Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

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Lecture No.:- 3

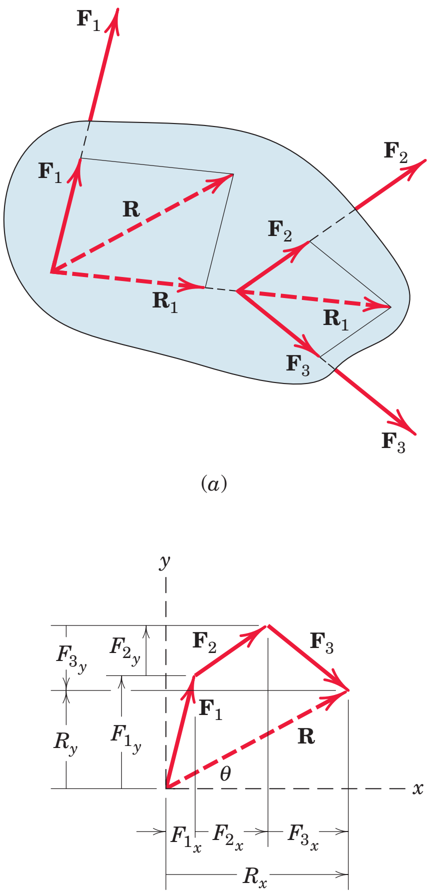
Lecture Title: [Resultants]

2/6 Resultants

The properties of force, moment, and couple were developed in the previous four articles. Now we are ready to describe the resultant action of a group or *system* of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduce the system to its simplest form to describe its action. The *resultant* of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.

The most common type of force system occurs when the forces all act in a single plane, say, the *x-y* plane, as illustrated by the system of three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ in Fig. 2/13*a*. We obtain the magnitude and direction of the resultant force $\mathbf{R}$ by forming the *force polygon* shown in part *b* of the figure, where the forces are added head-to-tail in any sequence. Thus, for any system of coplanar forces we may write



(*a*)

(*b*)

Figure 2/13

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$
$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x} \tag{2/9}$$

Graphically, the correct line of action of $\mathbf{R}$ may be obtained by preserving the correct lines of action of the forces and adding them by the parallelogram law. We see this in part *a* of the figure for the case of three forces where the sum $\mathbf{R}_1$ of $\mathbf{F}_2$ and $\mathbf{F}_3$ is added to $\mathbf{F}_1$ to obtain $\mathbf{R}$. The principle of transmissibility has been used in this process.

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs. 2/14*a* and *b*, where M_1, M_2, and M_3 are the couples resulting from the transfer of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ from their respective original lines of action to lines of action through point O.
2. Add all forces at O to form the resultant force $\mathbf{R}$, and add all couples to form the resultant couple M_O. We now have the single force–couple system, as shown in Fig. 2/14*c*.
3. In Fig. 2/14*d*, find the line of action of $\mathbf{R}$ by requiring $\mathbf{R}$ to have a moment of M_O about point O. Note that the force systems of Figs. 2/14*a* and 2/14*d* are equivalent, and that $\Sigma(Fd)$ in Fig. 2/14*a* is equal to Rd in Fig. 2/14*d*.

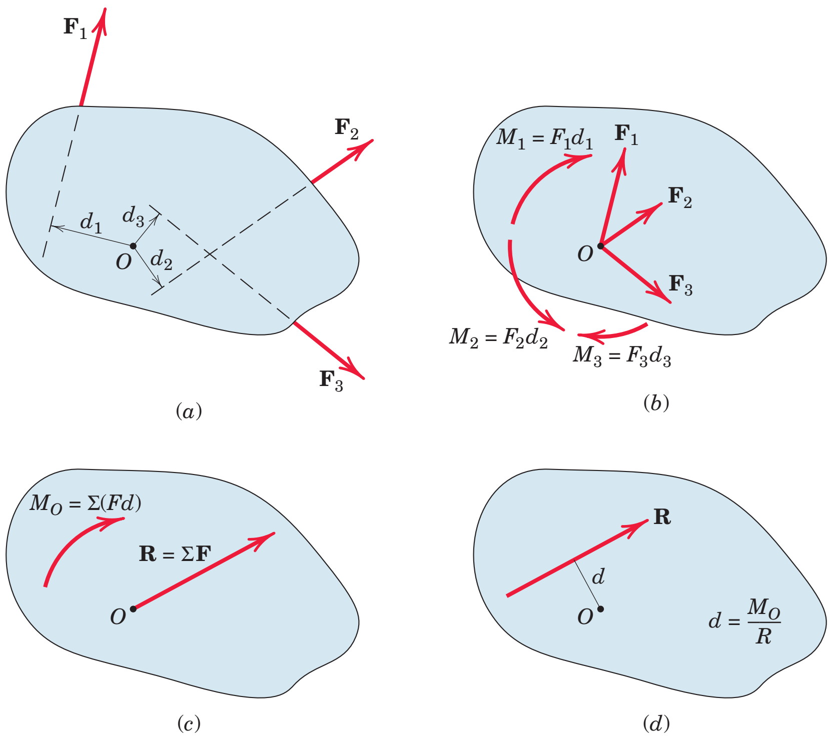


Figure 2/14

Principle of Moments

This process is summarized in equation form by

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$M_O = \Sigma M = \Sigma(Fd) \qquad \textbf{(2/10)}$$
$$Rd = M_O$$

The first two of Eqs. 2/10 reduce a given system of forces to a force–couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of $\mathbf{R}$, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of *noncurrent* force systems; we call this extension the *principle of moments*.

For a concurrent system of forces where the lines of action of all forces pass through a common point O, the moment sum ΣM_O about that point is zero. Thus, the line of action of the resultant $\mathbf{R} = \Sigma\mathbf{F}$, determined by the first of Eqs. 2/10, passes through point O. For a parallel force system, select a coordinate axis in the direction of the forces. If the resultant force $\mathbf{R}$ for a given force system is zero, the resultant of the system need not be zero because the resultant may be a couple. The three forces in Fig. 2/15, for instance, have a zero resultant force but have a resultant clockwise couple $M = F_3d$.

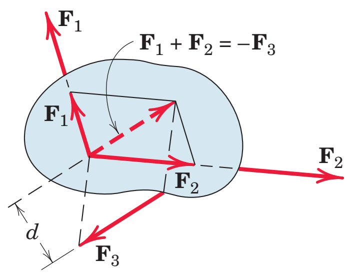
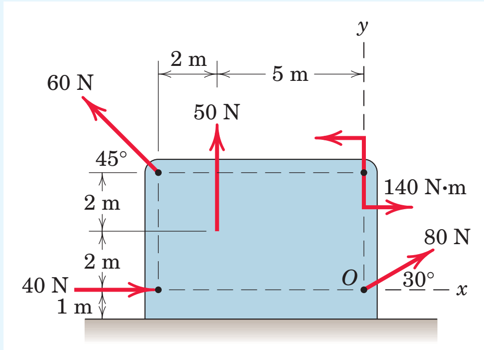


Figure 2/15

SAMPLE PROBLEM 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution. Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9$ N

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4$ N

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3$ N *Ans.*

$\left[\theta = \tan^{-1} \frac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1} \frac{132.4}{66.9} = 63.2°$ *Ans.*

1 $[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$= -237$ N·m

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.

We now determine the final line of action of **R** such that **R** alone represents the original system.

$[Rd = |M_O|]$ $\quad 148.3d = 237 \quad d = 1.600$ m *Ans.*

Hence, the resultant **R** may be applied at any point on the line which makes a 63.2° angle with the x-axis and is tangent at point A to a circle of 1.600-m radius with center O, as shown in part b of the figure. We apply the equation $Rd = M_O$ in an absolute-value sense (ignoring any sign of M_O) and let the physics of the situation, as depicted in Fig. *a*, dictate the final placement of **R**. Had M_O been counterclockwise, the correct line of action of **R** would have been the tangent at point B.

The resultant **R** may also be located by determining its intercept distance b to point C on the x-axis, Fig. *c*. With R_x and R_y acting through point C, only R_y exerts a moment about O so that

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$

Alternatively, the y-intercept could have been obtained by noting that the moment about O would be due to R_x only.

A more formal approach in determining the final line of action of **R** is to use the vector expression

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ is a position vector running from point O to any point on the line of action of **R**. Substituting the vector expressions for **r**, **R**, and $\mathbf{M}_O$ and carrying out the cross product result in

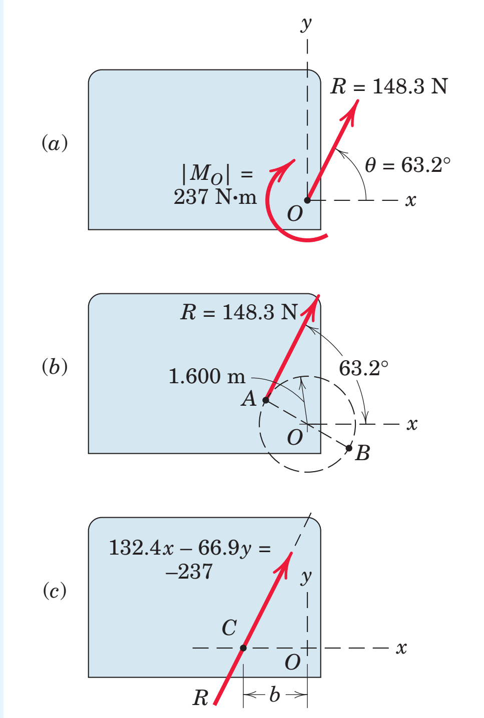
$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

Thus, the desired line of action, Fig. *c*, is given by

$$132.4x - 66.9y = -237$$

2 By setting $y = 0$, we obtain $x = -1.792$ m, which agrees with our earlier calculation of the distance b.



Helpful Hints

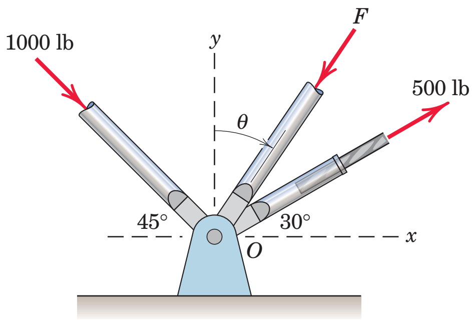
1 We note that the choice of point O as a moment center eliminates any moments due to the two forces which pass through O. Had the clockwise sign convention been adopted, M_O would have been +237 N·m, with the plus sign indicating a sense which agrees with the sign convention. Either sign convention, of course, leads to the conclusion of a clockwise moment M_O.

2 Note that the vector approach yields sign information automatically, whereas the scalar approach is more physically oriented. You should master both methods.

PROBLEMS

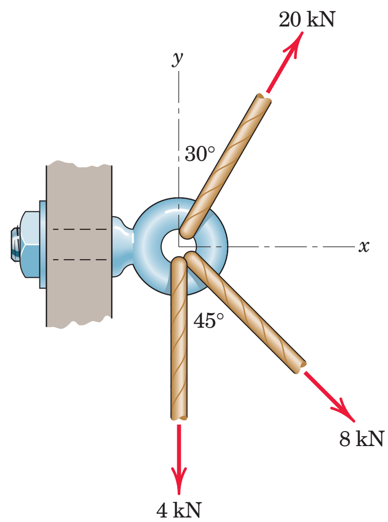
Introductory Problems

2/79 Two rods and one cable are attached to the support at O. If two of the forces are as shown, determine the magnitude F and direction θ of the third force so that the resultant of the three forces is vertically downward with a magnitude of 1200 lb.



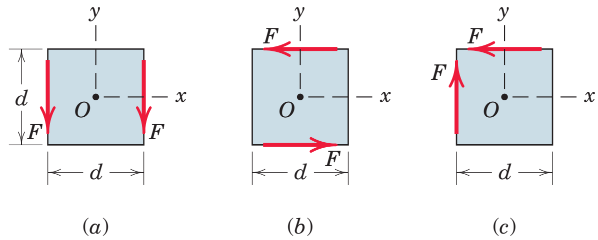
Problem 2/79

2/80 Determine the resultant **R** of the three tension forces acting on the eye bolt. Find the magnitude of **R** and the angle θ_x which **R** makes with the positive x-axis.



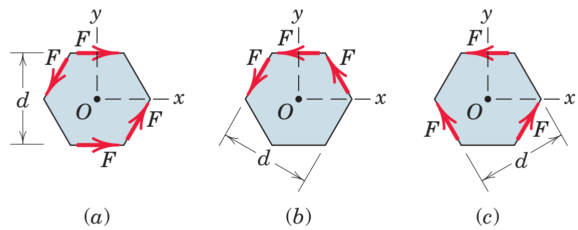
Problem 2/80

2/81 Determine the equivalent force–couple system at the center O for each of the three cases of forces being applied along the edges of a square plate of side d.



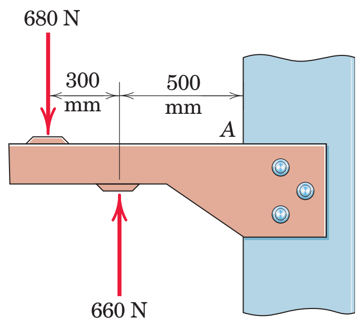
Problem 2/81

2/82 Determine the equivalent force–couple system at the origin O for each of the three cases of forces being applied along the edges of a regular hexagon of width d. If the resultant can be so expressed, replace this force–couple system with a single force.



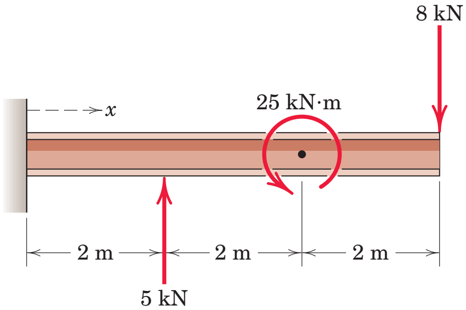
Problem 2/82

2/83 Where does the resultant of the two forces act?



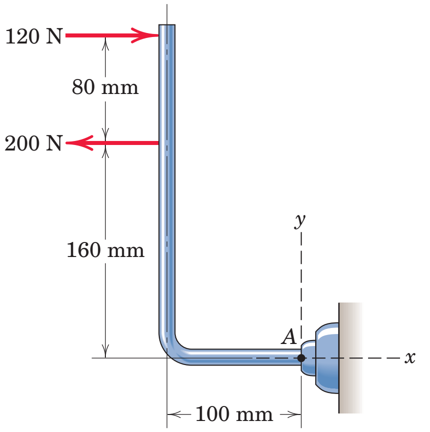
Problem 2/83

2/84 Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.



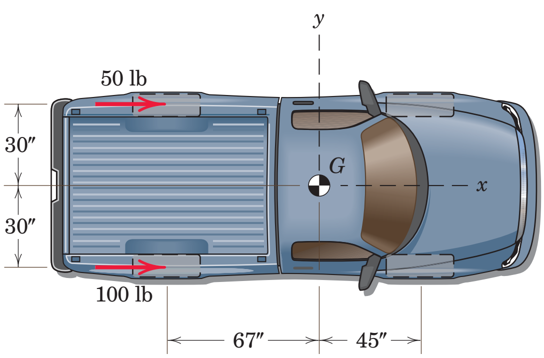
Problem 2/84

2/85 Replace the two forces acting on the bent pipe by a single equivalent force **R**. Specify the distance *y* from point *A* to the line of action of **R**.



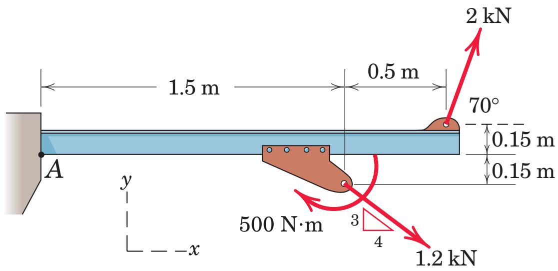
Problem 2/85

2/86 Under nonuniform and slippery road conditions, the two forces shown are exerted on the two rear-drive wheels of the pickup truck, which has a limited-slip rear differential. Determine the *y*-intercept of the resultant of this force system.



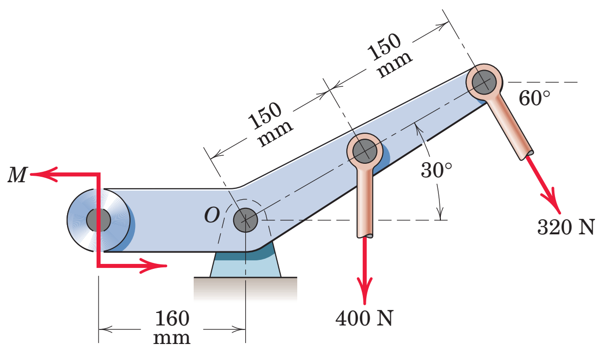
Problem 2/86

2/87 The flanged steel cantilever beam with riveted bracket is subjected to the couple and two forces shown, and their effect on the design of the attachment at *A* must be determined. Replace the two forces and couple by an equivalent couple *M* and resultant force **R** at *A*.



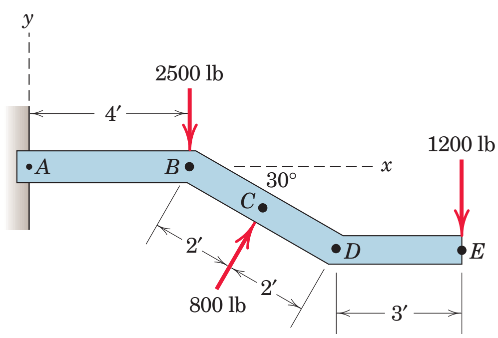
Problem 2/87

2/88 If the resultant of the two forces and couple *M* passes through point *O*, determine *M*.



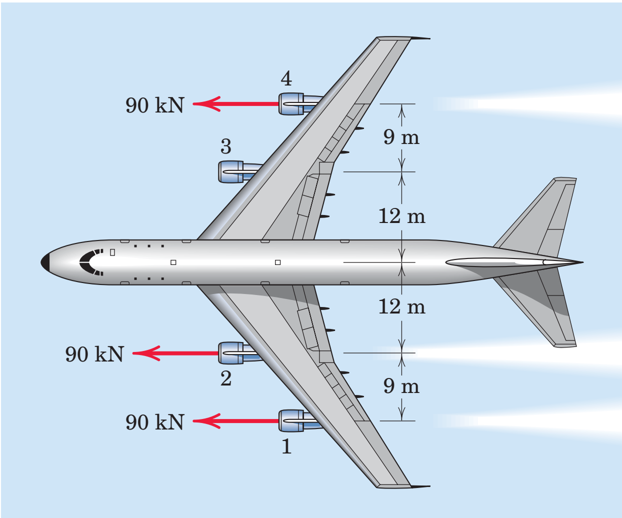
Problem 2/88

2/89 Replace the three forces which act on the bent bar by a force–couple system at the support point *A*. Then determine the *x*-intercept of the line of action of the stand-alone resultant force **R**.



Problem 2/89

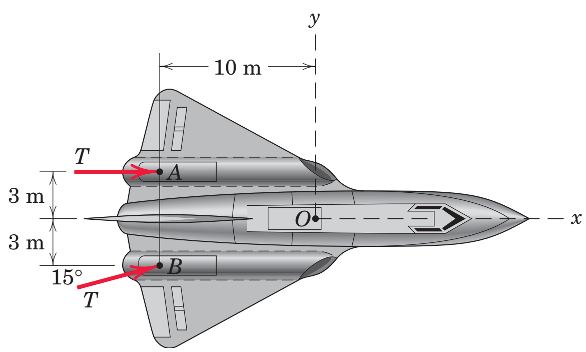
2/90 A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two-dimensional problem.



Problem 2/90

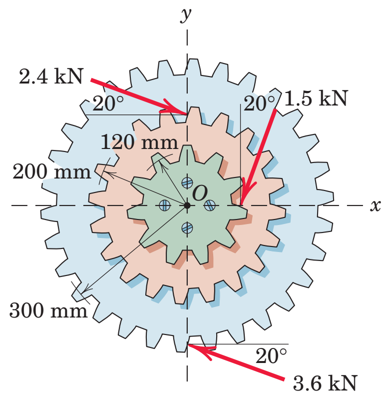
Representative Problems

2/91 The directions of the two thrust vectors of an experimental aircraft can be independently changed from the conventional forward direction within limits. For the thrust configuration shown, determine the equivalent force–couple system at point O. Then replace this force–couple system by a single force and specify the point on the x-axis through which the line of action of this resultant passes. These results are vital to assessing design performance.



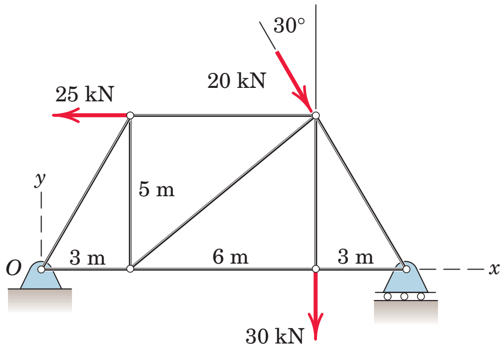
Problem 2/91

2/92 Determine the x- and y-axis intercepts of the line of action of the resultant of the three loads applied to the gearset.



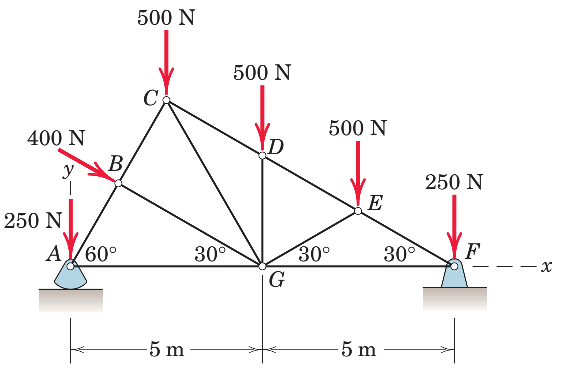
Problem 2/92

2/93 Determine the resultant **R** of the three forces acting on the simple truss. Specify the points on the x- and y-axes through which **R** must pass.



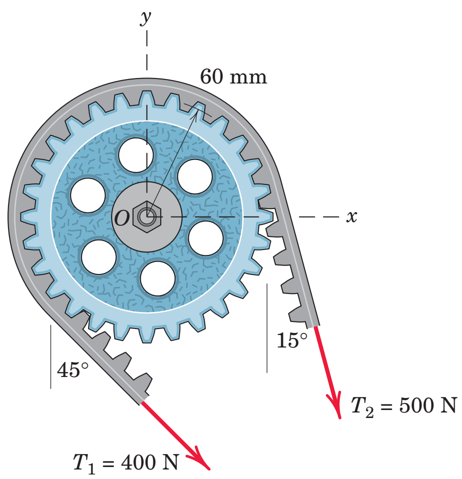
Problem 2/93

2/94 The asymmetric roof truss is of the type used when a near normal angle of incidence of sunlight onto the south-facing surface *ABC* is desirable for solar energy purposes. The five vertical loads represent the effect of the weights of the truss and supported roofing materials. The 400-N load represents the effect of wind pressure. Determine the equivalent force–couple system at *A*. Also, compute the *x*-intercept of the line of action of the system resultant treated as a single force **R**.



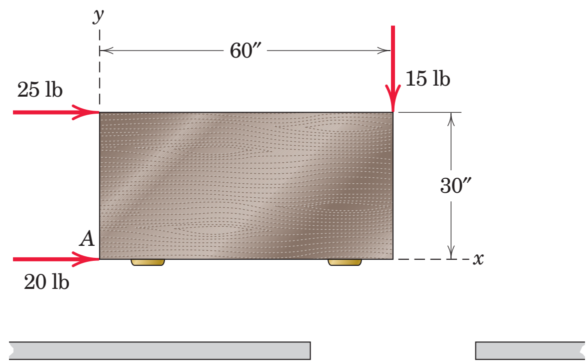
Problem 2/94

2/95 As part of a design test, the camshaft–drive sprocket is fixed and then the two forces shown are applied to a length of belt wrapped around the sprocket. Find the resultant of this system of two forces and determine where its line of action intersects both the *x*- and *y*-axes.



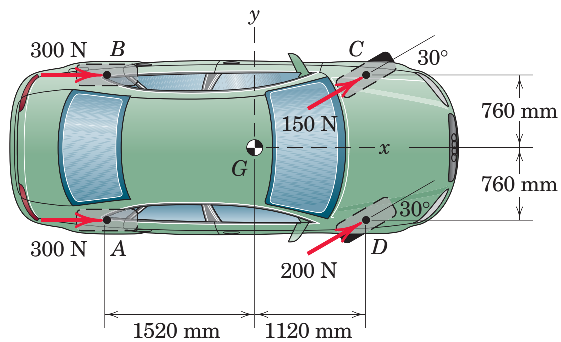
Problem 2/95

2/96 While sliding a desk toward the doorway, three students exert the forces shown in the overhead view. Determine the equivalent force–couple system at point *A*. Then determine the equation of the line of action of the resultant force.



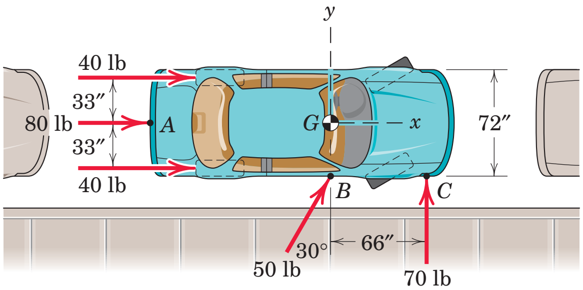
Problem 2/96

2/97 Under nonuniform and slippery road conditions, the four forces shown are exerted on the four drive wheels of the all-wheel-drive vehicle. Determine the resultant of this system and the *x*- and *y*-intercepts of its line of action. Note that the front and rear tracks are equal (i.e., $\overline{AB} = \overline{CD}$).



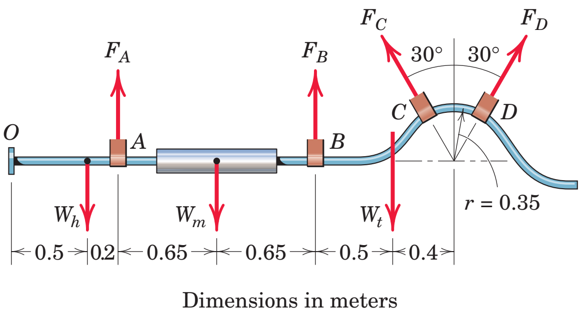
Problem 2/97

2/98 A rear-wheel-drive car is stuck in the snow between other parked cars as shown. In an attempt to free the car, three students exert forces on the car at points A, B, and C while the driver's actions result in a forward thrust of 40 lb acting parallel to the plane of rotation of each rear wheel. Treating the problem as two-dimensional, determine the equivalent force–couple system at the car center of mass G and locate the position x of the point on the car centerline through which the resultant passes. Neglect all forces not shown.



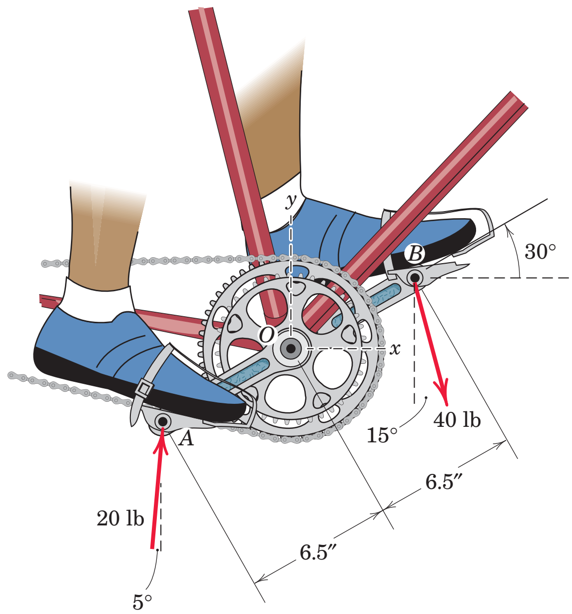
Problem 2/98

2/99 An exhaust system for a pickup truck is shown in the figure. The weights W_h, W_m, and W_t of the headpipe, muffler, and tailpipe are 10, 100, and 50 N, respectively, and act at the indicated points. If the exhaust-pipe hanger at point A is adjusted so that its tension F_A is 50 N, determine the required forces in the hangers at points B, C, and D so that the force–couple system at point O is zero. Why is a zero force–couple system at O desirable?



Problem 2/99

2/100 The pedal–chainwheel unit of a bicycle is shown in the figure. The left foot of the rider exerts the 40-lb force, while the use of toe clips allows the right foot to exert the nearly upward 20-lb force. Determine the equivalent force–couple system at point O. Also, determine the equation of the line of action of the system resultant treated as a single force **R**. Treat the problem as two-dimensional.



Problem 2/100