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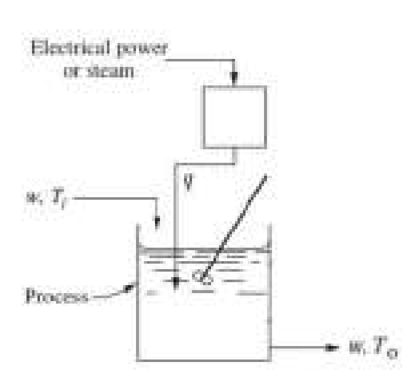
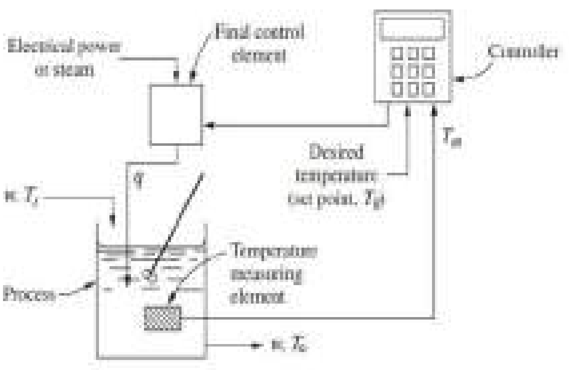
Development of Block Diagram

Each block in Fig. 8–2 represents the functional relationship existing between the input and output of a particular component. In previous chapters, such input-output relations were developed in the form of transfer functions. In block diagram representations of control systems, the variables selected are *deviation variables,* and inside each block is placed the transfer function relating the input-output pair of variables. Finally, the blocks are combined to give the overall block diagram. This is the procedure to be followed in developing Fig. 8–2.

1. Process

Consider first the block for the process. This block will be seen to differ somewhat from those presented in previous chapters in that two input variables are present (Ti , q); however, the procedure for developing the transfer function remains the same. Consider constant flow rate inpuut w (mass/time) in your solution





Energy balance at unsteady state around heating tank

$$\begin{bmatrix}\text{Rate of enthalpy} \\ \text{into heating vessel}\end{bmatrix} - \begin{bmatrix}\text{Rate of enthalpy} \\ \text{leaving heating vessel}\end{bmatrix} + \begin{bmatrix}\text{Rate of energy} \\ \text{input into heating vessel from heater}\end{bmatrix} = \begin{bmatrix}\text{Rate of accumlation} \\ \text{of enthalpy in heating vessel}\end{bmatrix}$$

$$w C_p (T_i - T_r) - w C_p (T_o - T_r) + q = \rho V C_p \frac{dT_o}{dt} \quad \text{——— (1)}$$

where :

T_r : reference temp. (25 °C)

Energy balance at steady state

$$\mp w C_p (T_{is} - T_r) \pm w C_p (T_{os} - T_r) \mp q_s = \mp \rho V C_p \frac{dT_{os}}{dt} = 0 \quad \text{——— (2)}$$

$$w C_p (T_i - T_{is}) - w C_p (T_o - T_{os}) + (q - q_s) = \rho V C_p \frac{d(T_o - T_{os})}{dt} \quad \text{——— (3)}$$

Let $\overline{\overline{T_i}} = T_i - T_{is}$

$\overline{\overline{T_o}} = T_o - T_{os}$ } sub. into Eq. (3)

Let $\bar{T}_i = T_i - T_{is}$

$\bar{T}_o = T_o - T_{os}$ } sub. into Eq. ③

$Q = q - q_s$

$$WC_p \bar{T}_i - WC_p \bar{T}_o + Q = \rho V C_p \frac{d\bar{T}_o}{dt}$$

$$\left[WC_p (\bar{T}_i - \bar{T}_o) + Q = \rho V C_p \frac{d\bar{T}_o}{dt} \right] \div \rho V C_p$$

$$\frac{W\cancel{C_p}}{\rho V \cancel{C_p}} (\bar{T}_i - \bar{T}_o) + \frac{1}{\rho V C_p} Q = \frac{d\bar{T}_o}{dt}$$

$$\frac{W}{\rho V} (\bar{T}_i - \bar{T}_o) + \frac{1}{\rho V C_p} Q = \frac{d\bar{T}_o}{dt}$$

$$\frac{d\bar{T}_o}{dt} = \frac{W}{\rho V} (\bar{T}_i - \bar{T}_o) + \frac{1}{\rho V C_p} Q$$ by taking Laplace inverse

$$s\bar{T}_o(s) - \cancel{\bar{T}(0)}^{\,0} = \frac{W}{\rho V} (\bar{T}_i(s) - \bar{T}_o(s)) + \frac{1}{\rho V C_p} Q(s)$$

$$s\bar{T}_o(s) = \frac{W}{\rho V} \bar{T}_i(s) - \frac{W}{\rho V} \bar{T}_o(s) + \frac{1}{\rho V C_p} Q(s)$$

$$s\bar{T}_o(s) + \frac{W}{\rho V} \bar{T}_o(s) = \frac{W}{\rho V} \bar{T}_i(s) + \frac{1}{\rho V C_p} Q(s)$$

$$\frac{W}{\rho V} \left(\frac{\rho V}{W} s \bar{T}_o(s) + \bar{T}_o(s) \right) = \frac{W}{\rho V} \bar{T}_i(s) + \frac{1}{\rho V C_p} Q(s)$$

$$\frac{\rho V}{W} s \bar{T}_o(s) + \bar{T}_o(s) = \frac{\rho V}{W} \left[\frac{W}{\rho V} \bar{T}_i(s) + \frac{1}{\rho V C_p} Q(s) \right]$$

$$\bar{T}_o(s) \left(\frac{\rho V}{W} s + 1 \right) = \bar{T}_i(s) + \frac{1}{WC_p} Q(s)$$

Since $\frac{\rho V}{W} = \frac{\frac{\cancel{kg}}{\cancel{m^3}} * \cancel{m^3}}{\frac{\cancel{kg}}{s}} \doteq \text{Sec} = \tau =$ time constant of the system

Therefore the above equation can be written as follows:

$$\bar{T}_o(s)(\tau s + 1) = \bar{T}_i(s) + \frac{1}{W} Q(s)$$

$$\boxed{\overline{T}_o(s) = \frac{1}{\tau s+1}\overline{T}_i(s) + \frac{1/wC_p}{\tau s+1} Q(s)}$$

Transfer function of the process

where

$\tau = \frac{\rho V}{w}$, time constant of the system

$$\frac{1}{wC_p} = \frac{1}{\frac{kg}{s}\cdot\frac{kJ}{kg\cdot C^\circ}} = \frac{^\circ C}{kJ/kg}$$, the gain for $Q(t) = K$

From the above transfer function, we can see that two independent quantities, the heater input $Q(s)$ and the inlet temperature $\overline{T}_i(s)$, can cause change in the outlet temperature.

- If there is a change in $Q(t)$ only, then $\overline{T}_i(t) = 0$ and the transfer function relating $\overline{T}_o$ to Q is

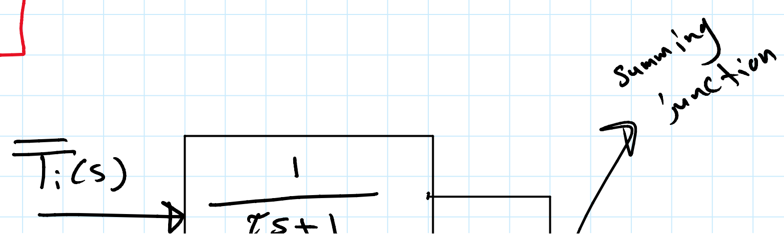
$$\overline{T}_o(s) = \frac{1}{\tau s+1}\cancelto{0}{\overline{T}_i(s)} + \frac{1/wC_p}{\tau s+1} Q(s)$$

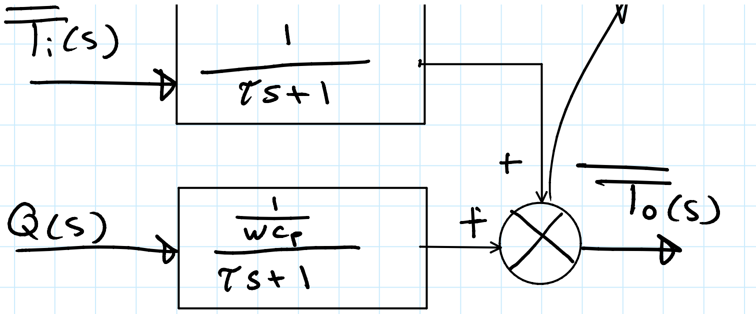
$$\boxed{\overline{T}_o(s) = \frac{1/wC_p}{\tau s+1} Q(s)}$$

- If there is a change $\overline{T}_i(t)$ only, then $Q(t) = 0$ and the transfer function relating $\overline{T}_o$ to $\overline{T}_i$

$$\overline{T}_o(s) = \frac{1}{\tau s+1}\overline{T}_i(s) + \cancelto{0}{\frac{1/wC_p}{\tau s+1} Q(s)}$$

$$\boxed{\overline{T}_o(s) = \frac{1}{\tau s+1}\overline{T}_i(s)}$$





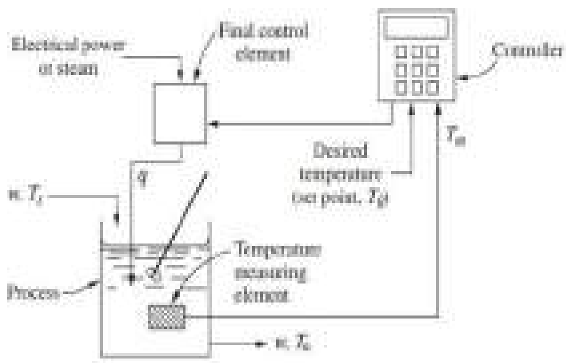
$1/wc_p = K$

$$\boxed{\overline{T}_o(s) = \frac{1}{\tau s+1}\overline{T}_i(s) + \frac{1/wc_p}{\tau s+1}Q(s)}$$

Q) According to the above transfer function which one is load and which one is process transfer function? why?

Solution : From the figure of closed loop system we can identify the transfer function of the load and the process

As we could see from the figure that final control element (i.e., control valve) is connected on q that means that transfer function with q is process transfer function.



2. Measuring Element

Temperature indicator

Pressure indicator

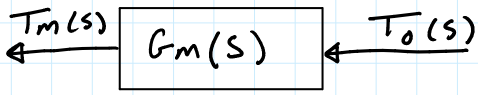
Flow rate indicator

Liquid level indicator

The temperature measuring element, which senses the bath temperature T_o and transmits a signal T_m to the controller (comparator), may exhibit some dynamic lag. From the discussion of the mercury thermometer in Chap. 4, we observed this lag to be first-order. In this example, we will assume that the temperature measuring element is a first-order system, for which the transfer function is

$$G_m(s) = \frac{T_m(s)}{\quad} = \frac{K_m}{\quad} \qquad (1)$$

$$G_m(s) = \frac{T_m(s)}{T_o(s)} = \frac{k_m}{\tau_m s + 1} \qquad (1)$$



where

k_m: the steady-state gain of measuring element

τ_m: the time constant or lag time of measuring element.

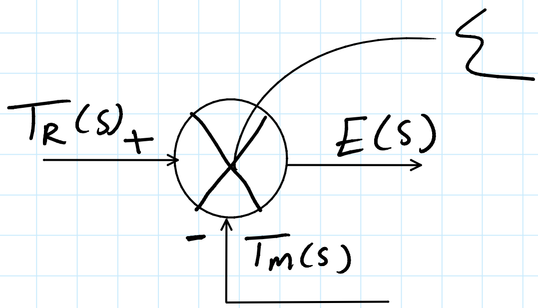
Note: Sometimes the time constant of measuring element (τ_m) is very small, Therefore, it can be neglected

$$G_m(s) = \frac{T_m(s)}{T_o(s)} = \frac{k_m}{\cancel{\tau_m}^{\,0} s + 1} = \frac{k_m}{1} = k_m$$

3. Comparator

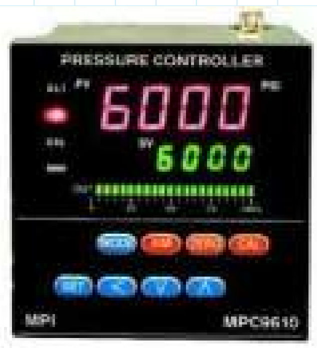
It is a part of controller and its action to compare between two signals; Set point (T_R) (i.e., desired value of output variable) and measuring element T_m

$$E(s) = T_R(s) - T_m(s)$$



4. Controller

Temperature controller



Pressure controller

Flow rate controller

Note:

$$E(s) \rightarrow \boxed{G_c(s)} \rightarrow P(s)$$

- The input to the controller is the error signal E(s).
- The output from the controller could be hydraulic, pneumatic, or an electric signal.

✓ The principle work of the controller is manipulating the error signal in such a way to reduce or eliminate the error signal.

There are many types of controllers that are used in industrial processes. The controllers can be classified according to their actions as follows:

1. Proportional controller (P)
2. Integral controller (I)
3. Derivative controller (D)
4. Proportional-derivative controller (PD)
5. Proportional-integral controller (PI)
6. Proportional-derivative-integral controller (PID)
7. On-off controller

Here, we consider a proportional controller, and the output signal is pneumatic P(s). Therefore, the transfer function of the controller G_c (s) will be expressed as follow:

$$G_c\,(s)=\frac{P(s)}{E(s)} = K_c$$

$$E(s) \rightarrow \boxed{G_c(s)} \rightarrow P(s)$$

5. Final control element (control valve)

Control valve

Solenoid valve

Variable transformer

The final control element could be a control valve (operates pneumatically), solenoid valve (operates electrically), or variable transformer (operates electrically).
Here we will consider control valve. Therefore, the input to the valve is pressure signal.
The output from the control valve will adjust the manipulating variable (i.e., heat input, Q)

$$G_v(s) = \frac{Q(s)}{P(s)} = \frac{K_v}{\tau_v s + 1} \quad \text{——— (1)}$$

$$P(s) \rightarrow \boxed{G_v(s)} \rightarrow Q(s)$$

where

K_v : the steady-state gain of final control element.

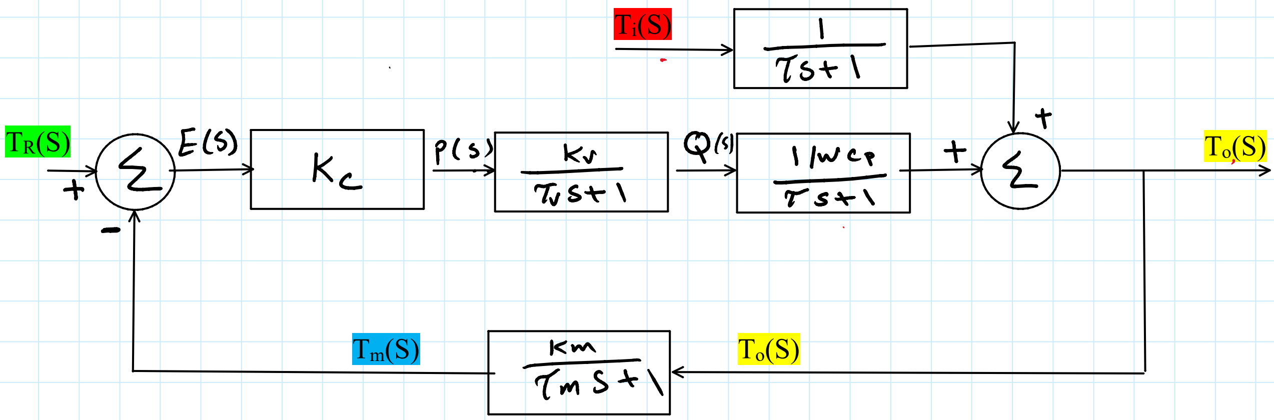
τ_v : the time constant or lag time of final control element.

Note: Sometimes the time constant of final control element (τ

Note: Sometimes the time constant of final control element (τ_v) is very small. Therefore, it can be neglected

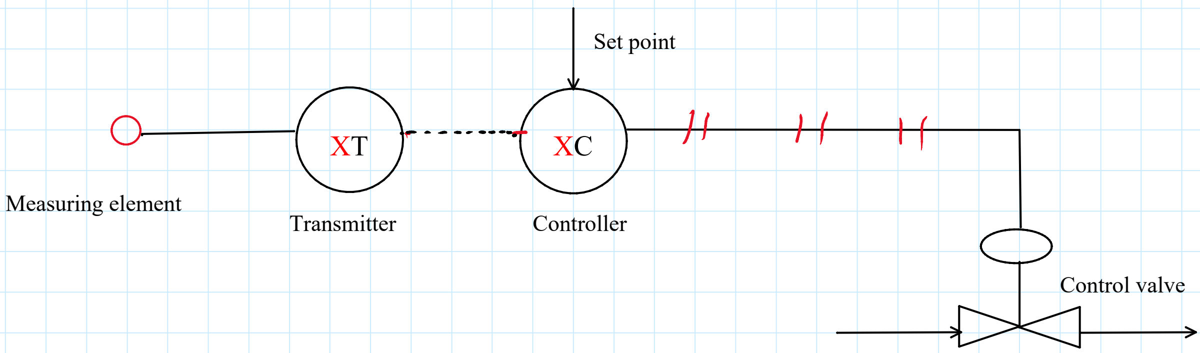
$$G_v(s) = \frac{Q(s)}{P(s)} = \frac{k_v}{\cancel{\tau_v s}^{\,0} + 1} = \frac{k_v}{1} = k_v$$

We have now completed the development of separate blocks. If these are combined according to Figure 8.2, we obtain the block diagram for complete control system as shown below.

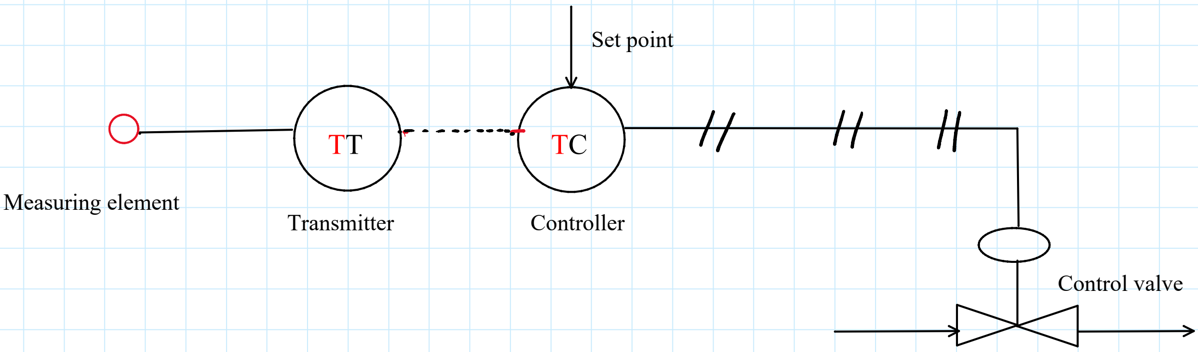


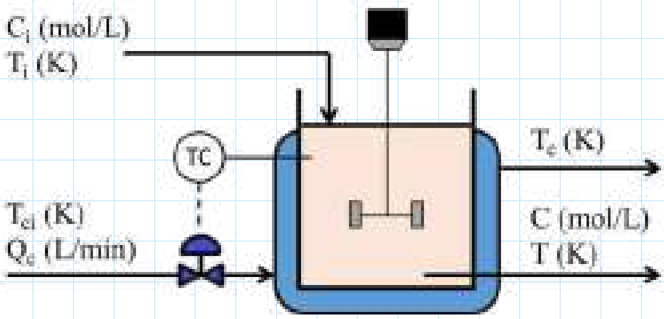
Global representation of a closed-loop system for different controlled variables

Generally, the closed-loop control system for any process can be expressed as shown in the figures:



X: The variable X could be Temperature (T), Flow (F), Pressure (P), level (L), concentration (C). For example, if we consider that want to represent colsed loop system for temperature control



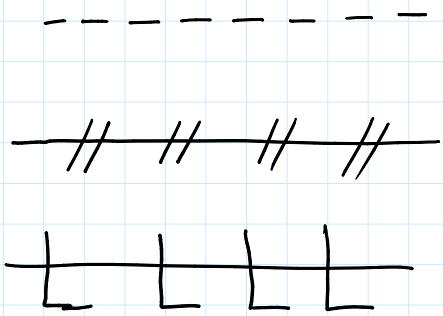


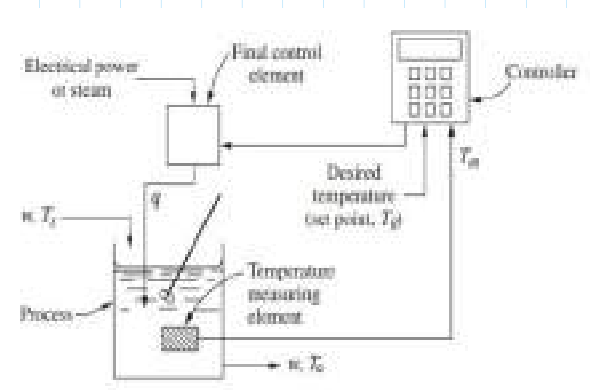
Note:

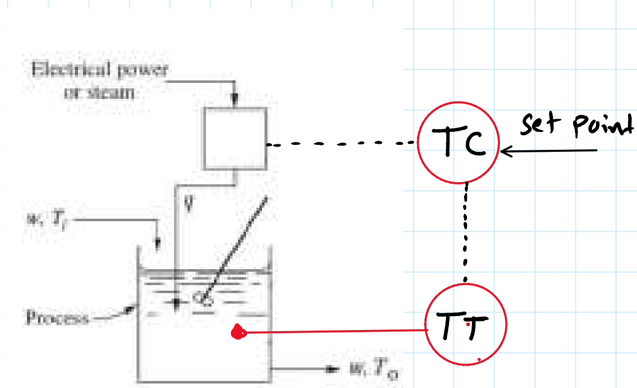
Electric signal

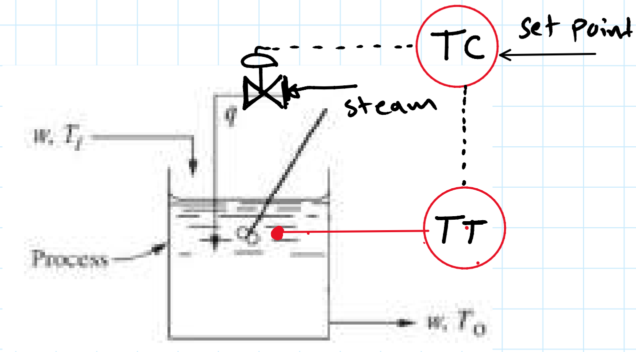
Pneumatic signal

Hydraulic signal

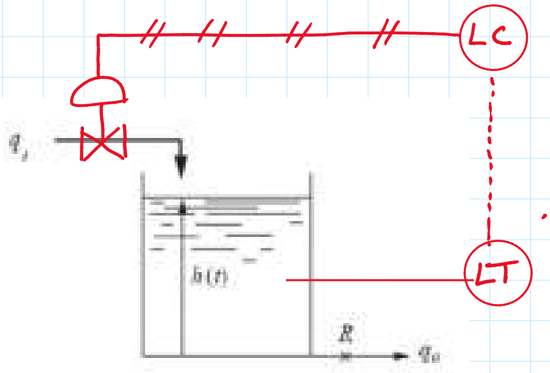








Example: For the liquid level system shown below find the following:
1. The transfer function of the process.
2. Sketch the signal flow block diagram.
3. The transfer function of $H(s)/H_{sp}(s)$.
4. The response of H if a unit step change occurs in set point and sketch it.
Use the following information in your solution:
$A=1\ m^2$, $R=2$, $q=10$ liter/minute, $V=5$ liters, $G_v=1.5$, $G_m=1$, $G_c=1$



Solution:

1. unsteady state mass balance around tank

In − out = Acc.

$$\rho q_i - \rho q_o = \frac{d\rho V}{dt} = \frac{d\rho A h}{dt}$$

since ρ & A are constants

$$\cancel{\rho}(q_i - q_o) = \cancel{\rho} A \frac{dh}{dt}$$

$$q_i - q_o = A\frac{dh}{dt}$$

Since

$$q_o = \frac{h}{R}$$

$$q_i - \frac{h}{R} = A\frac{dh}{dt} \qquad \text{—— (1)}$$

at st.st

$$\mp q_{is} \pm \frac{h_s}{R} = \mp A\frac{dh_s}{dt} = 0 \qquad \text{—— (2)}$$

$$(q_i - q_{is}) - \frac{(h-h_s)}{R} = A\frac{d(h-h_s)}{dt} \qquad \text{—— (3)}$$

Let

$$\left.\begin{aligned} Q &= q_i - q_{is} \\ H &= h - h_s \end{aligned}\right\} \text{ sub. into Eq. (3)}$$

$$Q - \frac{H}{R} = A\frac{dH}{dt}$$

$$\frac{RQ - H}{R} = A\frac{dH}{dt}$$

$$RA\frac{dH}{dt} = RQ - H$$

$$RA\frac{dH}{dt} + H = RQ$$

Since $\tau = RA = (2)(1) = 2$

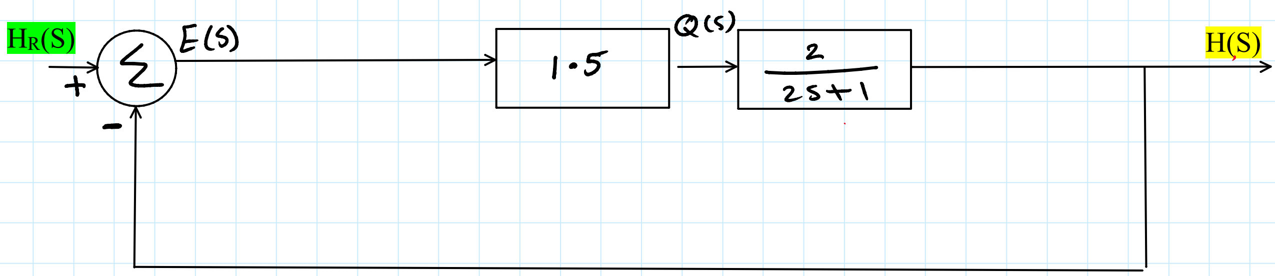
$$\tau\frac{dH}{dt} + H = RQ \quad \text{by taking Laplace transform for both sides}$$

$$\tau(sH(s) - \cancel{H(0)}^{\,0}) + H(s) = R\,Q(s)$$

$$H(s)(\tau s + 1) = R\,Q(s) \Rightarrow H(s) = \frac{1}{\tau s + 1}Q(s) \rightarrow$$ there is only process G_p, no load G_L exist

$$G_p(s) = \frac{H(s)}{Q(s)} = \frac{R}{\tau s + 1} = \frac{2}{2s + 1}$$

(2)



Note: Since $G_c = 1$ and $G_m = 1$, therefore they were not included in this block diagram.

(3) $\frac{H(s)}{H_R(s)} = ?$

$(1.5) \times \frac{2}{\ }$ $\quad \frac{3}{\ }$ $\quad \frac{3}{\ }$

③ $\dfrac{H(s)}{H_R(s)} = ?$

$$G(s) = \frac{H(s)}{H_R(s)} = \frac{(1.5)\times\frac{2}{2s+1}}{1+(1.5)\,\frac{2}{2s+1}} = \frac{\frac{3}{2s+1}}{1+\frac{3}{2s+1}} = \frac{\frac{3}{\cancel{2s+1}\;1}}{\frac{2s+1+3}{\cancel{2s+1}\;1}}$$

$$= \frac{3}{2s+4} = \frac{3}{4\left(\frac{2}{4}s+1\right)}$$

$$G(s) = \frac{H(s)}{H_R(s)} = \frac{\frac{3}{4}}{0.5s+1}$$

④ The response of *H* if a unit step change occurs in set point and sketch it.

Since $\dfrac{H(s)}{H_R(s)} = \dfrac{\frac{3}{4}}{0.5s+1}$

$H(s) = \dfrac{\frac{3}{4}}{0.5s+1}\,H_R(s)$, for unit step in set point $H_R(s) = \dfrac{1}{s}$

$H(s) = \dfrac{\frac{3}{4}}{0.5s+1} \times \dfrac{1}{s} = \dfrac{\frac{3}{4}}{s(0.5s+1)}$, by taking Laplace inverse

$H(t) = 0.75\left[1 - e^{-2t}\right]$