Al-Mustaqbal University

College of Engineering & Technology

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Lecture No.: 4

Lecture Title: [Moment]

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2/4 Moment

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*.

As a familiar example of the concept of moment, consider the pipe wrench of Fig. 2/8*a*. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude F of the force and the effective length d of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.

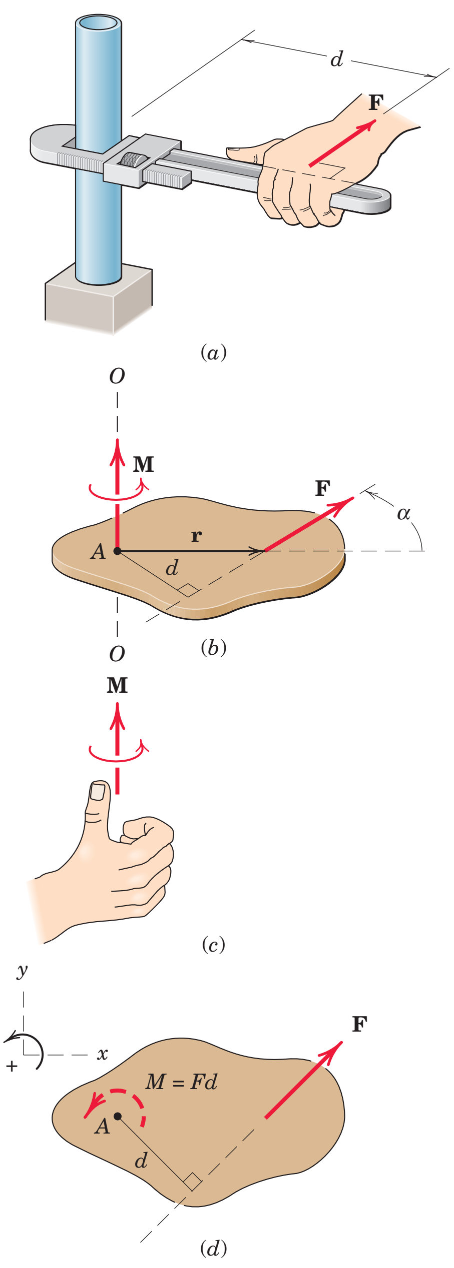


Figure 2/8

Moment about a Point

Figure 2/8*b* shows a two-dimensional body acted on by a force **F** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm d*, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$M = Fd \tag{2/5}$$

The moment is a vector **M** perpendicular to the plane of the body. The sense of **M** depends on the direction in which **F** tends to rotate the body. The right-hand rule, Fig. 2/8*c*, is used to identify this sense. We represent the moment of **F** about *O-O* as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency.

The moment **M** obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are newton-meters (N·m), and in the U.S. customary system are pound-feet (lb-ft).

When dealing with forces which all act in a given plane, we customarily speak of the moment *about a point*. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Thus, the moment of force **F** about point A in Fig. 2/8*d* has the magnitude $M = Fd$ and is counterclockwise.

Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (−) for clockwise moments, or vice versa. Sign consistency within a given problem is essential. For the sign convention of Fig. 2/8*d*, the moment of **F** about point A (or about the z-axis passing through point A) is positive. The curved arrow of the figure is a convenient way to represent moments in two-dimensional analysis.

The Cross Product

In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations. The moment of **F** about point *A* of Fig. 2/8*b* may be represented by the cross-product expression

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \tag{2/6}$$

where **r** is a position vector which runs from the moment reference point *A* to *any* point on the line of action of **F**. The magnitude of this expression is given by*

$$M = Fr \sin \alpha = Fd \tag{2/7}$$

which agrees with the moment magnitude as given by Eq. 2/5. Note that the moment arm $d = r \sin \alpha$ does not depend on the particular point on the line of action of **F** to which the vector **r** is directed. We establish the direction and sense of **M** by applying the right-hand rule to the sequence $\mathbf{r} \times \mathbf{F}$. If the fingers of the right hand are curled in the direction of rotation from the positive sense of **r** to the positive sense of **F**, then the thumb points in the positive sense of **M**.

We must maintain the sequence $\mathbf{r} \times \mathbf{F}$, because the sequence $\mathbf{F} \times \mathbf{r}$ would produce a vector with a sense opposite to that of the correct moment. As was the case with the scalar approach, the moment **M** may be thought of as the moment about point *A* or as the moment about the line *O-O* which passes through point *A* and is perpendicular to the plane containing the vectors **r** and **F**. When we evaluate the moment of a force about a given point, the choice between using the vector cross product or the scalar expression depends on how the geometry of the problem is specified. If we know or can easily determine the perpendicular distance between the line of action of the force and the moment center, then the scalar approach is generally simpler. If, however, **F** and **r** are not perpendicular and are easily expressible in vector notation, then the cross-product expression is often preferable.

In Section B of this chapter, we will see how the vector formulation of the moment of a force is especially useful for determining the moment of a force about a point in three-dimensional situations.

Varignon's Theorem

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

*See item 7 in Art. C/7 of Appendix C for additional information concerning the cross product.

To prove this theorem, consider the force **R** acting in the plane of the body shown in Fig. 2/9*a*. The forces **P** and **Q** represent any two nonrectangular components of **R**. The moment of **R** about point *O* is

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$, we may write

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

Using the distributive law for cross products, we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q} \qquad \textbf{(2/8)}$$

which says that the moment of **R** about *O* equals the sum of the moments about *O* of its components **P** and **Q**. This proves the theorem.

Varignon's theorem need not be restricted to the case of two components, but it applies equally well to three or more. Thus we could have used any number of concurrent components of **R** in the foregoing proof.*

Figure 2/9*b* illustrates the usefulness of Varignon's theorem. The moment of **R** about point *O* is *Rd*. However, if *d* is more difficult to determine than *p* and *q*, we can resolve **R** into the components **P** and **Q**, and compute the moment as

$$M_O = Rd = -pP + qQ$$

where we take the clockwise moment sense to be positive.

Sample Problem 2/5 shows how Varignon's theorem can help us to calculate moments.

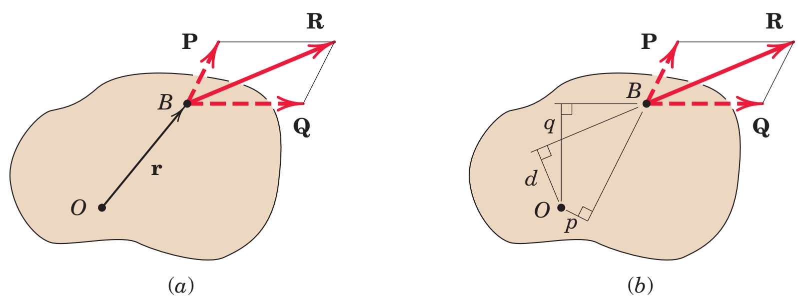
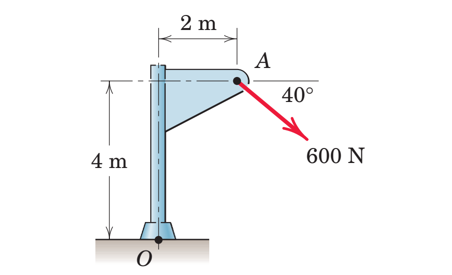


Figure 2/9

*As originally stated, Varignon's theorem was limited to the case of two concurrent components of a given force. See *The Science of Mechanics*, by Ernst Mach, originally published in 1883.

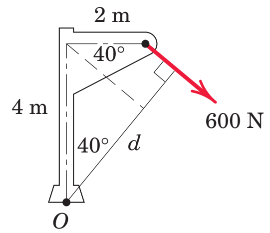
SAMPLE PROBLEM 2/5

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.



Solution. ***(I)*** The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

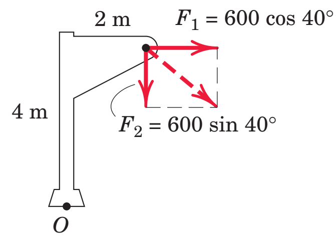


1 By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(II) Replace the force by its rectangular components at A,

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$



By Varignon's theorem, the moment becomes

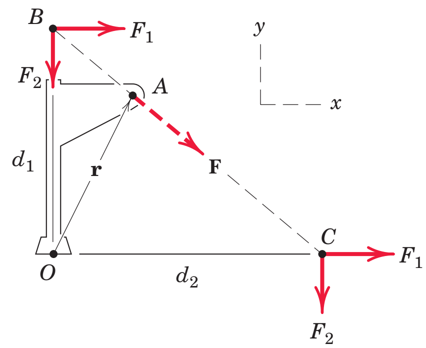
2 $$M_O = 460(4) + 386(2) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes

$$d_1 = 4 + 2 \tan 40° = 5.68 \text{ m}$$

and the moment is

$$M_O = 460(5.68) = 2610 \text{ N·m} \qquad \textit{Ans.}$$



3 ***(IV)*** Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$$d_2 = 2 + 4 \cot 40° = 6.77 \text{ m}$$

and the moment is

$$M_O = 386(6.77) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(V) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

4 $$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40° - \mathbf{j} \sin 40°)$$

$$= -2610\mathbf{k} \text{ N·m}$$

The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression is

$$M_O = 2610 \text{ N·m} \qquad \textit{Ans.}$$

Helpful Hints

1 The required geometry here and in similar problems should not cause difficulty if the sketch is carefully drawn.

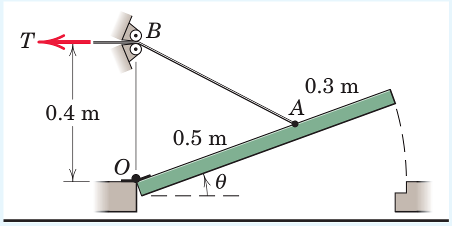
2 This procedure is frequently the shortest approach.

3 The fact that points B and C are not on the body proper should not cause concern, as the mathematical calculation of the moment of a force does not require that the force be on the body.

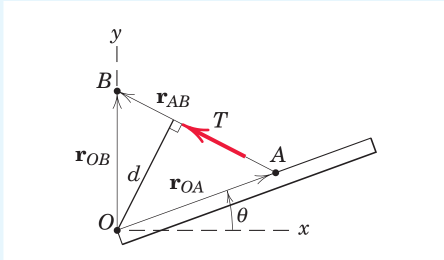
4 Alternative choices for the position vector $\mathbf{r}$ are $\mathbf{r} = d_1\mathbf{j} = 5.68\mathbf{j}$ m and $\mathbf{r} = d_2\mathbf{i} = 6.77\mathbf{i}$ m.

SAMPLE PROBLEM 2/6

The trap door OA is raised by the cable AB, which passes over the small frictionless guide pulleys at B. The tension everywhere in the cable is T, and this tension applied at A causes a moment M_O about the hinge at O. Plot the quantity M_O/T as a function of the door elevation angle θ over the range $0 \le \theta \le 90°$ and note minimum and maximum values. What is the physical significance of this ratio?



Solution. We begin by constructing a figure which shows the tension force **T** acting directly on the door, which is shown in an arbitrary angular position θ. It should be clear that the direction of **T** will vary as θ varies. In order to deal with this variation, we write a unit vector $\mathbf{n}_{AB}$ which "aims" **T**:



① $$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{\mathbf{r}_{OB} - \mathbf{r}_{OA}}{r_{AB}}$$

Using the x-y coordinates of our figure, we can write

② $$\mathbf{r}_{OB} = 0.4\mathbf{j} \text{ m and } \mathbf{r}_{OA} = 0.5(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) \text{ m}$$

So

$$\begin{aligned}\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA} &= 0.4\mathbf{j} - (0.5)(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})\\ &= -0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j} \text{ m}\end{aligned}$$

and

$$\begin{aligned}r_{AB} &= \sqrt{(0.5\cos\theta)^2 + (0.4 - 0.5\sin\theta)^2}\\ &= \sqrt{0.41 - 0.4\sin\theta} \text{ m}\end{aligned}$$

The desired unit vector is

$$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}$$

Our tension vector can now be written as

$$\mathbf{T} = T\mathbf{n}_{AB} = T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right]$$

③ The moment of **T** about point O, as a vector, is $\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{T}$, where $\mathbf{r}_{OB} = 0.4\mathbf{j}$ m, or

$$\begin{aligned}\mathbf{M}_O &= 0.4\mathbf{j} \times T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right]\\ &= \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}\mathbf{k}\end{aligned}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}$$

and the requested ratio is

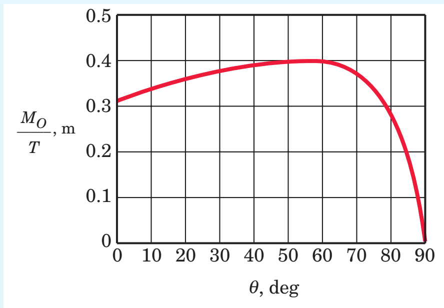
$$\frac{M_O}{T} = \frac{0.2\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}} \qquad \textit{Ans.}$$

which is plotted in the accompanying graph. The expression M_O/T is the moment arm d (in meters) which runs from O to the line of action of **T**. It has a maximum value of 0.4 m at $\theta = 53.1°$ (at which point **T** is horizontal) and a minimum value of 0 at $\theta = 90°$ (at which point **T** is vertical). The expression is valid even if T varies.

This sample problem treats moments in two-dimensional force systems, and it also points out the advantages of carrying out a solution for an arbitrary position, so that behavior over a range of positions can be examined.

Helpful Hints

① Recall that any unit vector can be written as a vector divided by its magnitude. In this case the vector in the numerator is a position vector.



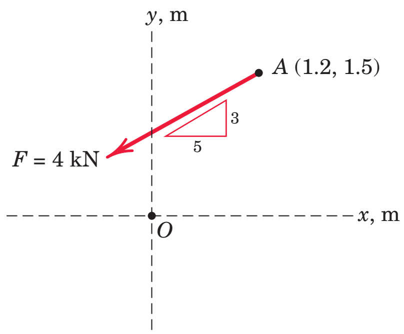
② Recall that any vector may be written as a magnitude times an "aiming" unit vector.

③ In the expression $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, the position vector $\mathbf{r}$ runs *from* the moment center *to* any point on the line of action of **F**. Here, $\mathbf{r}_{OB}$ is more convenient than $\mathbf{r}_{OA}$.

PROBLEMS

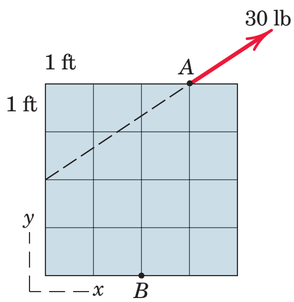
Introductory Problems

2/31 The 4-kN force **F** is applied at point A. Compute the moment of **F** about point O, expressing it both as a scalar and as a vector quantity. Determine the coordinates of the points on the x- and y-axes about which the moment of **F** is zero.



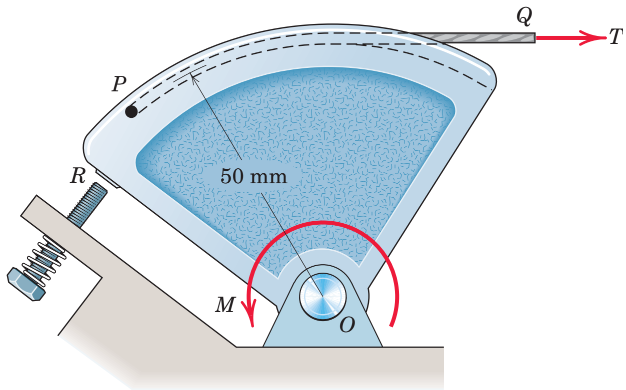
Problem 2/31

2/32 The rectangular plate is made up of 1-ft squares as shown. A 30-lb force is applied at point A in the direction shown. Calculate the moment M_B of the force about point B by at least two different methods.



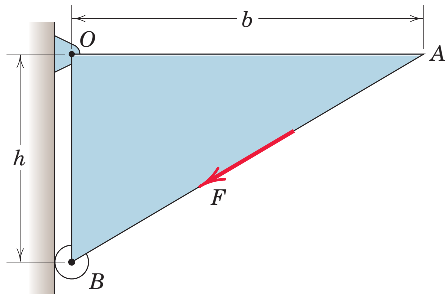
Problem 2/32

2/33 The throttle-control sector pivots freely at O. If an internal torsional spring exerts a return moment $M = 1.8$ N·m on the sector when in the position shown, for design purposes determine the necessary throttle-cable tension T so that the net moment about O is zero. Note that when T is zero, the sector rests against the idle-control adjustment screw at R.



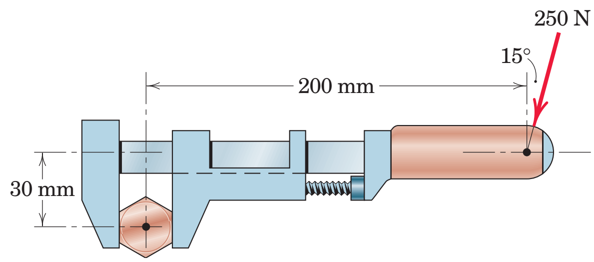
Problem 2/33

2/34 The force of magnitude F acts along the edge of the triangular plate. Determine the moment of **F** about point O.



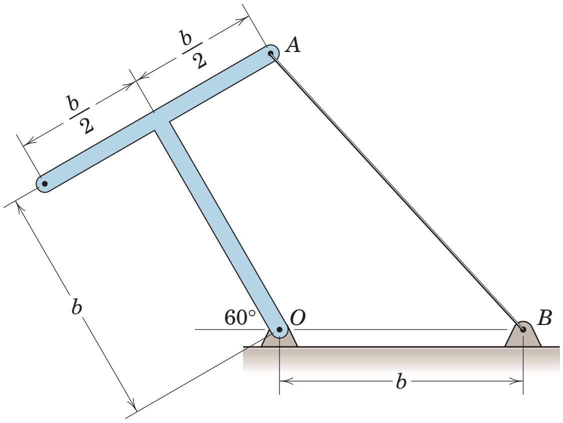
Problem 2/34

2/35 Calculate the moment of the 250-N force on the handle of the monkey wrench about the center of the bolt.



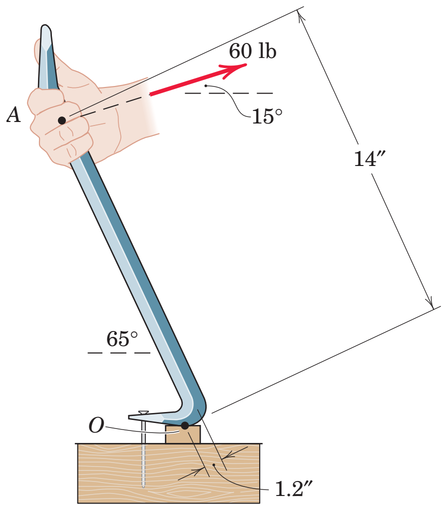
Problem 2/35

2/36 The tension in cable AB is 100 N. Determine the moment about O of this tension as applied to point A of the T-shaped bar. The dimension b is 600 mm.



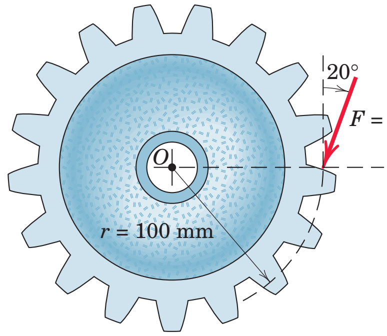
Problem 2/36

2/37 A prybar is used to remove a nail as shown. Determine the moment of the 60-lb force about the point O of contact between the prybar and the small support block.



Problem 2/37

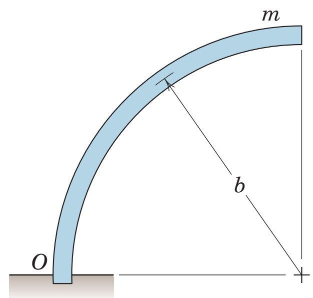
2/38 A force **F** of magnitude 60 N is applied to the gear. Determine the moment of **F** about point O.



Problem 2/38

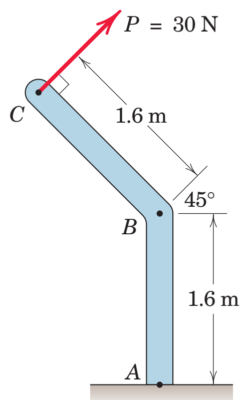
Representative Problems

2/39 The slender quarter-circular member of mass m is built-in at its support O. Determine the moment of its weight about point O. Use Table D/3 as necessary to determine the location of the mass center of the body.



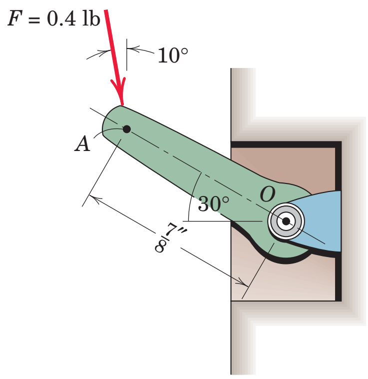
Problem 2/39

2/40 The 30-N force **P** is applied perpendicular to the portion BC of the bent bar. Determine the moment of **P** about point B and about point A.



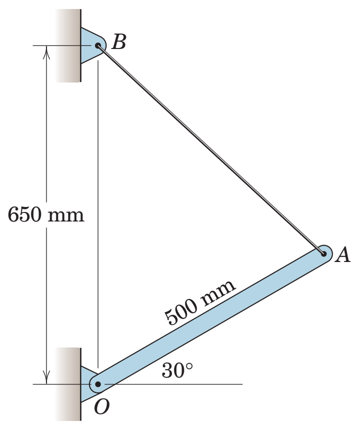
Problem 2/40

2/41 Compute the moment of the 0.4-lb force about the pivot O of the wall-switch toggle.



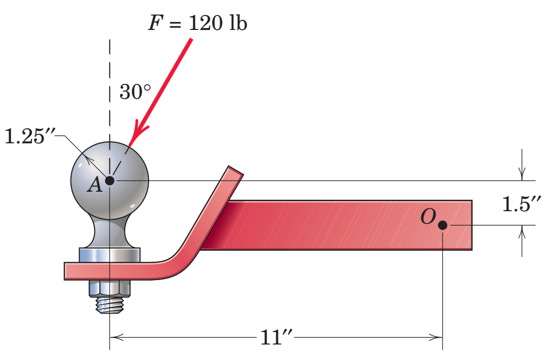
Problem 2/41

2/42 The cable AB carries a tension of 400 N. Determine the moment about O of this tension as applied to point A of the slender bar.



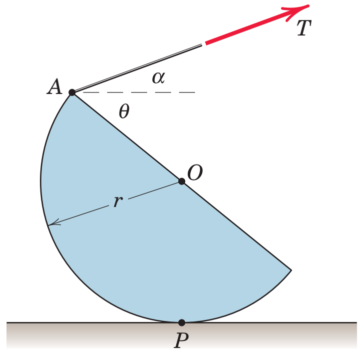
Problem 2/42

2/43 As a trailer is towed in the forward direction, the force $F = 120$ lb is applied as shown to the ball of the trailer hitch. Determine the moment of this force about point O.



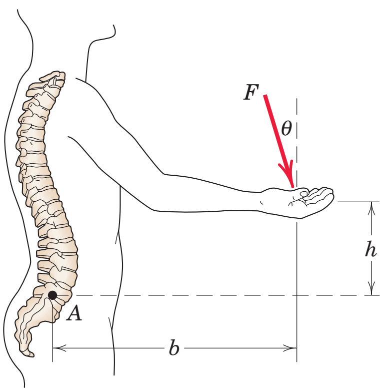
Problem 2/43

2/44 Determine the moments of the tension T about point P and about point O.



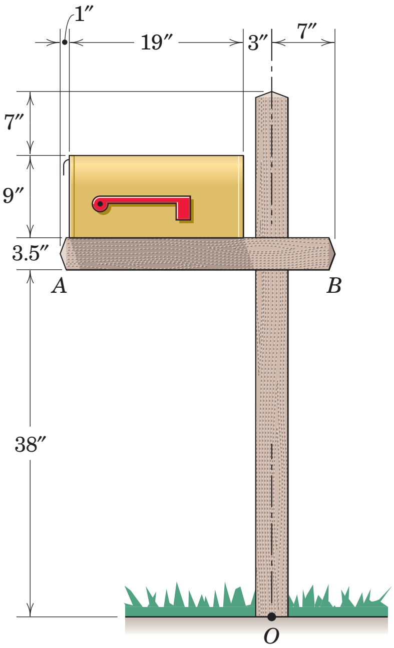
Problem 2/44

2/45 The lower lumbar region A of the spine is the part of the spinal column most susceptible to abuse while resisting excessive bending caused by the moment about A of a force F. For given values of F, b, and h, determine the angle θ which causes the most severe bending strain.



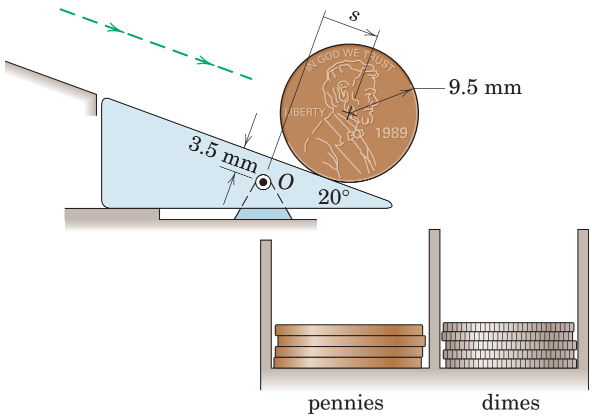
Problem 2/45

2/46 Determine the combined moment about O due to the weight of the mailbox and the cross member AB. The mailbox weighs 4 lb and the uniform cross member weighs 10 lb. Both weights act at the geometric centers of the respective items.



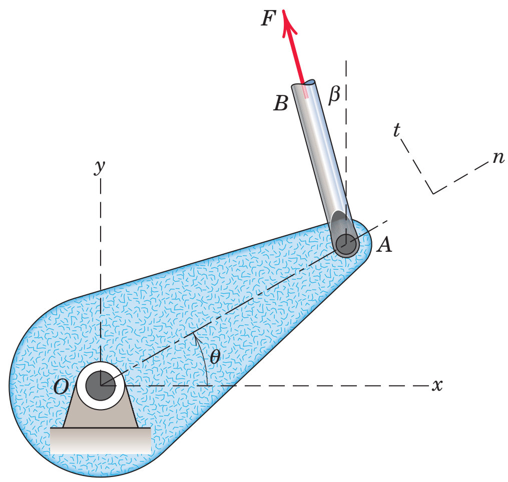
Problem 2/46

2/47 A portion of a mechanical coin sorter works as follows: Pennies and dimes roll down the 20° incline, the last triangular portion of which pivots freely about a horizontal axis through O. Dimes are light enough (2.28 grams each) so that the triangular portion remains stationary, and the dimes roll into the right collection column. Pennies, on the other hand, are heavy enough (3.06 grams each) so that the triangular portion pivots clockwise, and the pennies roll into the left collection column. Determine the moment about O of the weight of the penny in terms of the slant distance s in millimeters.



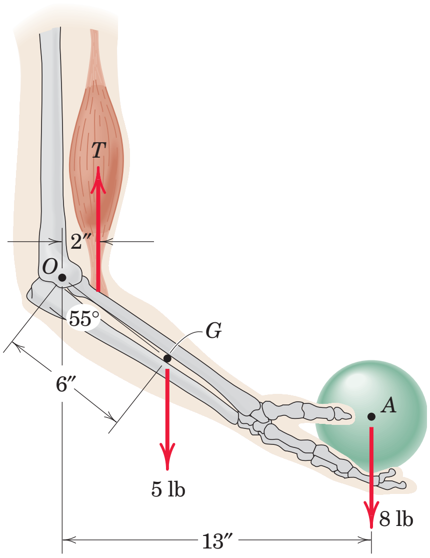
Problem 2/47

2/48 The crank of Prob. 2/10 is repeated here. If $\overline{OA} = 50$ mm, $\theta = 25°$, and $\beta = 55°$, determine the moment of the force $\mathbf{F}$ of magnitude $F = 20$ N about point O.



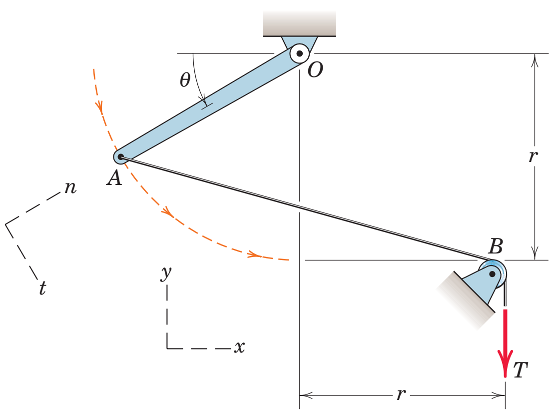
Problem 2/48

2/49 Elements of the lower arm are shown in the figure. The weight of the forearm is 5 lb with mass center at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the sphere. What must the biceps tension force be so that the overall moment about O is zero?



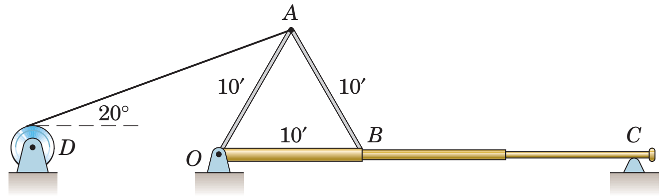
Problem 2/49

2/50 The mechanism of Prob. 2/16 is repeated here. For the conditions $\theta = 40°$, $T = 150$ N, and $r = 200$ mm, determine the moment about O of the tension T applied by cable AB to point A.



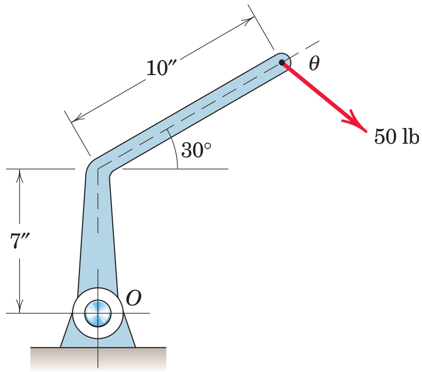
Problem 2/50

2/51 In order to raise the flagpole OC, a light frame OAB is attached to the pole and a tension of 780 lb is developed in the hoisting cable by the power winch D. Calculate the moment M_O of this tension about the hinge point O.



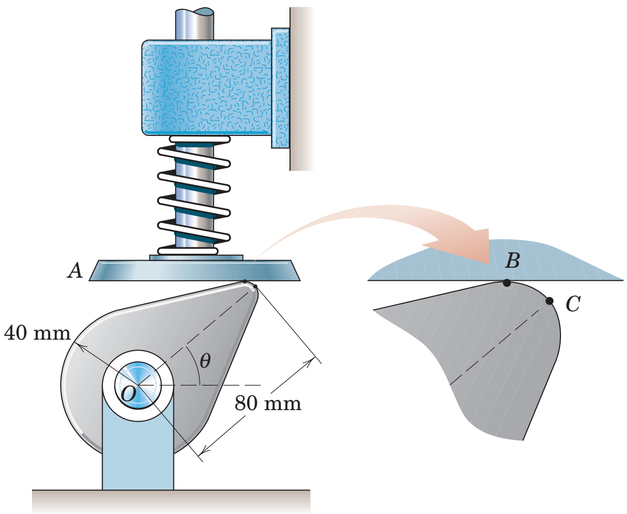
Problem 2/51

2/52 Determine the angle θ which will maximize the moment M_O of the 50-lb force about the shaft axis at O. Also compute M_O.



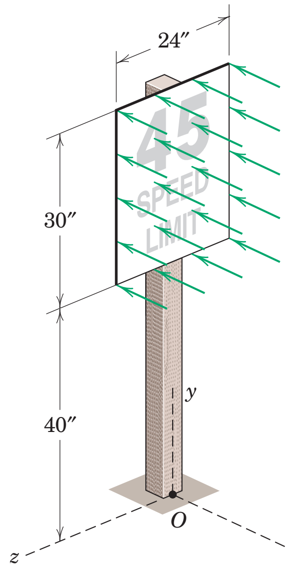
Problem 2/52

2/53 The spring-loaded follower A bears against the circular portion of the cam until the lobe of the cam lifts the plunger. The force required to lift the plunger is proportional to its vertical movement h from its lowest position. For design purposes determine the angle θ for which the moment of the contact force on the cam about the bearing O is a maximum. In the enlarged view of the contact, neglect the small distance between the actual contact point B and the end C of the lobe.



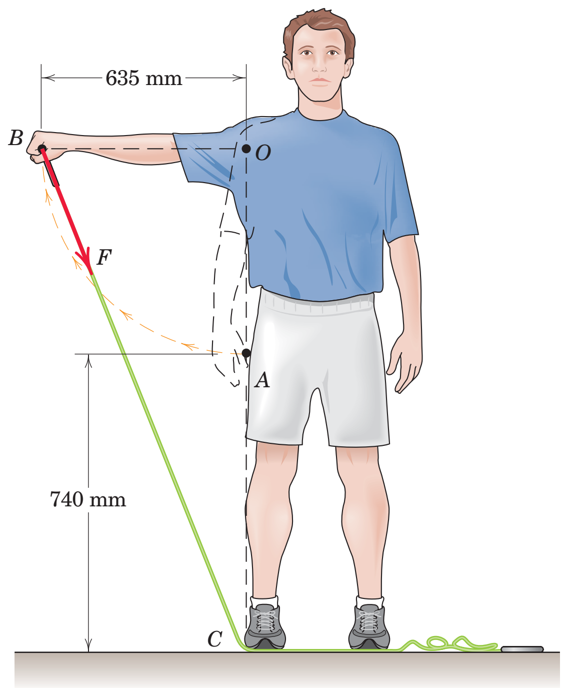
Problem 2/53

2/54 As the result of a wind blowing normal to the plane of the rectangular sign, a uniform pressure of 3.5 lb/ft^2 is exerted in the direction shown in the figure. Determine the moment of the resulting force about point O. Express your result as a vector using the coordinates shown.



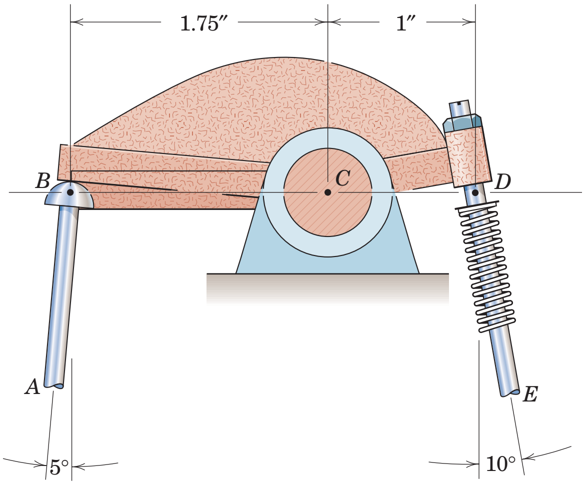
Problem 2/54

2/55 An exerciser begins with his arm in the relaxed vertical position OA, at which the elastic band is unstretched. He then rotates his arm to the horizontal position OB. The elastic modulus of the band is $k = 60$ N/m—that is, 60 N of force is required to stretch the band each additional meter of elongation. Determine the moment about O of the force which the band exerts on the hand B.



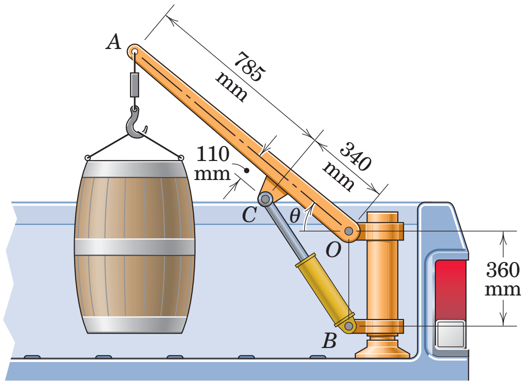
Problem 2/55

2/56 The rocker arm BD of an automobile engine is supported by a nonrotating shaft at C. If the design value of the force exerted by the pushrod AB on the rocker arm is 80 lb, determine the force which the valve stem DE must exert at D in order for the combined moment about point C to be zero. Compute the resultant of these two forces exerted on the rocker arm. Note that the points B, C, and D lie on a horizontal line and that both the pushrod and valve stem exert forces along their axes.



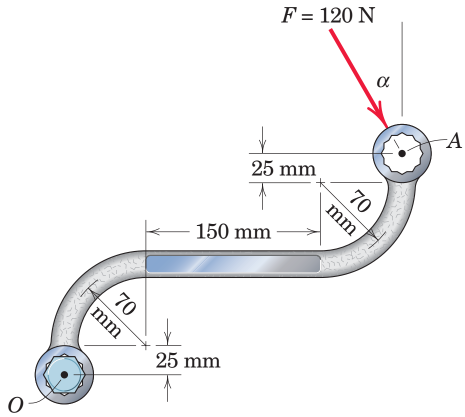
Problem 2/56

2/57 The small crane is mounted along the side of a pickup bed and facilitates the handling of heavy loads. When the boom elevation angle is $\theta = 40°$, the force in the hydraulic cylinder BC is 4.5 kN, and this force applied at point C is in the direction from B to C (the cylinder is in compression). Determine the moment of this 4.5-kN force about the boom pivot point O.



Problem 2/57

2/58 The 120-N force is applied as shown to one end of the curved wrench. If $\alpha = 30°$, calculate the moment of F about the center O of the bolt. Determine the value of α which would maximize the moment about O; state the value of this maximum moment.



Problem 2/58