جامـــــعة المـــــستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلـــوم
قـســـــــم علوم الذكاء الاصطناعي

المحاضرة الرابعة

المادة : mathematics
المرحلة : الاولى
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MATRICES المصفوفات

Inverse of a Matrix (معكوس المصفوفة)

A square matrix A has an inverse A^{-1} , if $A * A^{-1} = I$ where I I is the identity matrix. The formula to calculate the inverse:

$$A^{-1} = \frac{1}{\det(A)} Adj(A)$$

Where $\det(A)$ the determinant,

and $Adj(A)$ is the adjoint (or adjugate) matrix.

Adjoint of a Matrix (المصفوفة المرافقة):

- Calculate minors: For each element, compute the determinant of the sub matrix obtained by removing its row and column.
- Add cofactors: Multiply minors by the cofactor sign $(-1)^{i+j}$
- Transpose the cofactor matrix.

Example1: Given the 2×2 matrix:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

- $det(A) = (2 * 4) - (3 * 1) = 5$
- Cofactor matrix:

Cofactor $a_{11} = (-1)^{1+1} * |4| = 4$

Cofactor $a_{12} = (-1)^{1+2} * |1| = -1$

Cofactor $a_{21} = (-1)^{2+1} * |3| = -3$

Cofactor $a_{22} = (-1)^{2+2} * |2| = 2$

Cofactor matrix (A) $= \begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$:

- Transpose the cofactor matrix:

$$\text{Adj(A)} = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} Adj(A)$$

$$A^{-1} = \frac{1}{5} * \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

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Cramer's Rule for Solving Systems of Equations (قاعدة كرامر لحل نظام المعادلات)

Cramer's Rule solves systems of linear equations using determinants.

General Formula: For the system:

$$a_1 x - b_1 y = c_1$$

$$a_2 x - b_2 y = c_2$$

1- Compute the main determinant D

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

2- Compute the main determinant D_X, D_y

$$D_X = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$D_X = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

3- Solutions:

$$x = \frac{D_X}{D} \quad , \quad y = \frac{D_y}{D},$$

Example: For the system:

$$2x - 3y = 8$$

$$x - 2y = -1$$

1- Compute the main determinant D

$$D = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = (2 * -2) - (3 * 1) = -7$$

2- Compute the main determinant D_X, D_y

$$D_X = \begin{vmatrix} 8 & 3 \\ -1 & -2 \end{vmatrix} = -13$$

$$D_X = \begin{vmatrix} 2 & 8 \\ 1 & -1 \end{vmatrix} = -10$$

3- Solutions:

$$x = \frac{D_X}{D} = \frac{-13}{-7} \quad ,$$

$$y = \frac{D_y}{D} = \frac{-10}{-7},$$