جامـــــعـة المـــــسـتـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلـــــوم
قــســــــــــم علوم الذكاء الاصطناعي

المحاضرة الثانية

المادة : mathematics
المرحلة : الاولى
اسم الاستاذ: م.د. رياض حامد سلمان

MATRICES المصفوفات

The Trace of a Matrix:(اثر المصفوفة)

If there is a square matrix A (n x n), the **trace of this matrix** is defined as the sum of the diagonal elements in the square matrix. For example

$$A = \begin{pmatrix} -2 & 5 & 12 \\ 3 & 6 & -5 \\ 1 & 8 & 4 \end{pmatrix}$$

$$\because trace\ A = tr\ A = \sum_{i-1}^{n} a_{ii}$$

$$\therefore tr\ A = -2 + 6 + 4 = 8$$

The trace satisfies the following properties:

$$tr\left(A+B\right) = tr\ A + tr\ B$$

$$tr\ CA = C\ tr\ A$$

$$tr\left(AB\right) = tr\left(BA\right)$$

$$tr\ I_n = n$$

$$tr\left(IA\right) = tr\left(A\right)$$

Some Special Matrices:

Row Matrix (مصفوفة الصف)

A matrix that has only **one row** is called a Row Matrix, and it is sometimes referred to as a Row Vector. This matrix is denoted by the symbol (A) and its order is (1 x n). For example:

$$(A)_{1\times 3} = (3 \quad 6 \quad 5)$$

Column Matrix:

A matrix that has only one column is called a **Column Matrix**, and it is sometimes referred to as a **Column Vector**. This matrix is denoted by the symbol **[A]**, and its order is **(m x 1)**. For example:

$$:[A]_{4\times 1} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 7 \end{bmatrix}$$

Square Matrix (المصفوفة المربعة)

A square matrix is a matrix in which the number of rows equals the number of columns. For example, the following matrix:

$$\begin{pmatrix} 1 & -3 & 4 \\ 2 & 4 & 3 \\ 6 & 5 & 7 \end{pmatrix}$$

Diagonal Matrix (المصفوفة القطرية)

A diagonal matrix is a square matrix in which all elements are zero except for the elements on the main diagonal (the diagonal that runs from the top-left element to the bottom-right element $(a_{11} a_{22}, \dots, a_{mn})$ For example, the following matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 8 \end{pmatrix}$$

المصفوفة القياسية ومصفوفة الوحدة: Scalar and Unit Matrix

A diagonal matrix in which all the elements on its main diagonal are equal is called **a Scalar Matrix**. If the elements on the main diagonal of a scalar matrix are equal to one, it is called a **Unit Matrix** or **Identity Matrix**. This matrix is denoted by the symbol I_n where **n x n** represents the order of the matrix. For example:

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In general, if **A** is a square matrix of order **(m x m)** and I_{mxn} is the unit (identity) matrix of the same order, then:

$$IA = AI = A$$
$$I = I^2 = I^3 = ... = I^k$$

حيث k عدد صحيح موجب ويمكن إثبات ذلك بسهولة.

By multiplying any matrix **A** by the identity matrix **I**, the matrix **A** remains unchanged, provided that the multiplication of the two matrices follows the previously stated rules of matrix multiplication. For example:

فبضرب أي مصفوفة A في مصفوفة الوحدة تبقى المصفوفة A كما هي بدون تغيير بفرض قابلية ضرب المصفوفتين حسب قانون ضرب المصفوفات السابق، فمثلاً:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

كذلك إذا كانت:

$$A=\begin{pmatrix}1&2&3\\0&1&3\\3&0&1\end{pmatrix}$$

$$\therefore IA=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\times\begin{pmatrix}1&2&3\\0&1&3\\3&0&1\end{pmatrix}=\begin{pmatrix}1&2&3\\0&1&3\\3&0&1\end{pmatrix}=A$$

$$\therefore AI=\begin{pmatrix}1&2&3\\0&1&3\\3&0&1\end{pmatrix}\times\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}1&2&3\\0&1&3\\3&0&1\end{pmatrix}=A$$