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Chapter Eight
The control system for Linear Closed-Loop Systems

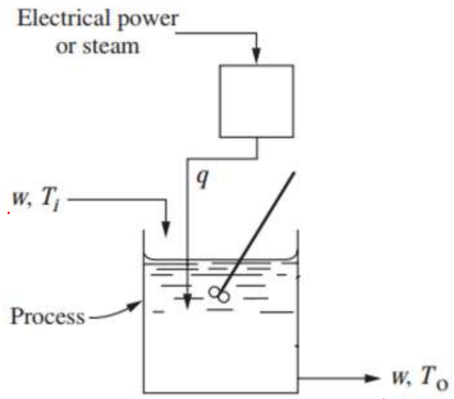
What the difference between the open-loop and closed-loop control system?

- An open-loop system is one in which the process is not controlled (i.e., manually control the process). In other words, T_o will be changed if any change occurs on T_i, q, W.

Variables of process

Input variables: T_i, q, W

Output variables: T_o

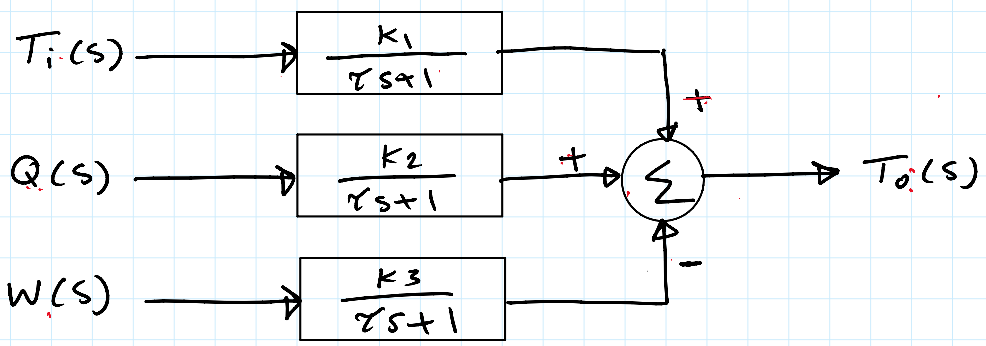


Transfer function for this open loop continuous stirred tank h

open loop system $T_o = f(T_i, q, W)$

$$T_o(s) = \frac{K_1}{\tau s+1} T_i(s) + \frac{K_2}{\tau s+1} Q(s) - \frac{K_3}{\tau s+1} W(s)$$

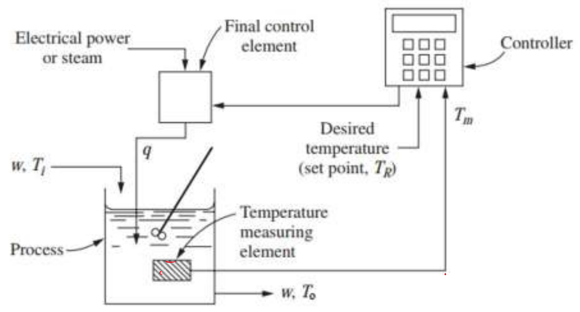
The signal flow Block diagram for system will be as shown below



- A closed-loop system is one in which the process is under control (i.e., automatic control). In other words, T_o will be remained constant if any change occurs on T_i, q, W.

Variables of process

- load variables: T_i, W
- Manipulating variable: Q
- Controlled variable: T_o



Components of a Control System

The system shown in Fig. 8–1 may be divided into the following components:

1. Process (stirred-tank heater).
2. Measuring element (thermocouple).
3. Controller.
4. Final control element (variable transformer or control valve).

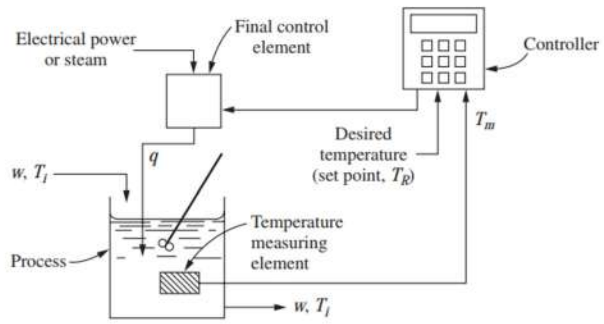
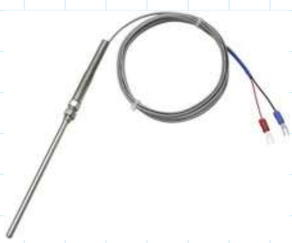


FIGURE 8–1
Control system for a stirred-tank heater.

Stirred-tank heater



Thermocouple

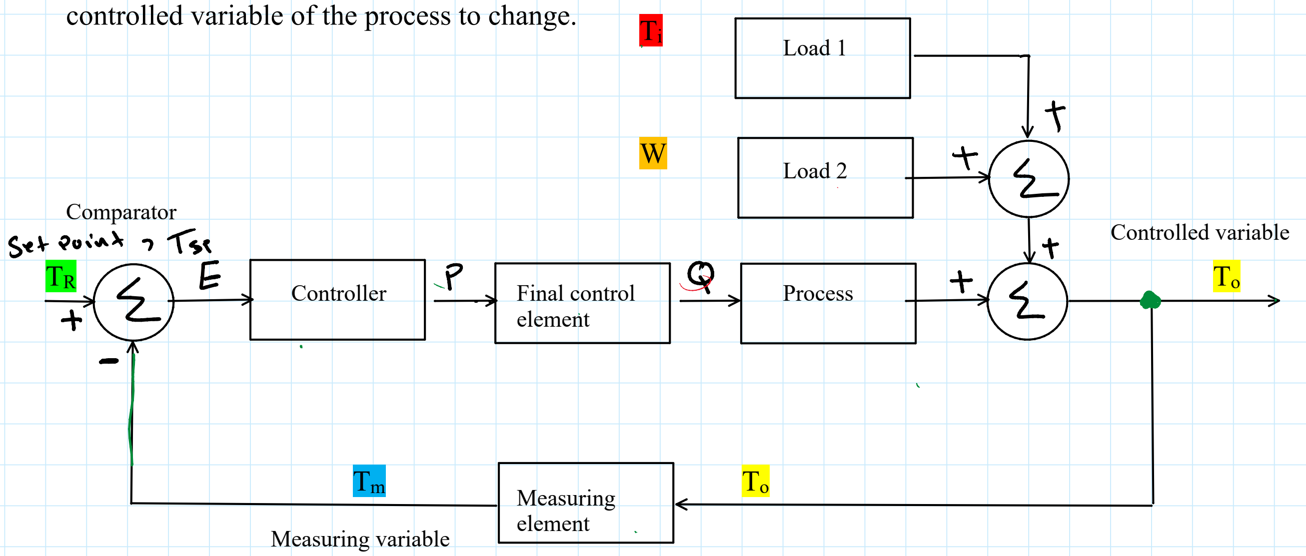
Controller

Variable transformer

Control valve

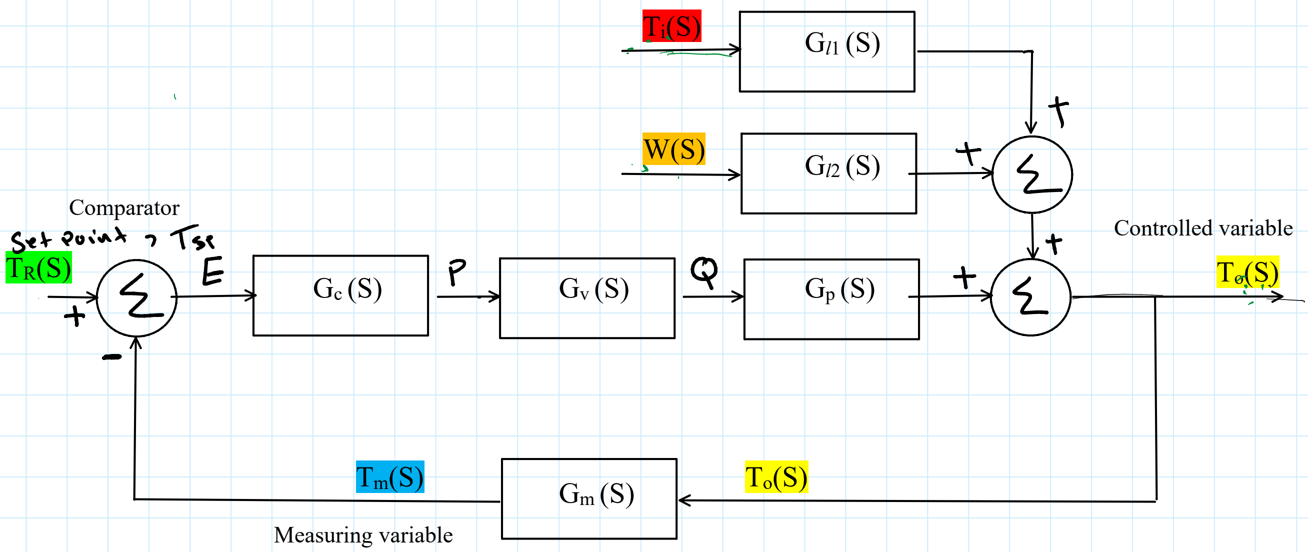
Signal flow block diagram for closed-loop control system

For computational purposes, it is convenient to represent the control system of Fig. 8–1 by means of the block diagram shown in Fig. 8–2. Such a diagram makes it much easier to visualize the relationships among the various signals. New terms, which appear in Fig. 8–2, are set point and load. The set point is a synonym for the desired value of the controlled variable. The load refers to a change in any variable that may cause the controlled variable of the process to change.



Trnsfer functions of closed loop control system

Let us replace the block diagram in terms of symbols



Where
G_c: Controller transfer function
G_v: Valve transfer function
G_p: Process transfer function
G_m: Measuring element transfer function
G_l: Load transfer function

$$G_1(s) = \frac{T_o(s)}{T_i(s)} = \frac{G_{L1}(s)}{1 + G_c(s)G_v(s)G_p(s)G_m(s)}$$

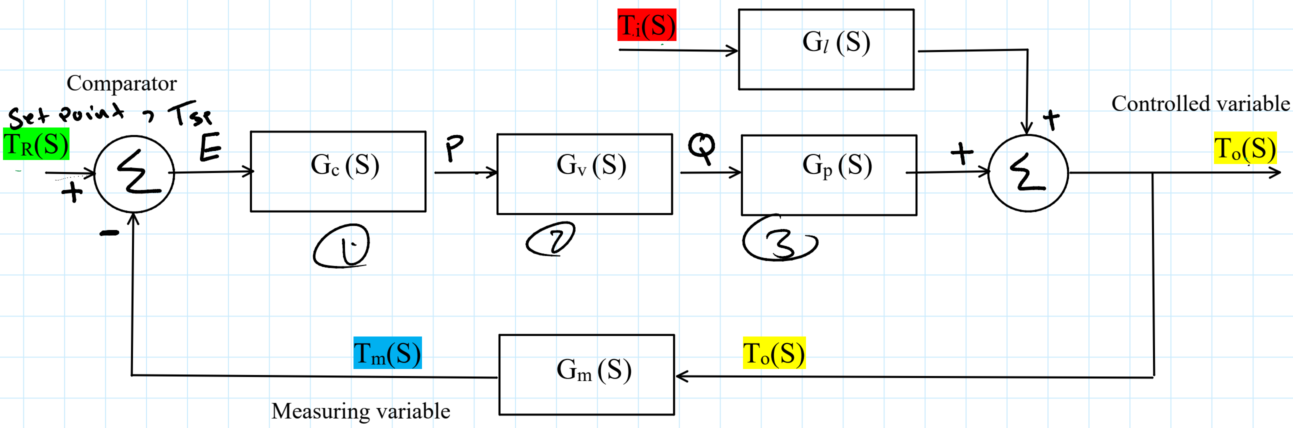
$$G_2(s) = \frac{T_o(s)}{W(s)} = \frac{G_{L2}(s)}{1 + G_c(s)\,G_v(s)\,G_p(s)\,G_m(s)}$$

$$G_3(s) = \frac{T_o(s)}{T_R(s)} = \frac{G_c(s)\, G_v(s)\, G_p(s)}{1 + G_c(s)\, G_v(s)\, G_p(s)\, G_m(s)}$$

Servo Problem Versus Regulator Problem

Regulator problem: It is a closed-loop system in which the load is variable, but the set point is constant.

Servo problem: It is a closed-loop system in which the load is constant, but the set point is variable.



For the regulator problem, the transfer function

$$G(s) = \frac{T_o(s)}{T_i(s)} = \frac{G_L(s)}{1 + G_c(s)\, G_v(s)\, G_p(s)\, G_m(s)}$$

For the Servo problem, the transfer function

$$G(s) = \frac{T_o(s)}{T_R(s)} = \frac{G_c(s)\, G_v(s)\, G_p(s)}{1 + G(s)\, G_v(s)\, G_p(s)\, G_m(s)}$$

Example 1: A closed loop control system consists of the following transfer functions:

* Process $T_o(s) = f(G_L(s), Q(s))$ where $T_i(s)$ is load variable and $Q(s)$ is manipulating variable.

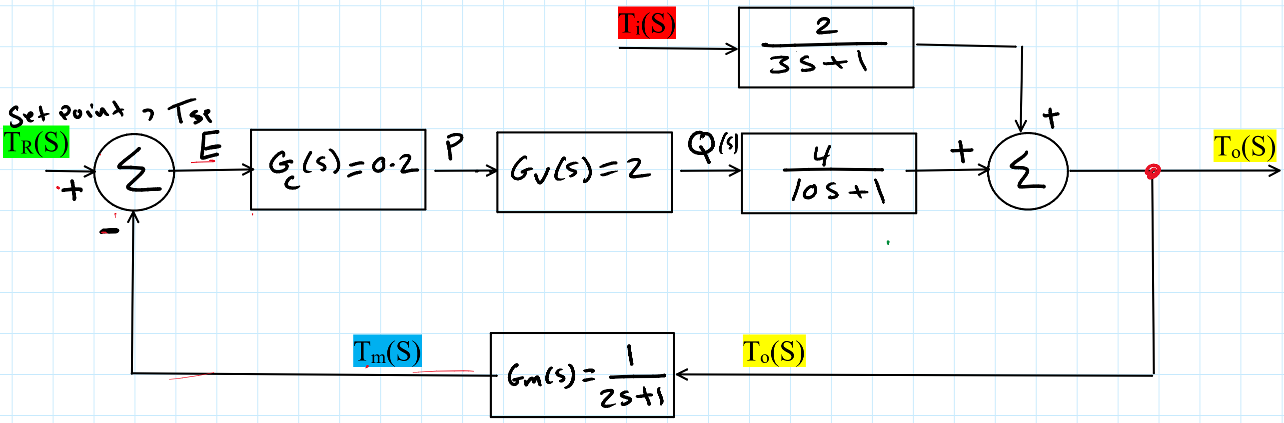
$$T_o(s) = \frac{2}{3s+1} T_i(s) + \frac{4}{10s+1} Q(s)$$

* Controller transfer function: $G_c(s) = 0.2$
* Final control element (i.e., valve) $= G_v(s) = 2$
* measurment transfer unction $G_m(s) = \frac{1}{}$

* Final control element (i.e., valve) = $G_v(s) = 2$

* measurment transfer unction $Gm(s) = \frac{1}{2s+1}$

Develop the signal flow block diagram for the above information



for regulator problem

$$G(s) = \frac{To(s)}{Ti(s)} = \frac{G_L(s)}{1 + G_c(s)\, G_v(s)\, G_p(s)\, Gm(s)}$$

$$G(s) = \frac{To(s)}{T_i(s)} = \frac{\frac{2}{3s+1}}{1 + (0.2)(2)\left(\frac{4}{10s+1}\right)\left(\frac{1}{2s+1}\right)}$$

$$G(s) = \frac{T_o(s)}{T_i(s)} = \frac{\frac{2}{3s+1}}{1 + \frac{1.6}{(10s+1)(2s+1)}}$$

for Servo problem

$$G(s) = \frac{To(s)}{TR(s)} = \frac{G_c(s)\, G_v(s)\, G_p(s)}{1 + G_c(s)\, G_v(s)\, G_p(s)\, G_m(s)}$$

$$G(s) = \frac{T_o(s)}{T_R(s)} = \frac{(0.2)(2)\left(\frac{4}{10s+1}\right)}{1 + (0.2)(2)\left(\frac{4}{10s+1}\right)\left(\frac{1}{2s+1}\right)}$$

Example 2: A close loop system in which ouput variable θ_o is function of both load variable θ_L and set point θ_{sp}, i.e., $\theta_o = f(\theta_L, \theta_{sp})$.

The system has the following elements with their transfer

of both load variable θ_L and set point θ_{sp}, i.e., $\theta_o = f(\theta_L, \theta_{sp})$.
The system has the following elements with their transfer functions:

- process: $G_p(s) = \dfrac{3}{10s+1}$
- Load : $G_L(s) = \dfrac{1}{10s+1}$
- Measurment: $G_m(s) = 1$
- Control value: $G_v(s) = 1.5$
- controller : $G_c(s) = 2$

Find the following:

1- Draw the block diagram for this closed loop.

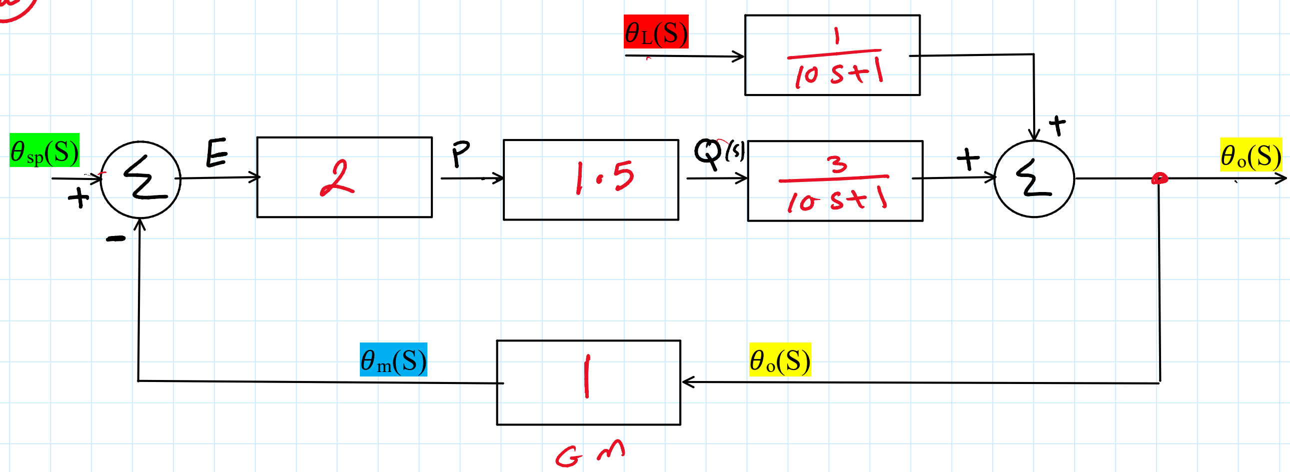
2. Find the transfer function if the system operates as servo.

3. Find the transfer function if the system operates as regulator.

4. Find the response, if a unit step change occurs in load and sketch it.

Solution:-

(a)



(b) Transfer function for Servo problem

$$G(s) = \frac{\theta_o(s)}{\theta_{sp}(s)} = \frac{G_c(s)\,G_v(s)\,G_p(s)}{1+G_c(s)\,G_v(s)\,G_p(s)\,G_m(s)} = \frac{(2)(1.5)\left(\frac{3}{10s+1}\right)}{1+(2)(1.5)\left(\frac{3}{10s+1}\right)(1)}$$

$\theta_o(s)$ $\quad \dfrac{9}{10s+1}$

$$\frac{\theta_o(s)}{\theta_{sp}(s)} = \frac{\frac{9}{10s+1}}{1+\frac{9}{10s+1}}$$

$$\frac{\theta_o(s)}{\theta_{sp}(s)} = \frac{\frac{9}{\cancel{10s+1}}}{\frac{10s+1+9}{\cancel{10s+1}}}$$

$$\frac{\theta_o(s)}{\theta_{sp}(s)} = \frac{9}{10s+10} = \frac{9}{10(s+1)}$$

$$= \frac{0.9}{s+1}$$

ⓒ Transfer function for Regular problem

$$G(s) = \frac{\theta_o(s)}{\theta_L(s)} = \frac{\frac{1}{10s+1}}{1+(2)(1.5)\left(\frac{3}{10s+1}\right)(1)} = \frac{\frac{1}{10s+1}}{1+\frac{9}{10s+1}}$$

$$\frac{\theta_o(s)}{\theta_L(s)} = \frac{\frac{1}{10s+1}}{\frac{10s+1+9}{10s+1}} = \frac{\frac{1}{\cancel{10s+1}}}{\frac{10s+10}{\cancel{10s+1}}}$$

$$\boxed{\frac{\theta_o(s)}{\theta_L(s)} = \frac{1}{10(s+1)} = \frac{0.1}{s+1}}$$

ⓓ For a unit step change in load

since $G(s) = \dfrac{\theta_o(s)}{\theta_L(s)} = \dfrac{0.1}{s+1}$

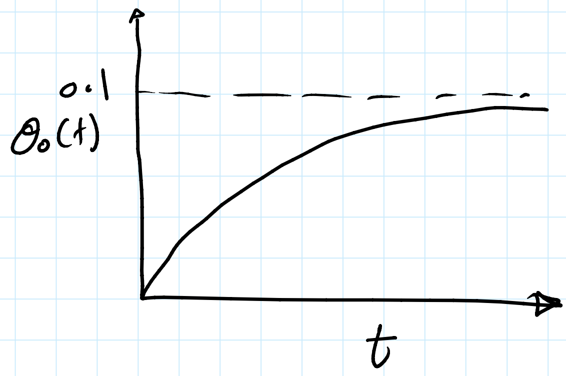
$$\theta_o(s) = \frac{0.1}{s+1} * \theta_L(s) \quad \text{———} \quad ①$$

Since $\theta_L(s) = \dfrac{1}{s}$ sub. into Eq. ①

$$\theta_o(s) = \frac{0.1}{s+1} * \frac{1}{s} = \frac{0.1}{s(s+1)}$$

$$\theta_o(s) = \frac{0.1}{s(s+1)}$$, by taking Laplace inverse for both sides

$$\theta_o(t) = 0.1(1 - e^{-t})$$



Important notes:

1- If $G_L(s) = G_p(s)$ then the signal Block diagram shown in figure 1 can be potted as displyed in figure 2.

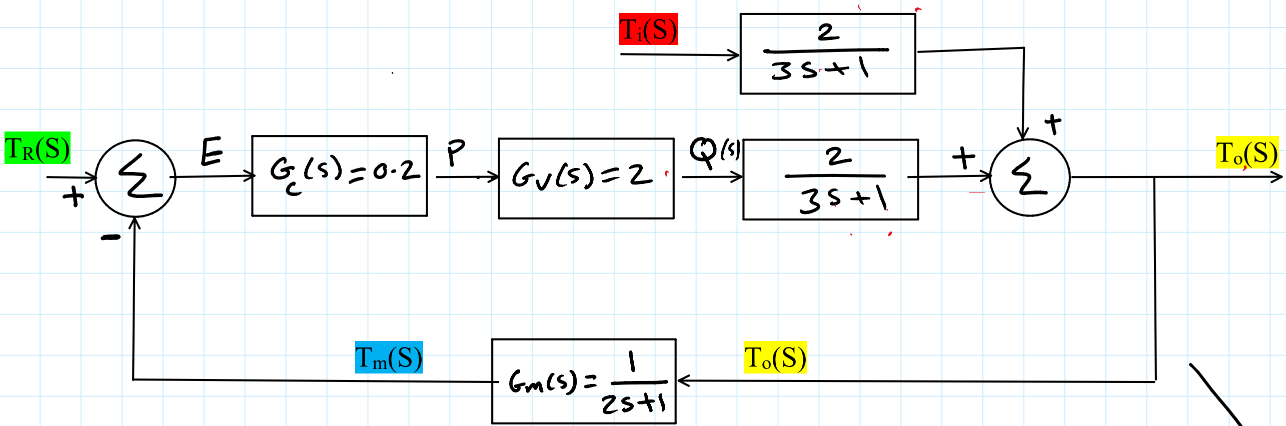
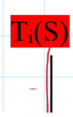


Figure 1

$$G(s) = \frac{T_o(s)}{T_i(s)} = \frac{G_L(s)}{1 + G_c(s)\,G_v(s)\,G_p(s)\,G_m(s)}$$

$$G(s) = \frac{\frac{2}{3s+1}}{1 + (0.2)(2)\left(\frac{2}{3s+1}\right)\left(\frac{1}{2s+1}\right)} = \frac{\frac{2}{3s+1}}{1 + \frac{0.8}{(2s+1)(3s+1)}}.$$



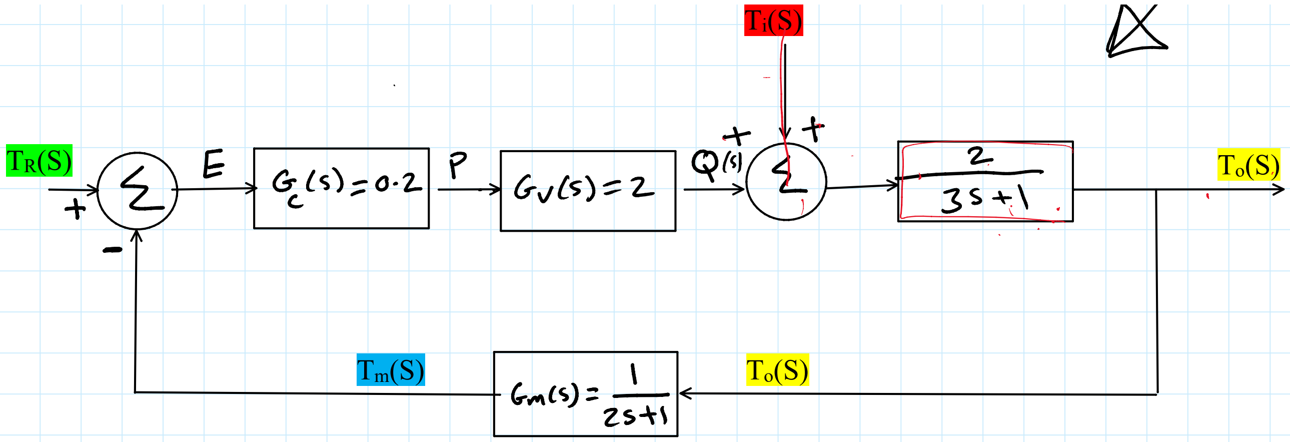


Figure 2

$$G(s)=\frac{T_o(s)}{T_i(s)}=\frac{\frac{2}{3s+1}}{1+(0.2)(2)\left(\frac{2}{3s+1}\right)\left(\frac{1}{2s+1}\right)}=\frac{\frac{2}{3s+1}}{1+\frac{0.8}{(2s+1)(3s+1)}}$$

2. If the denominators of $G_L(S)$ and $G_p(s)$ are equal, then the Block diagram of figure can be expressed as follows:

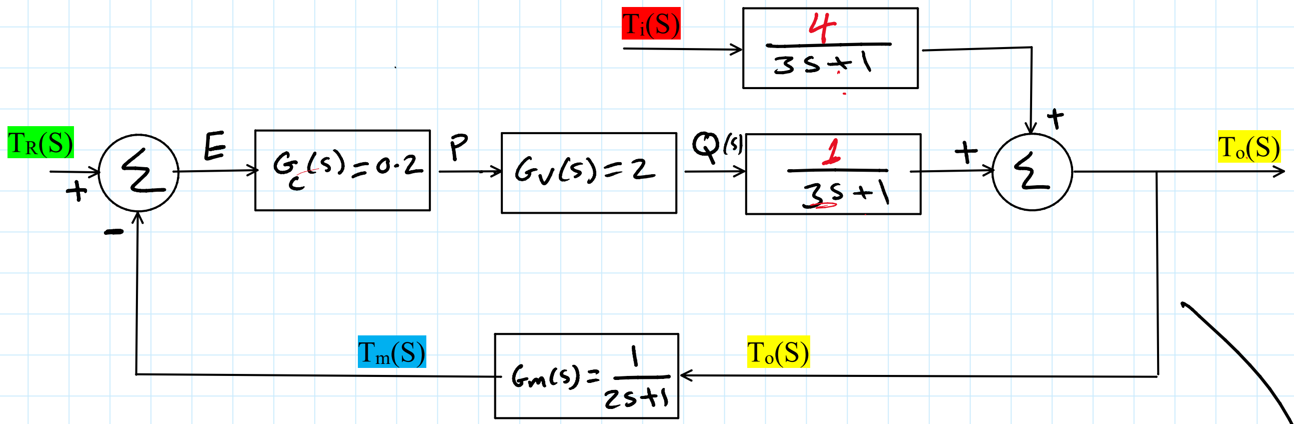
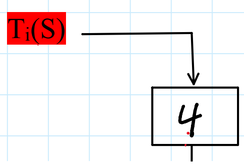


Figure 1

$$G(s)=\frac{T_o(s)}{T_i(s)}=\frac{G_L(s)}{1+G_c(s)\,G_v(s)\,G_p(s)\,G_m(s)}$$

$$G(s)=\frac{\frac{4}{3s+1}}{1+(0.2)(2)\left(\frac{2}{3s+1}\right)\left(\frac{1}{2s+1}\right)}=\frac{\frac{4}{3s+1}}{1+\frac{0.8}{(2s+1)(3s+1)}}$$



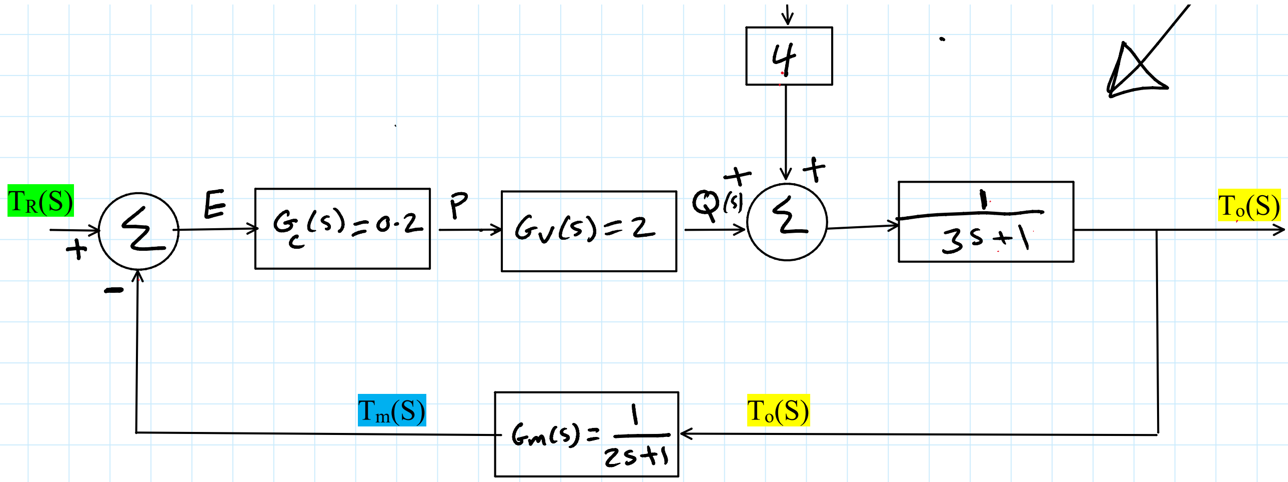


Figure 2

$$G(s)=\frac{(4)*\frac{1}{3s+1}}{1+(0.2)(2)\left(\frac{2}{3s+1}\right)\left(\frac{1}{2s+1}\right)}=\frac{\frac{4}{3s+1}}{1+\frac{0.8}{(2s+1)(3s+1)}}$$

3. If $G_m(s)=1$, the the block diagram of figure 1 can expressed as shown in figure 2.

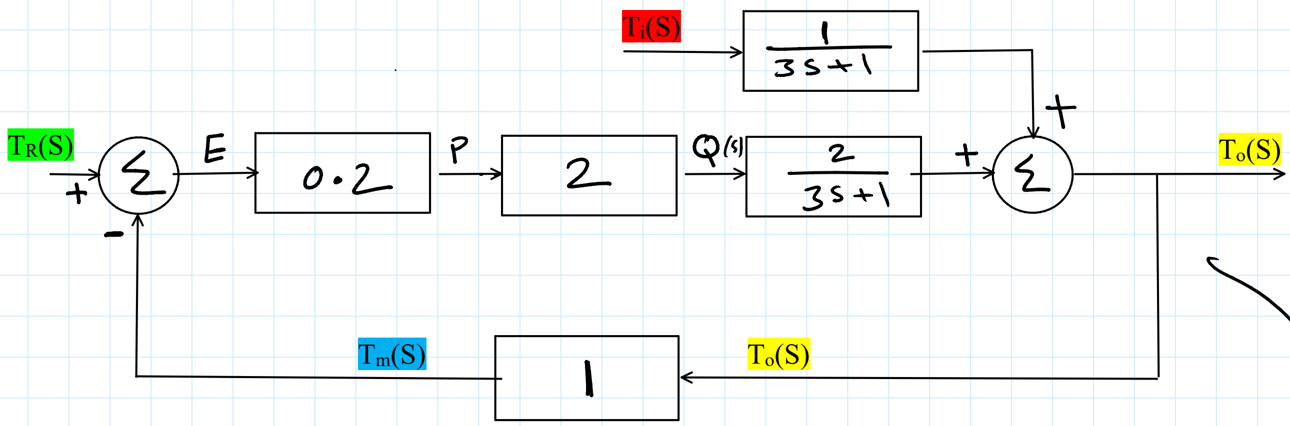
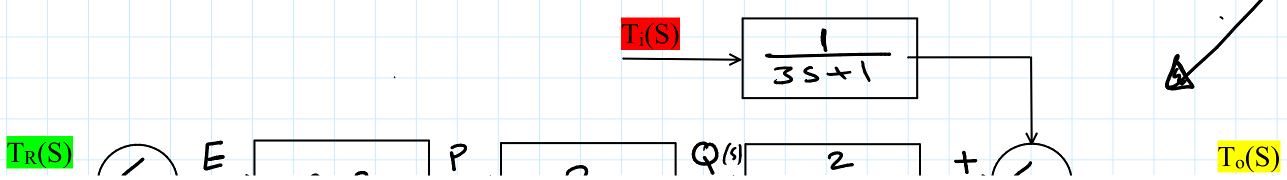
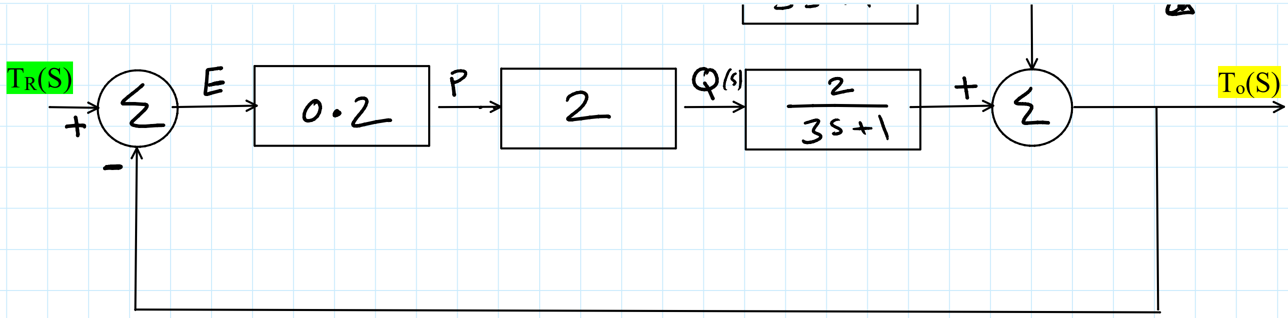


Figure 1

$$G(s)=\frac{T_o(s)}{T_i(s)}=\frac{G_1(s)}{1+G_c(s)\,G_v(s)\,G_p(s)\,G_m(s)}=\frac{\frac{1}{3s+1}}{1+(0.2)(2)\left(\frac{2}{3s+1}\right)(1)}$$

$$G(s)=\frac{T_o(s)}{T_i(s)}=\frac{\frac{1}{3s+1}}{1+\frac{0.8}{3s+1}}$$





$$G(s)=\frac{T_o(s)}{T_i(s)}=\frac{G_1(s)}{1+G_c(s)\,G_v(s)\,G_p(s)\,(1)}=\frac{\frac{1}{3s+1}}{1+(0.2)(2)\left(\frac{2}{3s+1}\right)}$$

$$G(s)=\frac{T_o(s)}{T_i(s)}=\frac{\frac{1}{3s+1}}{1+\frac{0.8}{3s+1}}$$

4. For calculation purposes, $G_c(s)$, $G_v(s)$ and $G_p(s)$ can be put as one block, as shown in below example.

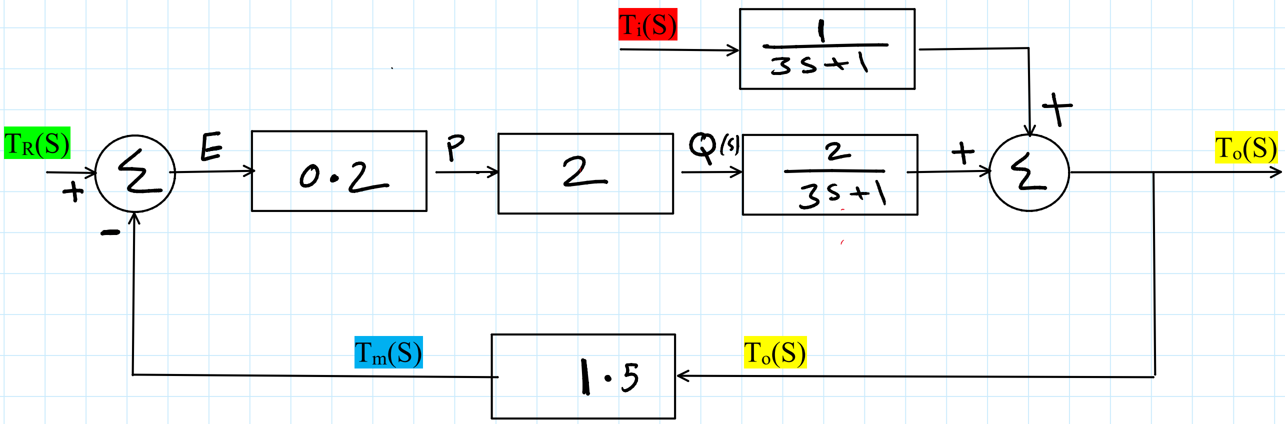
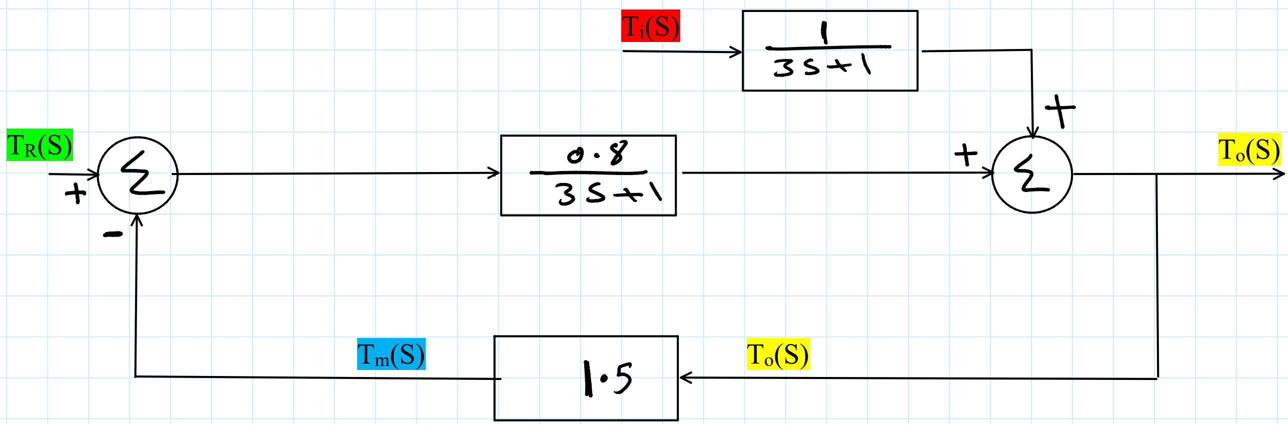


Figure 1



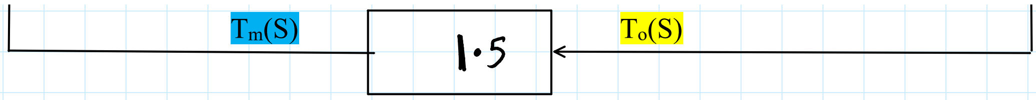


Figure 2