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**Lecture No.: - 2**

**Lecture Title: [Force Systems]**



# Force Systems

# 2

## CHAPTER OUTLINE

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### Section A Two-Dimensional Force Systems

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### Section B Three-Dimensional Force Systems

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## 2/1 Introduction

In this and the following chapters, we study the effects of forces which act on engineering structures and mechanisms. The experience gained here will help you in the study of mechanics and in other subjects such as stress analysis, design of structures and machines, and fluid flow. This chapter lays the foundation for a basic understanding not only of statics but also of the entire subject of mechanics, and you should master this material thoroughly.

## 2/2 Force

Before dealing with a group or *system* of forces, it is necessary to examine the properties of a single force in some detail. A force has been defined in Chapter 1 as an action of one body on another. In dynamics we will see that a force is defined as an action which tends to cause acceleration of a body. A force is a *vector quantity*, because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.

The action of the cable tension on the bracket in Fig. 2/1a is represented in the side view, Fig. 2/1b, by the force vector  $\mathbf{P}$  of magnitude  $P$ . The effect of this action on the bracket depends on  $P$ , the angle  $\theta$ , and the location of the point of application  $A$ . Changing any one of these three specifications will alter the effect on the bracket, such as the force

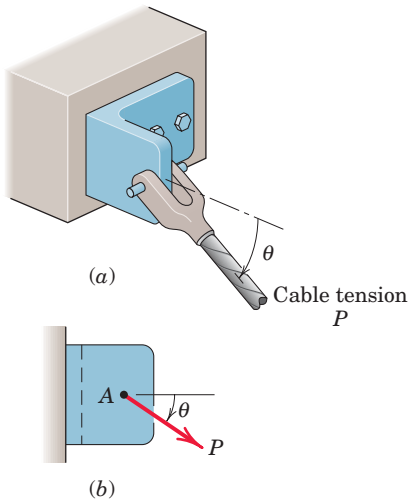


Figure 2/1

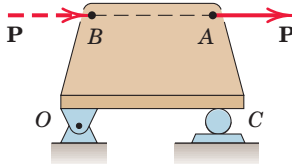
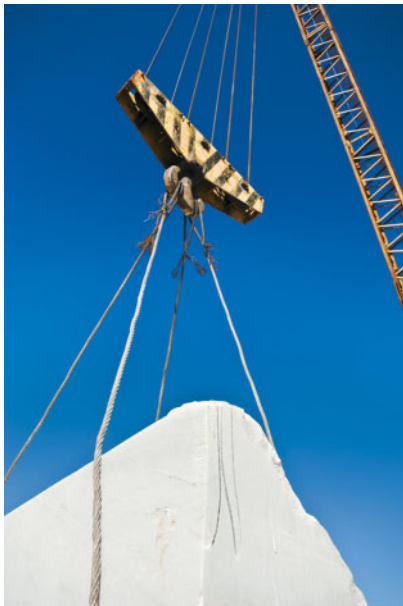


Figure 2/2



The forces associated with this lifting rig must be carefully identified, classified, and analyzed in order to provide a safe and effective working environment.

in one of the bolts which secure the bracket to the base, or the internal force and deformation in the material of the bracket at any point. Thus, the complete specification of the action of a force must include its *magnitude*, *direction*, and *point of application*, and therefore we must treat it as a fixed vector.

### External and Internal Effects

We can separate the action of a force on a body into two effects, *external* and *internal*. For the bracket of Fig. 2/1 the effects of  $\mathbf{P}$  external to the bracket are the reactive forces (not shown) exerted on the bracket by the foundation and bolts because of the action of  $\mathbf{P}$ . Forces external to a body can be either *applied* forces or *reactive* forces. The effects of  $\mathbf{P}$  internal to the bracket are the resulting internal forces and deformations distributed throughout the material of the bracket. The relation between internal forces and internal deformations depends on the material properties of the body and is studied in strength of materials, elasticity, and plasticity.

### Principle of Transmissibility

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force  $\mathbf{P}$  acting on the rigid plate in Fig. 2/2 may be applied at A or at B or at any other point on its line of action, and the net external effects of  $\mathbf{P}$  on the bracket will not change. The external effects are the force exerted on the plate by the bearing support at O and the force exerted on the plate by the roller support at C.

This conclusion is summarized by the *principle of transmissibility*, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force *external* to the *rigid* body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a *sliding* vector, and we need specify only the *magnitude*, *direction*, and *line of action* of the force, and not its *point of application*. Because this book deals essentially with the mechanics of rigid bodies, we will treat almost all forces as sliding vectors for the rigid body on which they act.

### Force Classification

Forces are classified as either *contact* or *body* forces. A contact force is produced by direct physical contact; an example is the force exerted on a body by a supporting surface. On the other hand, a body force is generated by virtue of the position of a body within a force field such as a gravitational, electric, or magnetic field. An example of a body force is your weight.

Forces may be further classified as either *concentrated* or *distributed*. Every contact force is actually applied over a finite area and is therefore really a distributed force. However, when the dimensions of the area are very small compared with the other dimensions of the

body, we may consider the force to be concentrated at a point with negligible loss of accuracy. Force can be distributed over an *area*, as in the case of mechanical contact, over a *volume* when a body force such as weight is acting, or over a *line*, as in the case of the weight of a suspended cable.

The *weight* of a body is the force of gravitational attraction distributed over its volume and may be taken as a concentrated force acting through the center of gravity. The position of the center of gravity is frequently obvious if the body is symmetric. If the position is not obvious, then a separate calculation, explained in Chapter 5, will be necessary to locate the center of gravity.

We can measure a force either by comparison with other known forces, using a mechanical balance, or by the calibrated movement of an elastic element. All such comparisons or calibrations have as their basis a primary standard. The standard unit of force in SI units is the newton (N) and in the U.S. customary system is the pound (lb), as defined in Art. 1/5.

### Action and Reaction

According to Newton's third law, the *action* of a force is always accompanied by an *equal and opposite reaction*. It is essential to distinguish between the action and the reaction in a pair of forces. To do so, we first *isolate* the body in question and then identify the force exerted *on* that body (not the force exerted *by* the body). It is very easy to mistakenly use the wrong force of the pair unless we distinguish carefully between action and reaction.

### Concurrent Forces

Two or more forces are said to be *concurrent at a point* if their lines of action intersect at that point. The forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  shown in Fig. 2/3a have a common point of application and are concurrent at the point A. Thus, they can be added using the parallelogram law in their common plane to obtain their sum or *resultant*  $\mathbf{R}$ , as shown in Fig. 2/3a. The resultant lies in the same plane as  $\mathbf{F}_1$  and  $\mathbf{F}_2$ .

Suppose the two concurrent forces lie in the same plane but are applied at two different points as in Fig. 2/3b. By the principle of transmissibility, we may move them along their lines of action and complete their vector sum  $\mathbf{R}$  at the point of concurrency A, as shown in Fig. 2/3b. We can replace  $\mathbf{F}_1$  and  $\mathbf{F}_2$  with the resultant  $\mathbf{R}$  without altering the external effects on the body upon which they act.

We can also use the triangle law to obtain  $\mathbf{R}$ , but we need to move the line of action of one of the forces, as shown in Fig. 2/3c. If we add the same two forces as shown in Fig. 2/3d, we correctly preserve the magnitude and direction of  $\mathbf{R}$ , but we lose the correct line of action, because  $\mathbf{R}$  obtained in this way does not pass through A. Therefore this type of combination should be avoided.

We can express the sum of the two forces mathematically by the vector equation

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$

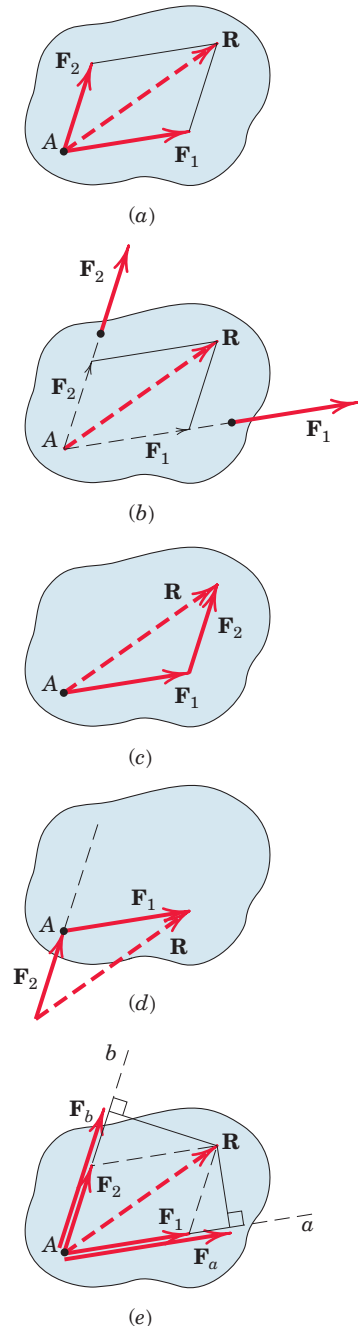


Figure 2/3

### Vector Components

In addition to combining forces to obtain their resultant, we often need to replace a force by its *vector components* in directions which are convenient for a given application. The vector sum of the components must equal the original vector. Thus, the force  $\mathbf{R}$  in Fig. 2/3a may be replaced by, or *resolved* into, two vector components  $\mathbf{F}_1$  and  $\mathbf{F}_2$  with the specified directions by completing the parallelogram as shown to obtain the magnitudes of  $\mathbf{F}_1$  and  $\mathbf{F}_2$ .

The relationship between a force and its vector components along given axes must not be confused with the relationship between a force and its perpendicular\* projections onto the same axes. Figure 2/3e shows the perpendicular projections  $\mathbf{F}_a$  and  $\mathbf{F}_b$  of the given force  $\mathbf{R}$  onto axes  $a$  and  $b$ , which are parallel to the vector components  $\mathbf{F}_1$  and  $\mathbf{F}_2$  of Fig. 2/3a. Figure 2/3e shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections  $\mathbf{F}_a$  and  $\mathbf{F}_b$  is not the vector  $\mathbf{R}$ , because the parallelogram law of vector addition must be used to form the sum. The components and projections of  $\mathbf{R}$  are equal only when the axes  $a$  and  $b$  are perpendicular.

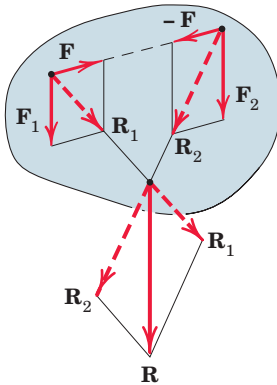


Figure 2/4

### A Special Case of Vector Addition

To obtain the resultant when the two forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  are parallel as in Fig. 2/4, we use a special case of addition. The two vectors are combined by first adding two equal, opposite, and collinear forces  $\mathbf{F}$  and  $-\mathbf{F}$  of convenient magnitude, which taken together produce no external effect on the body. Adding  $\mathbf{F}_1$  and  $\mathbf{F}$  to produce  $\mathbf{R}_1$ , and combining with the sum  $\mathbf{R}_2$  of  $\mathbf{F}_2$  and  $-\mathbf{F}$  yield the resultant  $\mathbf{R}$ , which is correct in magnitude, direction, and line of action. This procedure is also useful for graphically combining two forces which have a remote and inconvenient point of concurrency because they are almost parallel.

It is usually helpful to master the analysis of force systems in two dimensions before undertaking three-dimensional analysis. Thus the remainder of Chapter 2 is subdivided into these two categories.

## SECTION A TWO-DIMENSIONAL FORCE SYSTEMS

### 2/3 Rectangular Components

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector  $\mathbf{F}$  of Fig. 2/5 may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \quad (2/1)$$

where  $\mathbf{F}_x$  and  $\mathbf{F}_y$  are *vector components* of  $\mathbf{F}$  in the  $x$ - and  $y$ -directions. Each of the two vector components may be written as a scalar times the

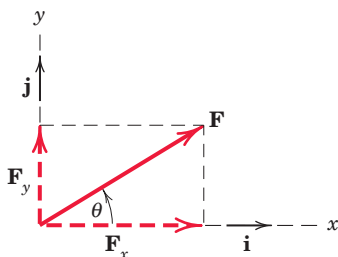


Figure 2/5

\*Perpendicular projections are also called *orthogonal* projections.

appropriate unit vector. In terms of the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  of Fig. 2/5,  $\mathbf{F}_x = F_x \mathbf{i}$  and  $\mathbf{F}_y = F_y \mathbf{j}$ , and thus we may write

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad (2/2)$$

where the scalars  $F_x$  and  $F_y$  are the  $x$  and  $y$  scalar components of the vector  $\mathbf{F}$ .

The scalar components can be positive or negative, depending on the quadrant into which  $\mathbf{F}$  points. For the force vector of Fig. 2/5, the  $x$  and  $y$  scalar components are both positive and are related to the magnitude and direction of  $\mathbf{F}$  by

$$\begin{aligned} F_x &= F \cos \theta & F &= \sqrt{F_x^2 + F_y^2} \\ F_y &= F \sin \theta & \theta &= \tan^{-1} \frac{F_y}{F_x} \end{aligned} \quad (2/3)$$

### Conventions for Describing Vector Components

We express the magnitude of a vector with lightface italic type in print; that is,  $|\mathbf{F}|$  is indicated by  $F$ , a quantity which is always *nonnegative*. However, the scalar components, also denoted by lightface italic type, will include sign information. See Sample Problems 2/1 and 2/3 for numerical examples which involve both positive and negative scalar components.

When both a force and its vector components appear in a diagram, it is desirable to show the vector components of the force with dashed lines, as in Fig. 2/5, and show the force with a solid line, or vice versa. With either of these conventions it will always be clear that a force and its components are being represented, and not three separate forces, as would be implied by three solid-line vectors.

Actual problems do not come with reference axes, so their assignment is a matter of arbitrary convenience, and the choice is frequently up to the student. The logical choice is usually indicated by the way in which the geometry of the problem is specified. When the principal dimensions of a body are given in the horizontal and vertical directions, for example, you would typically assign reference axes in these directions.

### Determining the Components of a Force

Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the  $x$ -axis, and the origin of coordinates need not be on the line of action of a force. Therefore, it is essential that we be able to determine the correct components of a force no matter how the axes are oriented or how the angles are measured. Figure 2/6 suggests a few typical examples of vector resolution in two dimensions.

Memorization of Eqs. 2/3 is not a substitute for understanding the parallelogram law and for correctly projecting a vector onto a reference axis. A neatly drawn sketch always helps to clarify the geometry and avoid error.

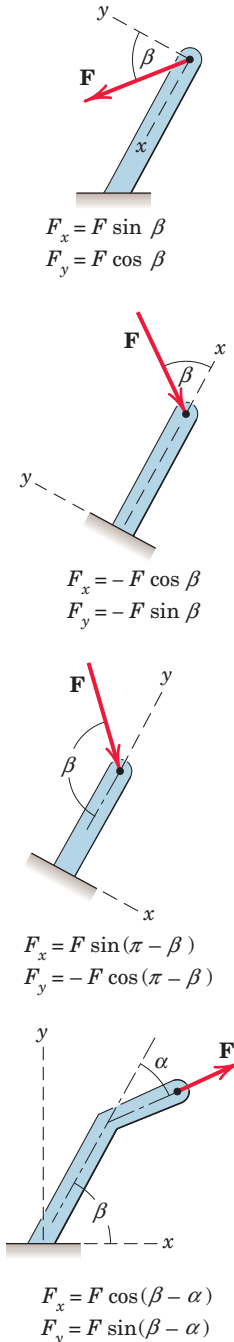


Figure 2/6

Rectangular components are convenient for finding the sum or resultant  $\mathbf{R}$  of two forces which are concurrent. Consider two forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  which are originally concurrent at a point  $O$ . Figure 2/7 shows the line of action of  $\mathbf{F}_2$  shifted from  $O$  to the tip of  $\mathbf{F}_1$  according to the triangle rule of Fig. 2/3. In adding the force vectors  $\mathbf{F}_1$  and  $\mathbf{F}_2$ , we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1x}\mathbf{i} + F_{1y}\mathbf{j}) + (F_{2x}\mathbf{i} + F_{2y}\mathbf{j})$$

or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1x} + F_{2x})\mathbf{i} + (F_{1y} + F_{2y})\mathbf{j}$$

from which we conclude that

$$R_x = F_{1x} + F_{2x} = \Sigma F_x \quad (2/4)$$

$$R_y = F_{1y} + F_{2y} = \Sigma F_y$$

The term  $\Sigma F_x$  means “the algebraic sum of the  $x$  scalar components”. For the example shown in Fig. 2/7, note that the scalar component  $F_{2y}$  would be negative.

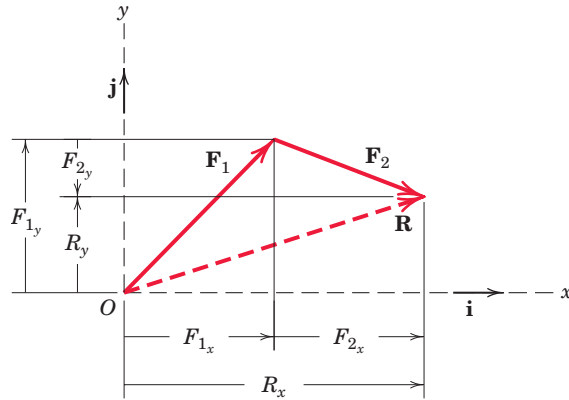


Figure 2/7



The structural elements in the foreground transmit concentrated forces to the brackets at both ends.



### SAMPLE PROBLEM 2/1

The forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$ , and  $\mathbf{F}_3$ , all of which act on point A of the bracket, are specified in three different ways. Determine the  $x$  and  $y$  scalar components of each of the three forces.

**Solution.** The scalar components of  $\mathbf{F}_1$ , from Fig. *a*, are

$$F_{1x} = 600 \cos 35^\circ = 491 \text{ N} \quad \text{Ans.}$$

$$F_{1y} = 600 \sin 35^\circ = 344 \text{ N} \quad \text{Ans.}$$

The scalar components of  $\mathbf{F}_2$ , from Fig. *b*, are

$$F_{2x} = -500\left(\frac{4}{5}\right) = -400 \text{ N} \quad \text{Ans.}$$

$$F_{2y} = 500\left(\frac{3}{5}\right) = 300 \text{ N} \quad \text{Ans.}$$

Note that the angle which orients  $\mathbf{F}_2$  to the  $x$ -axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the  $x$  scalar component of  $\mathbf{F}_2$  is negative by inspection.

The scalar components of  $\mathbf{F}_3$  can be obtained by first computing the angle  $\alpha$  of Fig. *c*.

$$\alpha = \tan^{-1} \left[ \frac{0.2}{0.4} \right] = 26.6^\circ$$

$$\textcircled{1} \text{ Then, } F_{3x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N} \quad \text{Ans.}$$

$$F_{3y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N} \quad \text{Ans.}$$

Alternatively, the scalar components of  $\mathbf{F}_3$  can be obtained by writing  $\mathbf{F}_3$  as a magnitude times a unit vector  $\mathbf{n}_{AB}$  in the direction of the line segment  $AB$ . Thus,

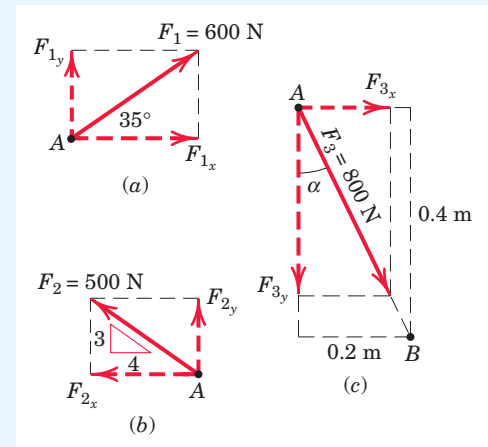
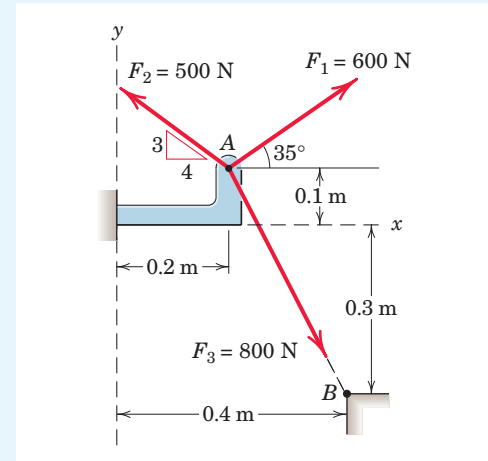
$$\begin{aligned} \textcircled{2} \quad \mathbf{F}_3 &= F_3 \mathbf{n}_{AB} = F_3 = \frac{\overrightarrow{AB}}{AB} = 800 \left[ \frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}} \right] \\ &= 800 [0.447\mathbf{i} - 0.894\mathbf{j}] \\ &= 358\mathbf{i} - 716\mathbf{j} \text{ N} \end{aligned}$$

The required scalar components are then

$$F_{3x} = 358 \text{ N} \quad \text{Ans.}$$

$$F_{3y} = -716 \text{ N} \quad \text{Ans.}$$

which agree with our previous results.



### Helpful Hints

- ① You should carefully examine the geometry of each component determination problem and not rely on the blind use of such formulas as  $F_x = F \cos \theta$  and  $F_y = F \sin \theta$ .
- ② A unit vector can be formed by dividing any vector, such as the geometric position vector  $\overrightarrow{AB}$ , by its length or magnitude. Here we use the overarrow to denote the vector which runs from A to B and the overbar to determine the distance between A and B.



## SAMPLE PROBLEM 2/2

Combine the two forces **P** and **T**, which act on the fixed structure at *B*, into a single equivalent force **R**.

- Graphical solution.** The parallelogram for the vector addition of forces **T** and **P** is constructed as shown in Fig. *a*. The scale used here is 1 in. = 800 lb; a scale of 1 in. = 200 lb would be more suitable for regular-size paper and would give greater accuracy. Note that the angle  $\alpha$  must be determined prior to construction of the parallelogram. From the given figure

$$\tan \alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6 \sin 60^\circ}{3 + 6 \cos 60^\circ} = 0.866 \quad \alpha = 40.9^\circ$$

Measurement of the length *R* and direction  $\theta$  of the resultant force **R** yields the approximate results

$$R = 525 \text{ lb} \quad \theta = 49^\circ \quad \text{Ans.}$$

- Geometric solution.** The triangle for the vector addition of **T** and **P** is shown in Fig. *b*. The angle  $\alpha$  is calculated as above. The law of cosines gives

$$R^2 = (600)^2 + (800)^2 - 2(600)(800) \cos 40.9^\circ = 274,300$$

$$R = 524 \text{ lb} \quad \text{Ans.}$$

From the law of sines, we may determine the angle  $\theta$  which orients **R**. Thus,

$$\frac{600}{\sin \theta} = \frac{524}{\sin 40.9^\circ} \quad \sin \theta = 0.750 \quad \theta = 48.6^\circ \quad \text{Ans.}$$

**Algebraic solution.** By using the *x-y* coordinate system on the given figure, we may write

$$R_x = \Sigma F_x = 800 - 600 \cos 40.9^\circ = 346 \text{ lb}$$

$$R_y = \Sigma F_y = -600 \sin 40.9^\circ = -393 \text{ lb}$$

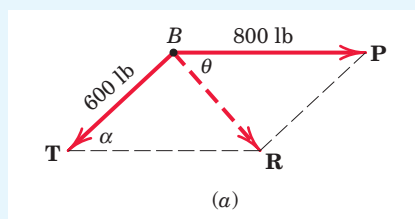
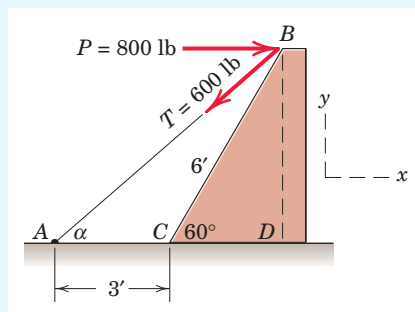
The magnitude and direction of the resultant force **R** as shown in Fig. *c* are then

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ lb} \quad \text{Ans.}$$

$$\theta = \tan^{-1} \frac{|R_y|}{|R_x|} = \tan^{-1} \frac{393}{346} = 48.6^\circ \quad \text{Ans.}$$

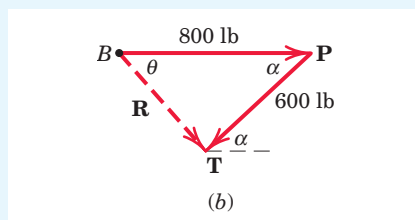
The resultant **R** may also be written in vector notation as

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j} = 346 \mathbf{i} - 393 \mathbf{j} \text{ lb} \quad \text{Ans.}$$

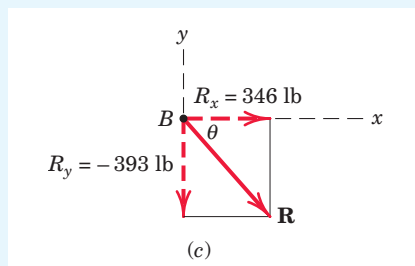


## Helpful Hints

- 1 Note the repositioning of **P** to permit parallelogram addition at *B*.



- 2 Note the repositioning of **F** so as to preserve the correct line of action of the resultant **R**.



### SAMPLE PROBLEM 2/3

The 500-N force  $\mathbf{F}$  is applied to the vertical pole as shown. (1) Write  $\mathbf{F}$  in terms of the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  and identify both its vector and scalar components. (2) Determine the scalar components of the force vector  $\mathbf{F}$  along the  $x'$ - and  $y'$ -axes. (3) Determine the scalar components of  $\mathbf{F}$  along the  $x$ - and  $y'$ -axes.

**Solution. Part (1).** From Fig. *a* we may write  $\mathbf{F}$  as

$$\begin{aligned}\mathbf{F} &= (F \cos \theta)\mathbf{i} - (F \sin \theta)\mathbf{j} \\ &= (500 \cos 60^\circ)\mathbf{i} - (500 \sin 60^\circ)\mathbf{j} \\ &= (250\mathbf{i} - 433\mathbf{j}) \text{ N}\end{aligned}\quad \text{Ans.}$$

The scalar components are  $F_x = 250$  N and  $F_y = -433$  N. The vector components are  $\mathbf{F}_x = 250\mathbf{i}$  N and  $\mathbf{F}_y = -433\mathbf{j}$  N.

**Part (2).** From Fig. *b* we may write  $\mathbf{F}$  as  $\mathbf{F} = 500\mathbf{i}'$  N, so that the required scalar components are

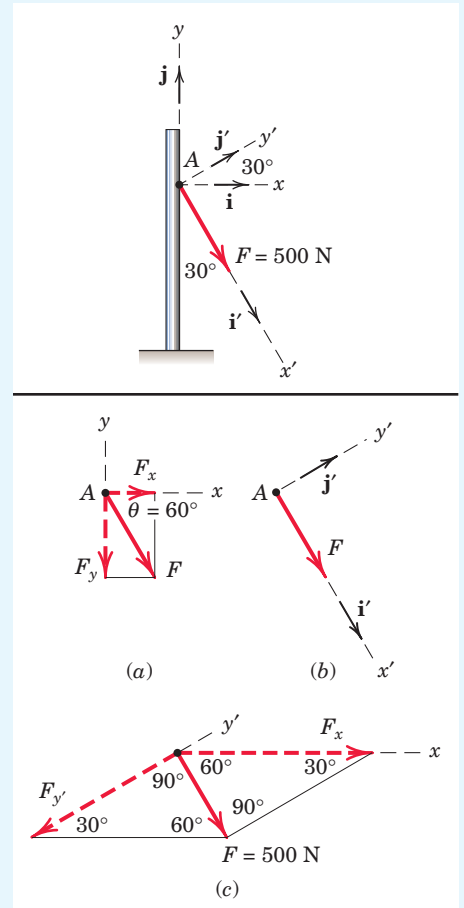
$$F_{x'} = 500 \text{ N} \quad F_{y'} = 0 \quad \text{Ans.}$$

**Part (3).** The components of  $\mathbf{F}$  in the  $x$ - and  $y'$ -directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. *c*. The magnitudes of the components may be calculated by the law of sines. Thus,

$$\begin{aligned}\frac{|F_x|}{\sin 90^\circ} &= \frac{500}{\sin 30^\circ} & |F_x| &= 1000 \text{ N} \\ \frac{|F_{y'}|}{\sin 60^\circ} &= \frac{500}{\sin 30^\circ} & |F_{y'}| &= 866 \text{ N}\end{aligned}$$

The required scalar components are then

$$F_x = 1000 \text{ N} \quad F_{y'} = -866 \text{ N} \quad \text{Ans.}$$



#### Helpful Hint

- 1 Obtain  $F_x$  and  $F_{y'}$  graphically and compare your results with the calculated values.

### SAMPLE PROBLEM 2/4

Forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  act on the bracket as shown. Determine the projection  $F_b$  of their resultant  $\mathbf{R}$  onto the  $b$ -axis.

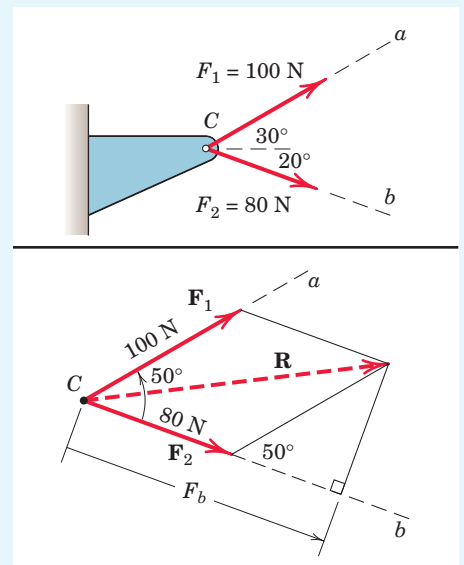
**Solution.** The parallelogram addition of  $\mathbf{F}_1$  and  $\mathbf{F}_2$  is shown in the figure. Using the law of cosines gives us

$$R^2 = (80)^2 + (100)^2 - 2(80)(100) \cos 130^\circ \quad R = 163.4 \text{ N}$$

The figure also shows the orthogonal projection  $F_b$  of  $\mathbf{R}$  onto the  $b$ -axis. Its length is

$$F_b = 80 + 100 \cos 50^\circ = 144.3 \text{ N} \quad \text{Ans.}$$

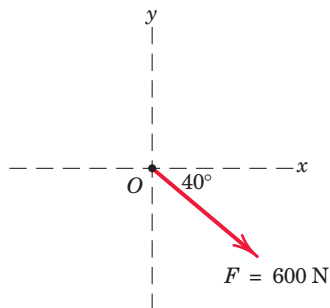
Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the  $a$ -axis had been perpendicular to the  $b$ -axis, then the projections and components of  $\mathbf{R}$  would have been equal.



## PROBLEMS

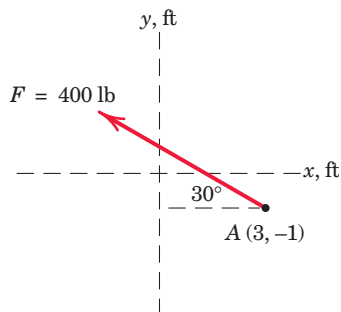
## Introductory Problems

- 2/1** The force  $\mathbf{F}$  has a magnitude of 600 N. Express  $\mathbf{F}$  as a vector in terms of the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ . Identify the  $x$  and  $y$  scalar components of  $\mathbf{F}$ .



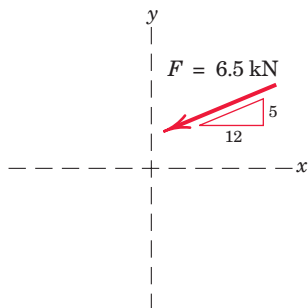
Problem 2/1

- 2/2** The magnitude of the force  $\mathbf{F}$  is 400 lb. Express  $\mathbf{F}$  as a vector in terms of the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ . Identify both the scalar and vector components of  $\mathbf{F}$ .



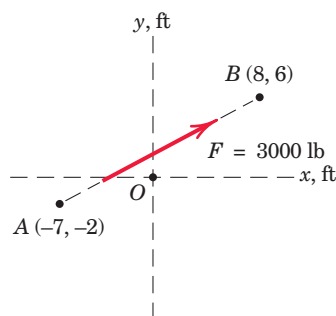
Problem 2/2

- 2/3** The slope of the 6.5-kN force  $\mathbf{F}$  is specified as shown in the figure. Express  $\mathbf{F}$  as a vector in terms of the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ .



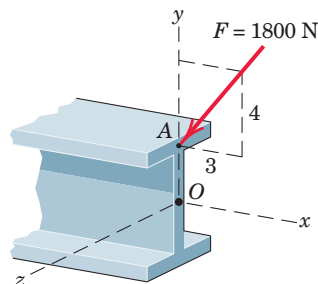
Problem 2/3

- 2/4** The line of action of the 3000-lb force runs through the points  $A$  and  $B$  as shown in the figure. Determine the  $x$  and  $y$  scalar components of  $\mathbf{F}$ .



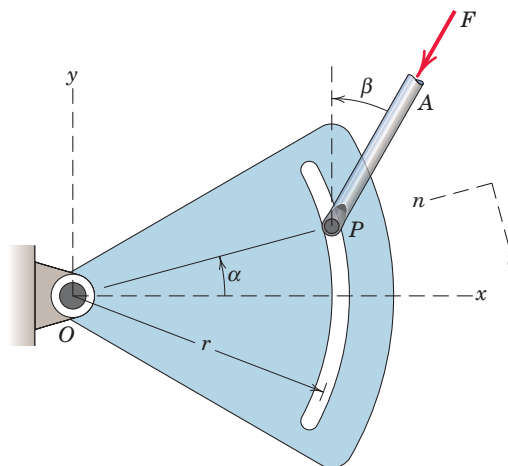
Problem 2/4

- 2/5** The 1800-N force  $\mathbf{F}$  is applied to the end of the I-beam. Express  $\mathbf{F}$  as a vector using the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ .



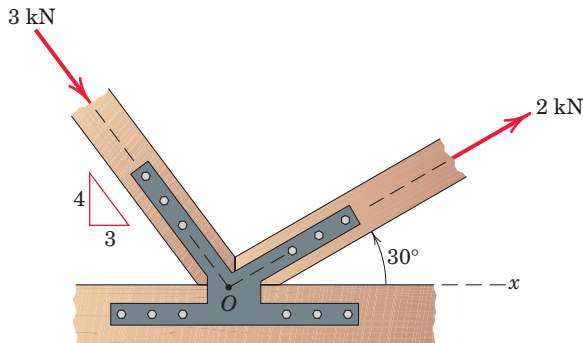
Problem 2/5

- 2/6** The control rod  $AP$  exerts a force  $\mathbf{F}$  on the sector as shown. Determine both the  $x$ - $y$  and the  $n$ - $t$  components of the force.



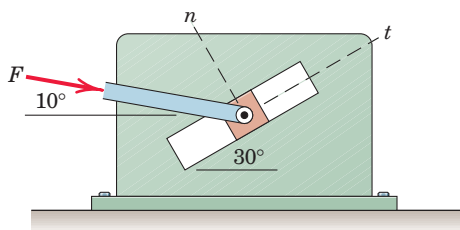
Problem 2/6

- 2/7** The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint  $O$ . Determine the magnitude of the resultant  $\mathbf{R}$  of the two forces and the angle  $\theta$  which  $\mathbf{R}$  makes with the positive  $x$ -axis.



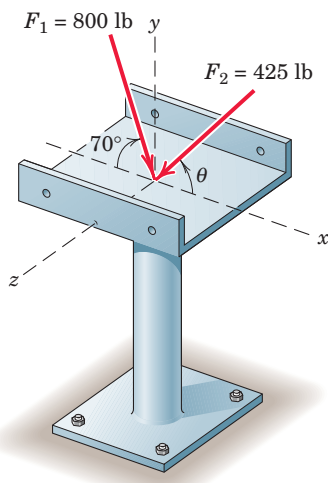
Problem 2/7

- 2/8** The  $t$ -component of the force  $\mathbf{F}$  is known to be 75 N. Determine the  $n$ -component and the magnitude of  $\mathbf{F}$ .



Problem 2/8

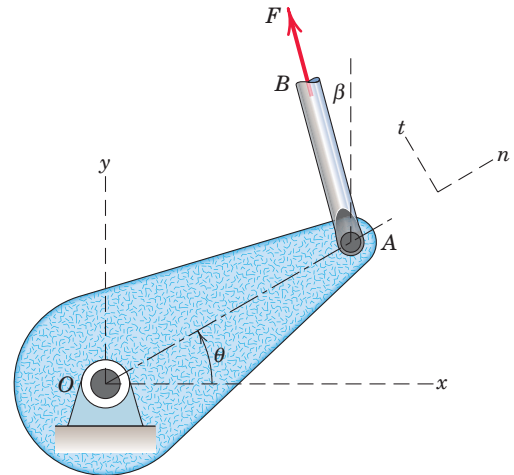
- 2/9** Two forces are applied to the construction bracket as shown. Determine the angle  $\theta$  which makes the resultant of the two forces vertical. Determine the magnitude  $R$  of the resultant.



Problem 2/9

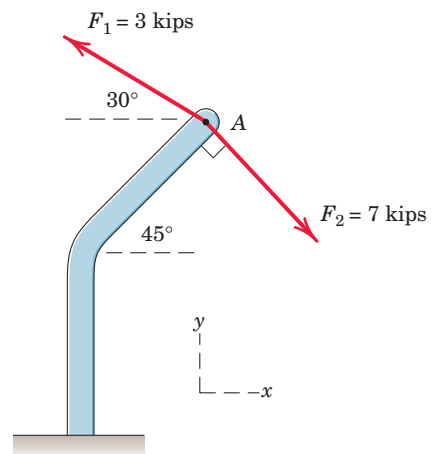
### Representative Problems

- 2/10** Determine the  $n$ - and  $t$ -components of the force  $\mathbf{F}$  which is exerted by the rod  $AB$  on the crank  $OA$ . Evaluate your general expression for  $F = 100$  N and (a)  $\theta = 30^\circ$ ,  $\beta = 10^\circ$  and (b)  $\theta = 15^\circ$ ,  $\beta = 25^\circ$ .



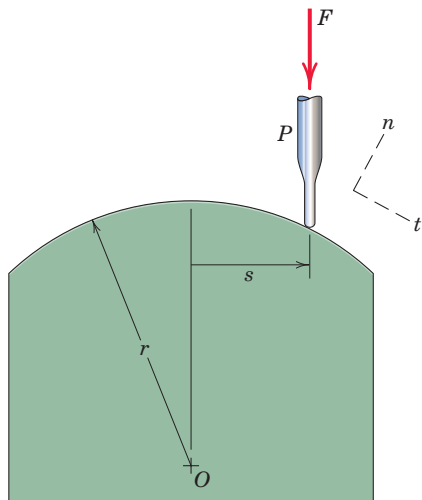
Problem 2/10

- 2/11** The two forces shown act at point  $A$  of the bent bar. Determine the resultant  $\mathbf{R}$  of the two forces.



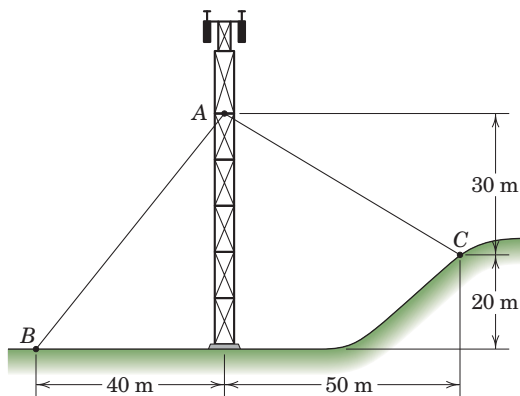
Problem 2/11

- 2/12** A small probe  $P$  is gently forced against the circular surface with a vertical force  $\mathbf{F}$  as shown. Determine the  $n$ - and  $t$ -components of this force as functions of the horizontal position  $s$ .



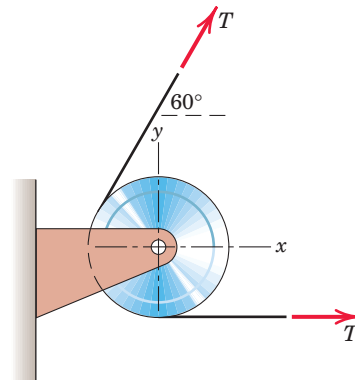
Problem 2/12

- 2/13** The guy cables  $AB$  and  $AC$  are attached to the top of the transmission tower. The tension in cable  $AB$  is 8 kN. Determine the required tension  $T$  in cable  $AC$  such that the net effect of the two cable tensions is a downward force at point  $A$ . Determine the magnitude  $R$  of this downward force.



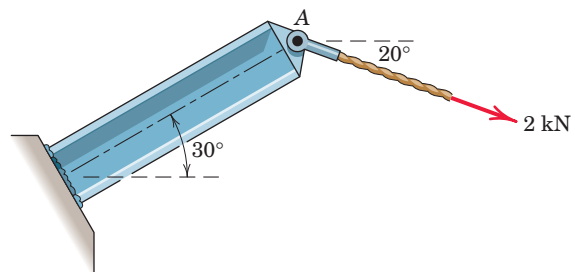
Problem 2/13

- 2/14** If the equal tensions  $T$  in the pulley cable are 400 N, express in vector notation the force  $\mathbf{R}$  exerted on the pulley by the two tensions. Determine the magnitude of  $\mathbf{R}$ .



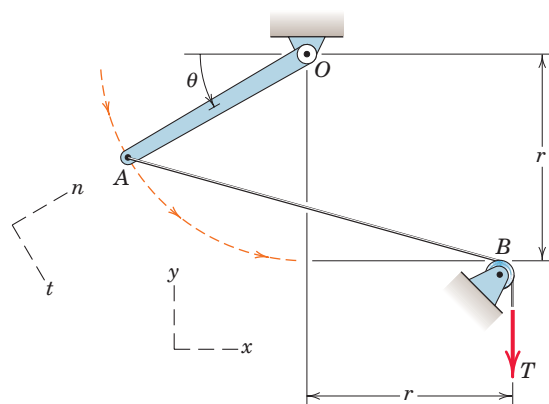
Problem 2/14

- 2/15** To satisfy design limitations it is necessary to determine the effect of the 2-kN tension in the cable on the shear, tension, and bending of the fixed I-beam. For this purpose replace this force by its equivalent of two forces at  $A$ ,  $F_t$  parallel and  $F_n$  perpendicular to the beam. Determine  $F_t$  and  $F_n$ .



Problem 2/15

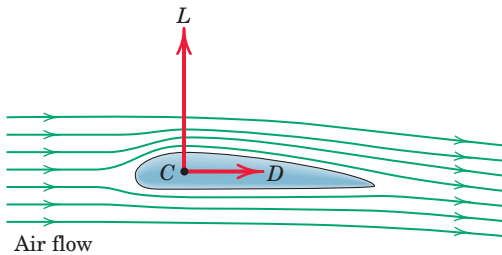
- 2/16** Determine the  $x$ - and  $y$ -components of the tension  $T$  which is applied to point  $A$  of the bar  $OA$ . Neglect the effects of the small pulley at  $B$ . Assume that  $r$  and  $\theta$  are known.



Problem 2/16

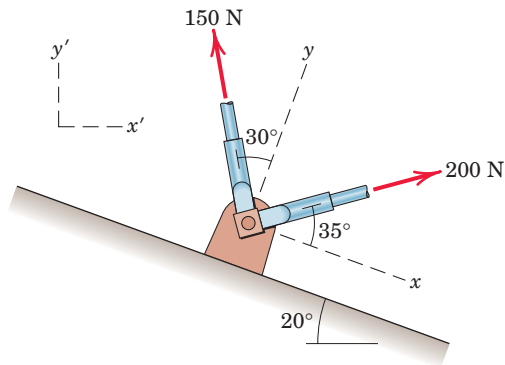
**2/17** Refer to the mechanism of the previous problem. Develop general expressions for the  $n$ - and  $t$ -components of the tension  $T$  applied to point  $A$ . Then evaluate your expressions for  $T = 100$  N and  $\theta = 35^\circ$ .

**2/18** The ratio of the lift force  $L$  to the drag force  $D$  for the simple airfoil is  $L/D = 10$ . If the lift force on a short section of the airfoil is 50 lb, compute the magnitude of the resultant force  $\mathbf{R}$  and the angle  $\theta$  which it makes with the horizontal.



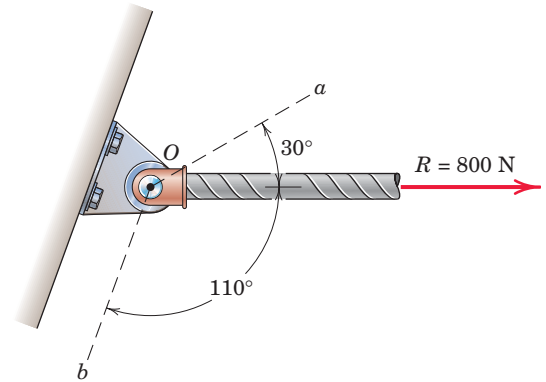
**Problem 2/18**

**2/19** Determine the resultant  $\mathbf{R}$  of the two forces applied to the bracket. Write  $\mathbf{R}$  in terms of unit vectors along the  $x$ - and  $y$ -axes shown.



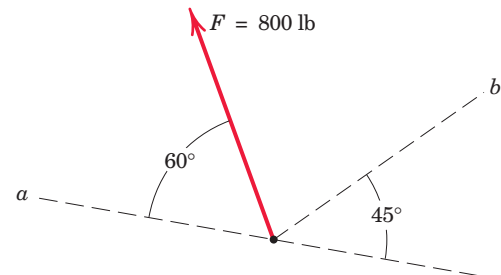
**Problem 2/19**

**2/20** Determine the scalar components  $R_a$  and  $R_b$  of the force  $\mathbf{R}$  along the nonrectangular axes  $a$  and  $b$ . Also determine the orthogonal projection  $P_a$  of  $\mathbf{R}$  onto axis  $a$ .



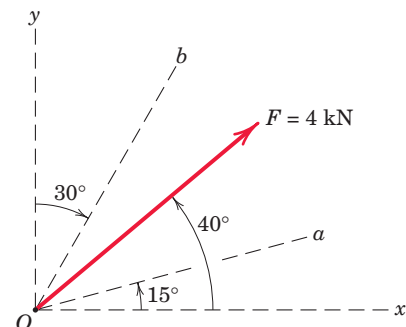
**Problem 2/20**

**2/21** Determine the components of the 800-lb force  $\mathbf{F}$  along the oblique axes  $a$  and  $b$ . Also, determine the projections of  $\mathbf{F}$  onto the  $a$ - and  $b$ -axes.



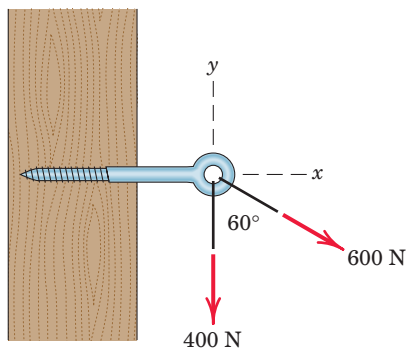
**Problem 2/21**

**2/22** Determine the components  $F_a$  and  $F_b$  of the 4-kN force along the oblique axes  $a$  and  $b$ . Determine the projections  $P_a$  and  $P_b$  of  $\mathbf{F}$  onto the  $a$ - and  $b$ -axes.



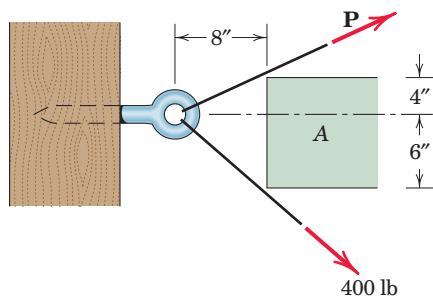
**Problem 2/22**

- 2/23** Determine the resultant  $\mathbf{R}$  of the two forces shown by (a) applying the parallelogram rule for vector addition and (b) summing scalar components.



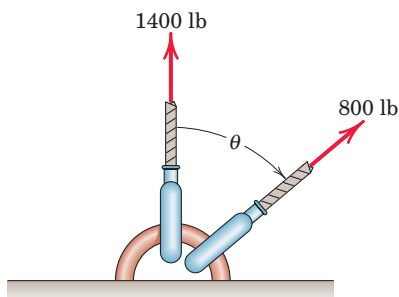
**Problem 2/23**

- 2/24** It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction  $A$  prevents direct access, so that two forces, one 400 lb and the other  $\mathbf{P}$ , are applied by cables as shown. Compute the magnitude of  $\mathbf{P}$  necessary to ensure a resultant  $\mathbf{T}$  directed along the spike. Also find  $T$ .



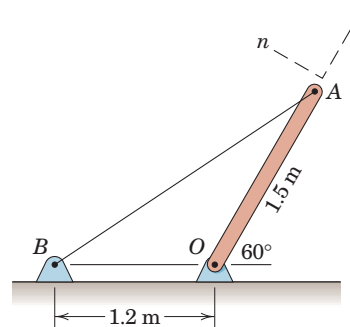
**Problem 2/24**

- 2/25** At what angle  $\theta$  must the 800-lb force be applied in order that the resultant  $\mathbf{R}$  of the two forces have a magnitude of 2000 lb? For this condition, determine the angle  $\beta$  between  $\mathbf{R}$  and the vertical.



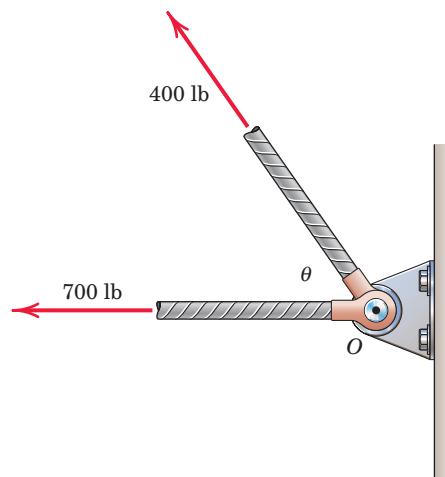
**Problem 2/25**

- 2/26** The cable  $AB$  prevents bar  $OA$  from rotating clockwise about the pivot  $O$ . If the cable tension is 750 N, determine the  $n$ - and  $t$ -components of this force acting on point  $A$  of the bar.



**Problem 2/26**

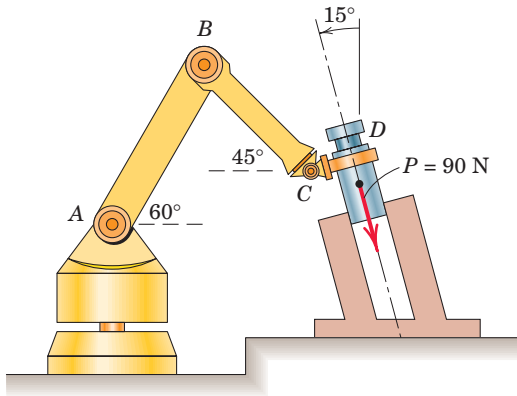
- 2/27** At what angle  $\theta$  must the 400-lb force be applied in order that the resultant  $\mathbf{R}$  of the two forces have a magnitude of 1000 lb? For this condition what will be the angle  $\beta$  between  $\mathbf{R}$  and the horizontal?



**Problem 2/27**

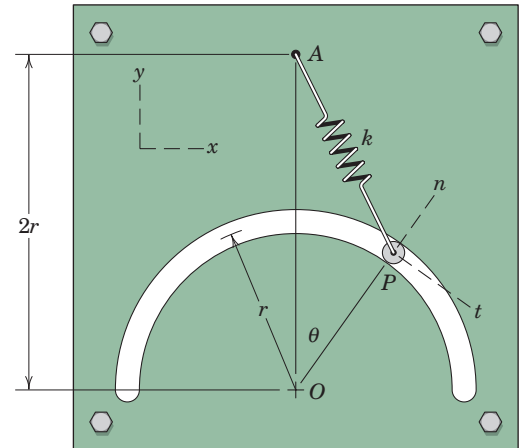


- 2/28** In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90-N force  $P$  on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts on the robot along axes (a) parallel and perpendicular to the arm  $AB$ , and (b) parallel and perpendicular to the arm  $BC$ .



Problem 2/28

- 2/29** The unstretched length of the spring is  $r$ . When pin  $P$  is in an arbitrary position  $\theta$ , determine the  $x$ - and  $y$ -components of the force which the spring exerts on the pin. Evaluate your general expressions for  $r = 400$  mm,  $k = 1.4$  kN/m, and  $\theta = 40^\circ$ . (Note: The force in a spring is given by  $F = k\delta$ , where  $\delta$  is the extension from the unstretched length.)



Problem 2/29

- 2/30** Refer to the figure and statement of Prob. 2/29. When pin  $P$  is in the position  $\theta = 20^\circ$ , determine the  $n$ - and  $t$ -components of the force  $F$  which the spring of modulus  $k = 1.4$  kN/m exerts on the pin. The distance  $r = 400$  mm.