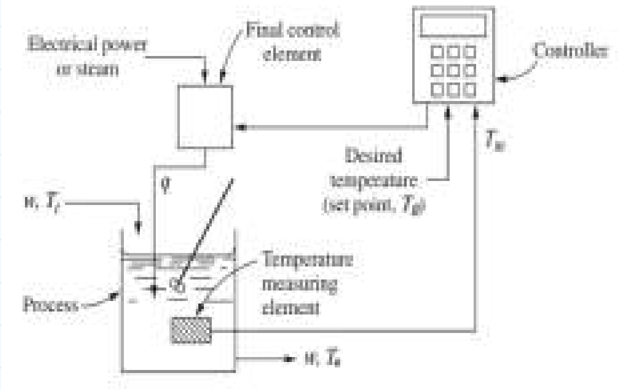


FIGURE 9–4
Equivalent block for transducer, controller, and converter.

The offset in proportional controller

Offset: It is the steady-state value of the error. In other word, the offset represents the difference between the new steady-state value and the original value (the set point)

It is impossible to completely eliminate the error through the use of a proportional controller. For example, if the set point is 80°C and a disturbance occurs that drops the temperature to 70°C, if we use only a proportional controller, then we will never be able to get the tank temperature to exactly 80°C. Once the sytem stabilizes again, the temperature will not be exactly 80°C, but perhaps 77°C or 83°C. There will always be some residual steady state error (called offset). For a home water heater, this is probably good enough; the exact temperature is not that critical. In an industrial process, this may not be adequate, and we have to resort to a bit more complicated controller to drive the error to zero.



How to calculate the offset mathematically?

1- For the Regulator closed loop system, the offset can be calculated as follows:

Offset = final value of Controlled variable.

$$\text{offset} = \lim_{t \to \infty} y(t)$$

or

$$\text{offset} = \lim_{s \to 0} s\, y(s)$$

2. For the servo closed loop, the offset can be calculated as follows:

Offset = change in set Point − final value of controlled variable

$$\text{offset} = \Delta \text{ set point} - \lim_{t \to \infty} y(t)$$

or

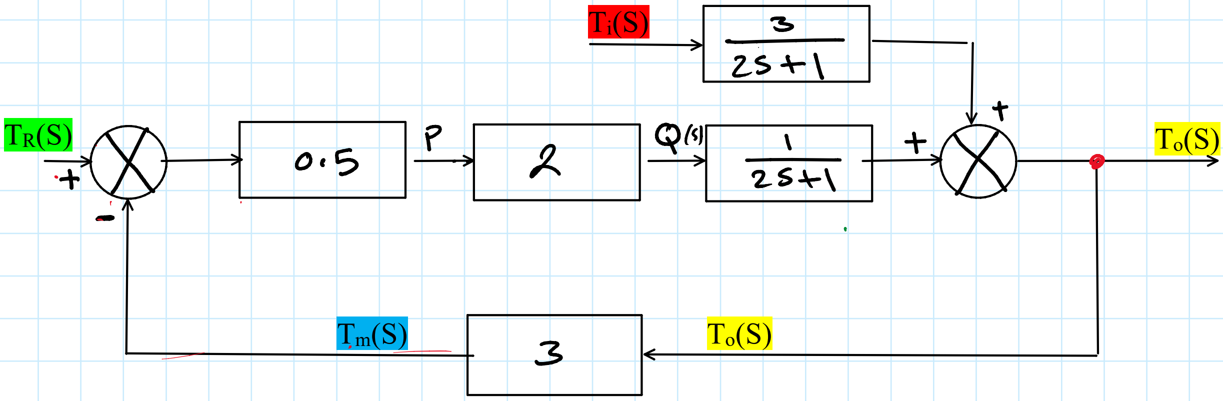
$$\text{offset} = \Delta \text{ set point} - \lim_{s \to 0} y(s)$$

Example 2: For the below signal flow block diagram closed-loop system, find the following:

1. For a unit step change in T_i, find the following:
a. time constant (τ)
b. the final value of response
c. the offset

d. sketch the response

2. Repeat the (1) when the value of K_c=1 and K_c=2.
3. What is the effect of increasing the value of controller gain (K_c) on time constant and the offset?



Solution:-

$$\frac{T_o(s)}{T_i(s)} = \frac{G_L}{1 + G_c G_v G_p G_m} = \frac{\frac{3}{2s+1}}{1 + (0.5)(2)\left(\frac{1}{2s+1}\right)(3)}$$

$$= \frac{\frac{3}{2s+1}}{1 + \frac{3}{2s+1}}$$

$$= \frac{\frac{3}{\cancel{2s+1}}}{\frac{2s+1+3}{\cancel{2s+1}}}$$

$$= \frac{3}{2s+4} = \frac{3}{4(0.5s+1)}$$

$$\left.\begin{aligned} \frac{T_o(s)}{T_i(s)} &= \frac{0.75}{0.5s+1} \\ &\quad \frac{k}{\tau s+1} \end{aligned}\right\} \text{ by comparison}$$

$$\therefore \quad \tau = 0.5$$

since

$$\frac{T_o(s)}{T_i(s)} = \frac{0.75}{0.5s+1}$$

$$T_o(s) = \frac{0.75}{0.5s+1} \cdot T_i(s)$$

for a unit step change in T_i

$$T_i(s) = \frac{1}{s}$$ sub. into the above eq.

$\therefore$

$$T_o(s) = \frac{0.75}{(0.5s+1)} * \frac{1}{s}$$

$$T_o(s) = \frac{0.75}{s(0.5s+1)}$$

$$Y(t) = KA\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$Y(t) = 0.75(1)\left[1 - e^{-\frac{t}{0.5}}\right]$$

$$Y(t) = 0.75\left[1 - e^{-2t}\right]$$

By taking Laplace inverse for above equation

$$T_o(t) = 0.75\left[1 - e^{-2t}\right]$$

To find the final value of response, we can use the following equation:

$$U.V. = \text{ultimate value} = \lim_{t \to \infty} Y(t) = \lim_{t \to 0} 0.75\left[1 - e^{-2t}\right]$$

$$T_o(\infty) = 0.75\left[1 - e^{-\infty}\right] = 0.75$$

Finding offset

since

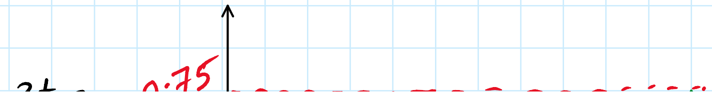
$$\text{offset} = \lim_{t \to \infty} y(t)$$

or

$$\text{offset} = \lim_{s \to 0} s\,y(s)$$

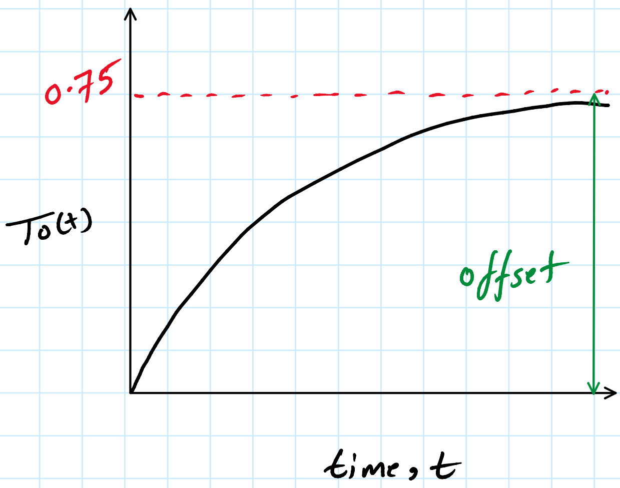
$$\therefore \quad \text{offset} = \lim_{t \to \infty} T_o(\infty) = \lim_{t \to \infty}\left(0.75\left[1 - e^{-2t}\right]\right) = 0.75$$

Sketching the response



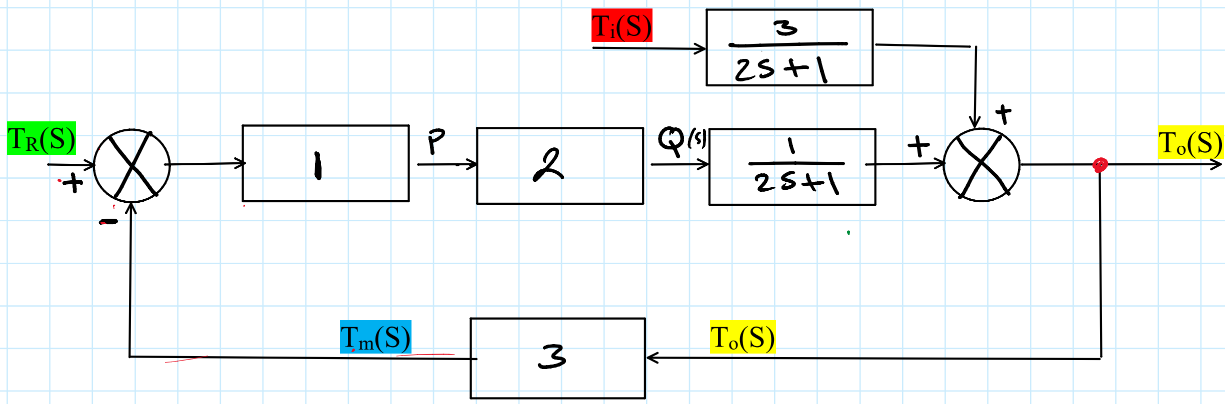
Sketching the response

$$T_o(t) = 0.75\left[1 - e^{-2t}\right]$$



Hence, for $K_c = 0.5 \rightarrow \tau = 0.5$ offset $= 0.75$

now what about $K_c = 1$



$$\frac{T_o(s)}{T_i(s)} = \frac{G_L}{1 + G_c G_v G_p G_m} = \frac{\frac{3}{2s+1}}{1 + (1)(2)\left(\frac{1}{2s+1}\right)(3)}$$

$$= \frac{\frac{3}{2s+1}}{1 + \frac{6}{2s+1}}$$

$$= \frac{\frac{3}{\cancel{2s+1}}\ 1}{\frac{2s+1+6}{\cancel{2s+1}}\ 1}$$

$$= \frac{3}{2s+7} = \frac{3}{7(0.29s+1)}$$

$$\frac{T_o(s)}{T_i(s)} = \frac{0.43}{0.29s+1}$$? by comparsion

$$\frac{T_o(s)}{T_i(s)} = \frac{0.43}{0.29s+1} \quad \left.\frac{K}{\tau s+1}\right\} \text{ by comparsion}$$

$\therefore \quad \tau = 0.29$

Since

$$\frac{T_o(s)}{T_i(s)} = \frac{0.43}{0.29s+1}$$

$$T_o(s) = \frac{0.43}{0.29s+1} T_i(s)$$

for a unit step change $T_i(s) = \frac{1}{s}$

$\therefore$

$$T_o(s) = \frac{0.43}{0.29s+1} \cdot \frac{1}{s}$$

$$T_o(s) = \frac{0.43}{s(0.29s+1)}$$

since the response for 1st order transfer when Input as step change

$$y(t) = KA\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$T_o(t) = 0.43\left[1 - e^{-\frac{t}{0.29}}\right]$$

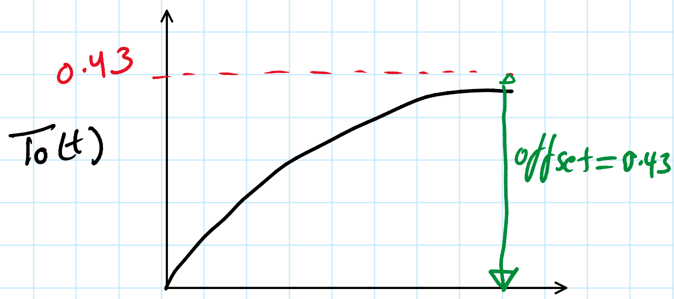
By taking Laplace inverse for the above equation, we obtain

$$T_o(t) = 0.43\left[1 - e^{-\frac{t}{0.29}}\right]$$

final value of response $= T_o(\infty) = \lim_{t\to\infty} 0.43\left[1 - e^{-\frac{\infty}{0.29}}\right] = 0.43$

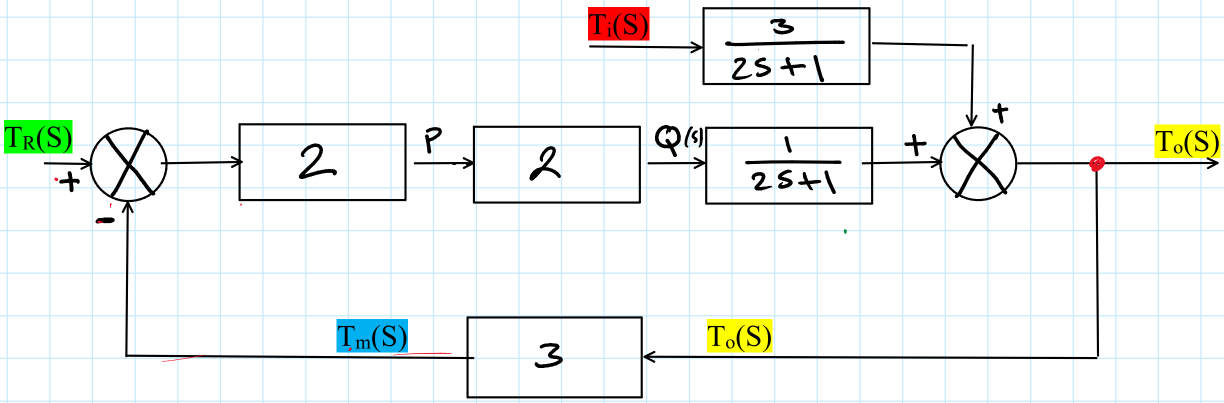
offset $= T_o(\infty) = 0.43$

$\therefore$ for $K_c = 1 \Rightarrow \tau = 0.29$, offset $= 0.43$



t

now for $K_c = 2 \Rightarrow \tau = ?$ offset $= ?$



$$\frac{T_o(s)}{T_i(s)} = \frac{G_L}{1+G_c G_v G_p G_m} = \frac{\frac{3}{2s+1}}{1+(2)(2)\left(\frac{1}{2s+1}\right)(3)}$$

$$= \frac{\frac{3}{2s+1}}{1+\frac{12}{2s+1}}$$

$$= \frac{\frac{3}{\cancel{2s+1}}}{\frac{2s+1+12}{\cancel{2s+1}}}$$

$$= \frac{3}{2s+13} = \frac{3}{13(0.15s+1)}$$

$$\frac{T_o(s)}{T_i(s)} = \frac{0.23}{0.15s+1} \Bigg\} \text{by comparsion}$$

$$\frac{K}{\tau s+1}$$

$\therefore \tau = 0.15$

Since $\dfrac{T_o(s)}{T_i(s)} = \dfrac{0.23}{0.15s+1}$

$$T_o(s) = \frac{0.23}{0.15s+1} T_i(s)$$

for a unit step change $T_i(s) = \frac{1}{s}$

for a unit step change $T_i(s) = \frac{1}{s}$

$$\therefore \quad T_o(s) = \frac{0.23}{0.15\,s+1} * \frac{1}{s}$$

$$T_o(s) = \frac{0.23}{s\,(0.15\,s+1)}$$

By taking Laplace inverse for the above equation, we obtain

$$T_o(t) = 0.23\left[1 - e^{-\frac{t}{0.15}}\right]$$

Since the response for 1st order transfer when Input as step change

$$y(t) = KA\left[1 - e^{-\frac{t}{\tau}}\right]$$

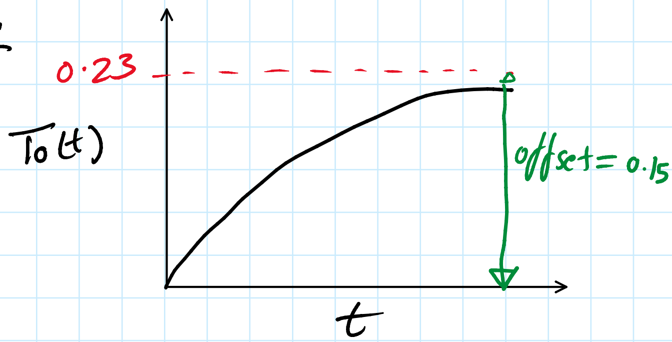
$$T_o(t) = 0.23\left[1 - e^{-\frac{t}{0.15}}\right]$$

final value of response $= T_o(\infty) = \lim_{t \to \infty} 0.23\left[1 - e^{-\frac{\infty}{0.15}}\right] = 0.23$

offset $= T_o(\infty) = 0.23$

$\therefore$ for $K_c = 2 \Rightarrow \tau = 0.15$, offset $= 0.23$

Table: Summary of the effect of Kc on τ + offset



Kc	τ	offset
0.5	0.5	0.75
1	0.29	0.43
2	0.15	0.23

faster response ✓ ok

decrease offset ✓ ok

Note: According to the results presented in the above table, one can notice that as the value of gain (Kc) increases, the values of time constant and offset decrease, which is preferred in the control system.

2. ON/OFF Controller

A special case of proportional control is on/off control. It considers a discontinuous controller.
If the gain Kc is made very high, the valve will move from one extreme position to another if the process deviates only slightly from the set point. This very sensitive action is called the on/off action because the valve is either fully open (on) or fully closed (off); i.e., the valve acts as a switch. This is a very simple controller and is exemplified by the thermostat used in a home-heating system, refrigerator, Iron and so on.

✓ The thermostat on the water heater is called an "on/off" type of controller. Depending on the value of the error signal, the output from the controller is either "full-on" or "full off," and the fuel valve is fully open or fully closed; there are no intermediate values of the output.