Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

Subject Name: Statics

1st Class, First Semester

Subject Code: [UOMU011011]

Academic Year: 2024-2025

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Lecture No.: 5

Lecture Title: [Couple Moment]

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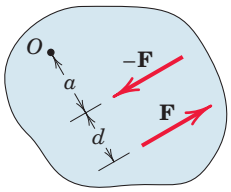
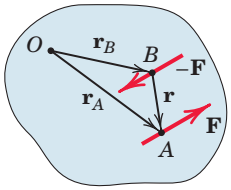


(a)



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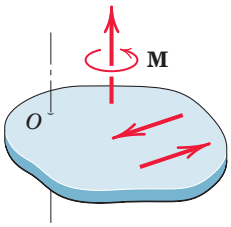
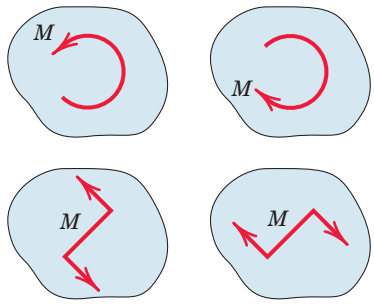


(c)



(d)

Figure 2/10

2/5 Couple

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ a distance d apart, as shown in Fig. 2/10*a*. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple $\mathbf{M}$. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

$$M = Fd$$

Its direction is counterclockwise when viewed from above for the case illustrated. Note especially that the magnitude of the couple is independent of the distance a which locates the forces with respect to the moment center O. It follows that the moment of a couple has the same value for all moment centers.

Vector Algebra Method

We may also express the moment of a couple by using vector algebra. With the cross-product notation of Eq. 2/6, the combined moment about point O of the forces forming the couple of Fig. 2/10*b* is

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

where $\mathbf{r}_A$ and $\mathbf{r}_B$ are position vectors which run from point O to arbitrary points A and B on the lines of action of $\mathbf{F}$ and $-\mathbf{F}$, respectively. Because $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, we can express $\mathbf{M}$ as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Here again, the moment expression contains no reference to the moment center O and, therefore, is the same for all moment centers. Thus, we may represent $\mathbf{M}$ by a free vector, as shown in Fig. 2/10*c*, where the direction of $\mathbf{M}$ is normal to the plane of the couple and the sense of $\mathbf{M}$ is established by the right-hand rule.

Because the couple vector $\mathbf{M}$ is always perpendicular to the plane of the forces which constitute the couple, in two-dimensional analysis we can represent the sense of a couple vector as clockwise or counterclockwise by one of the conventions shown in Fig. 2/10*d*. Later, when we deal with couple vectors in three-dimensional problems, we will make full use of vector notation to represent them, and the mathematics will automatically account for their sense.

Equivalent Couples

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. Figure 2/11

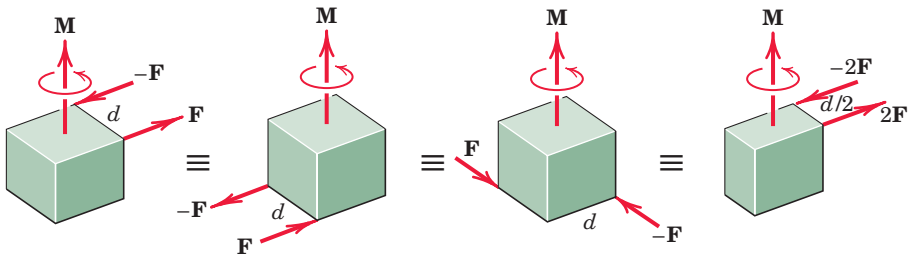


Figure 2/11

shows four different configurations of the same couple **M**. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

Force-Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig. 2/12, where the given force **F** acting at point A is replaced by an equal force **F** at some point B and the counterclockwise couple $M = Fd$. The transfer is seen in the middle figure, where the equal and opposite forces **F** and $-\mathbf{F}$ are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple $M = Fd$, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. 2/12 is referred to as a *force–couple system*.

By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force. Replacement of a force by an equivalent force–couple system, and the reverse procedure, have many applications in mechanics and should be mastered.

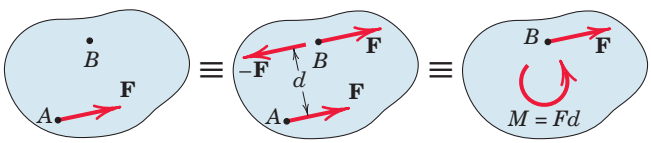
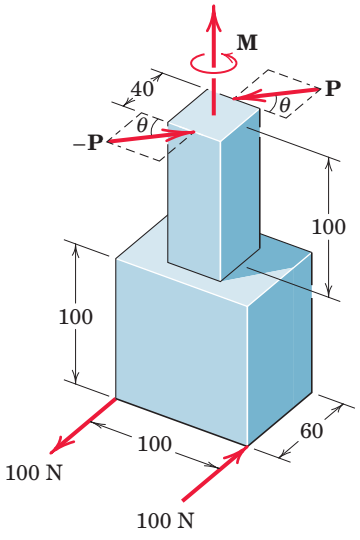


Figure 2/12

SAMPLE PROBLEM 2/7

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and $-\mathbf{P}$, each of which has a magnitude of 400 N. Determine the proper angle θ.



Dimensions in millimeters

Solution. The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $\qquad M = 100(0.1) = 10\ \text{N}\cdot\text{m}$

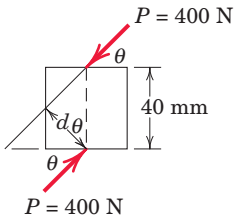
The forces **P** and $-\mathbf{P}$ produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

1 Equating the two expressions gives

$$10 = (400)(0.040)\cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3^\circ \qquad \textit{Ans.}$$

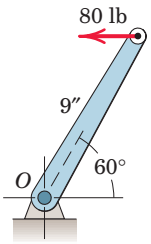


Helpful Hint

1 Since the two equal couples are parallel free vectors, the only dimensions which are relevant are those which give the perpendicular distances between the forces of the couples.

SAMPLE PROBLEM 2/8

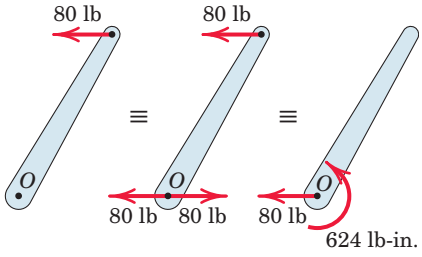
Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at O and a couple.



Solution. We apply two equal and opposite 80-lb forces at O and identify the counterclockwise couple

$[M = Fd]$ $\qquad M = 80(9\sin 60^\circ) = 624\ \text{lb-in.}$ *Ans.*

1 Thus, the original force is equivalent to the 80-lb force at O and the 624-lb-in. couple as shown in the third of the three equivalent figures.



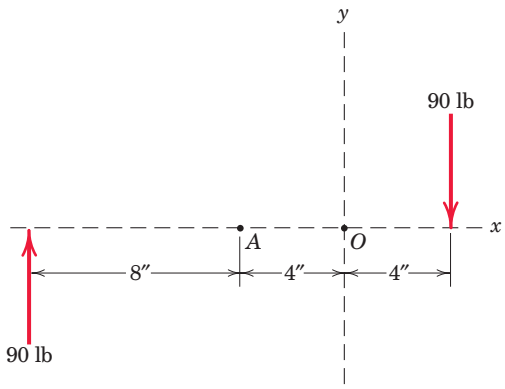
Helpful Hint

1 The reverse of this problem is often encountered, namely, the replacement of a force and a couple by a single force. Proceeding in reverse is the same as replacing the couple by two forces, one of which is equal and opposite to the 80-lb force at O. The moment arm to the second force would be $M/F = 624/80 = 7.79$ in., which is 9 sin 60°, thus determining the line of action of the single resultant force of 80 lb.

PROBLEMS

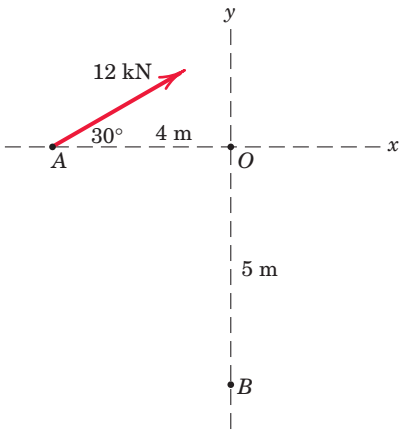
Introductory Problems

2/59 Compute the combined moment of the two 90-lb forces about (*a*) point *O* and (*b*) point *A*.



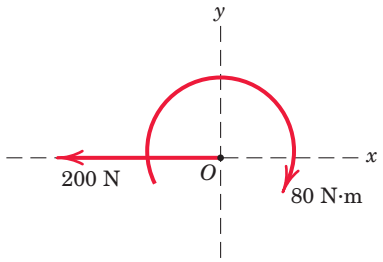
Problem 2/59

2/60 Replace the 12-kN force acting at point *A* by a force–couple system at (*a*) point *O* and (*b*) point *B*.



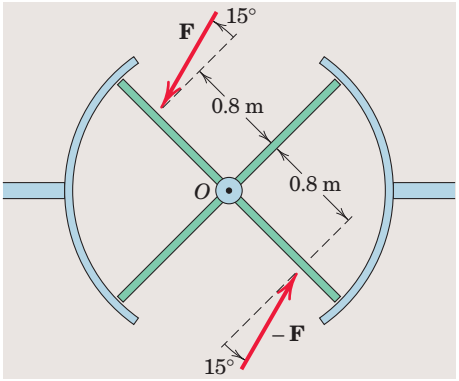
Problem 2/60

2/61 Replace the force–couple system at point *O* by a single force. Specify the coordinate y_A of the point on the *y*-axis through which the line of action of this resultant force passes.



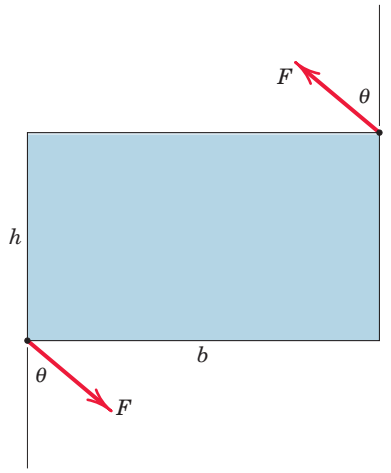
Problem 2/61

2/62 The top view of a revolving entrance door is shown. Two persons simultaneously approach the door and exert force of equal magnitudes as shown. If the resulting moment about the door pivot axis at *O* is 25 N·m, determine the force magnitude *F*.



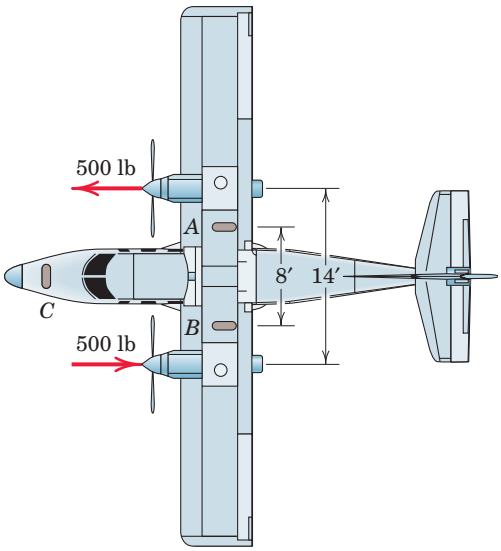
Problem 2/62

2/63 Determine the moment associated with the couple applied to the rectangular plate. Reconcile the results with those for the individual special cases of $\theta = 0$, $b = 0$, and $h = 0$.



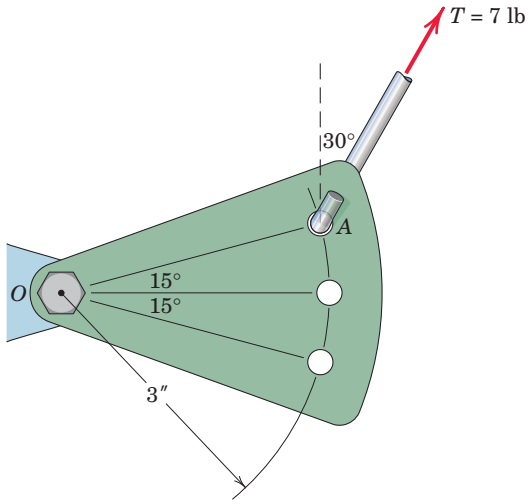
Problem 2/63

2/64 As part of a test, the two aircraft engines are revved up and the propeller pitches are adjusted so as to result in the fore and aft thrusts shown. What force F must be exerted by the ground on each of the main braked wheels at A and B to counteract the turning effect of the two propeller thrusts? Neglect any effects of the nose wheel C, which is turned 90° and unbraked.



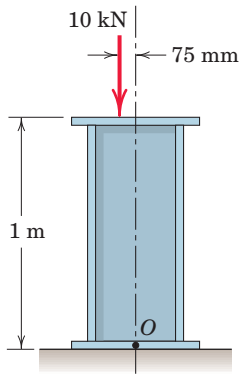
Problem 2/64

2/65 The 7-lb force is applied by the control rod on the sector as shown. Determine the equivalent force–couple system at O.



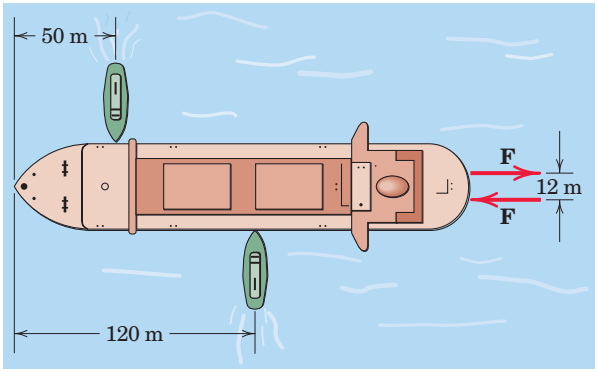
Problem 2/65

2/66 Replace the 10-kN force acting on the steel column by an equivalent force–couple system at point O. This replacement is frequently done in the design of structures.



Problem 2/66

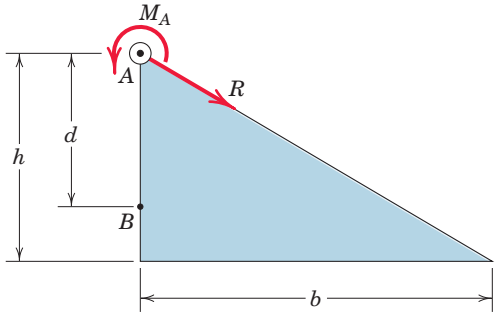
2/67 Each propeller of the twin-screw ship develops a full-speed thrust of 300 kN. In maneuvering the ship, one propeller is turning full speed ahead and the other full speed in reverse. What thrust P must each tug exert on the ship to counteract the effect of the ship's propellers?



Problem 2/67

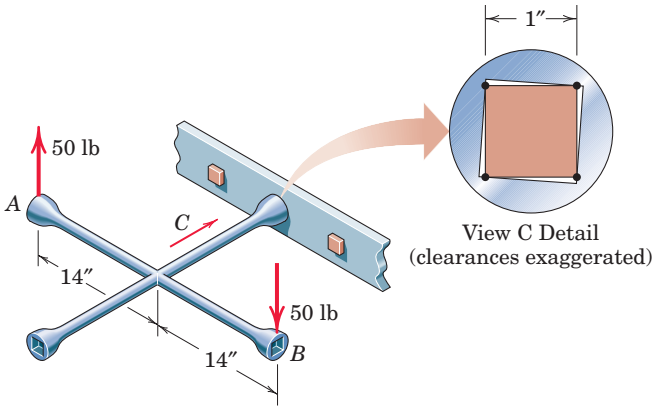
Representative Problems

2/68 The force–couple system at A is to be replaced by a single equivalent force acting at a point B on the vertical edge (or its extension) of the triangular plate. Determine the distance d between A and B.



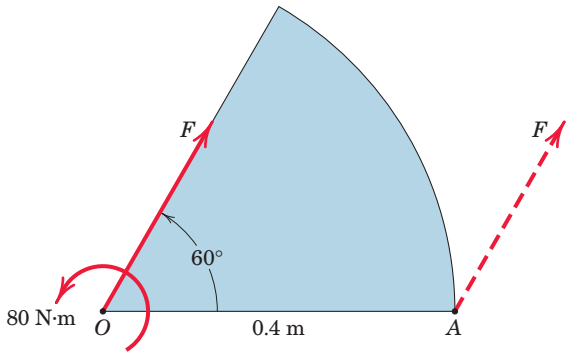
Problem 2/68

2/69 A lug wrench is used to tighten a square-head bolt. If 50-lb forces are applied to the wrench as shown, determine the magnitude F of the equal forces exerted on the four contact points on the 1-in. bolt head so that their external effect on the bolt is equivalent to that of the two 50-lb forces. Assume that the forces are perpendicular to the flats of the bolt head.



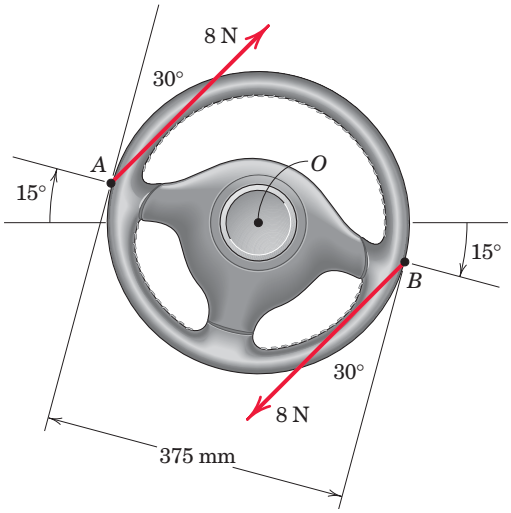
Problem 2/69

2/70 A force–couple system acts at O on the 60° circular sector. Determine the magnitude of the force F if the given system can be replaced by a stand-alone force at corner A of the sector.



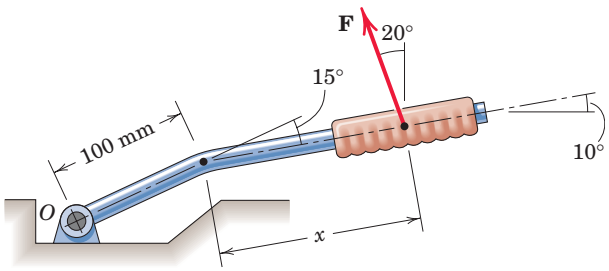
Problem 2/70

2/71 During a steady right turn, a person exerts the forces shown on the steering wheel. Note that each force consists of a tangential component and a radially-inward component. Determine the moment exerted about the steering column at *O*.



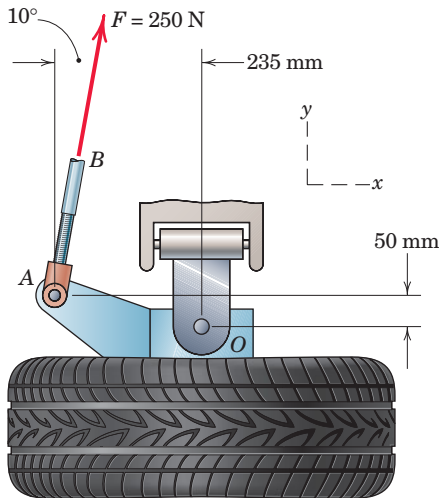
Problem 2/71

2/72 A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point *O*.



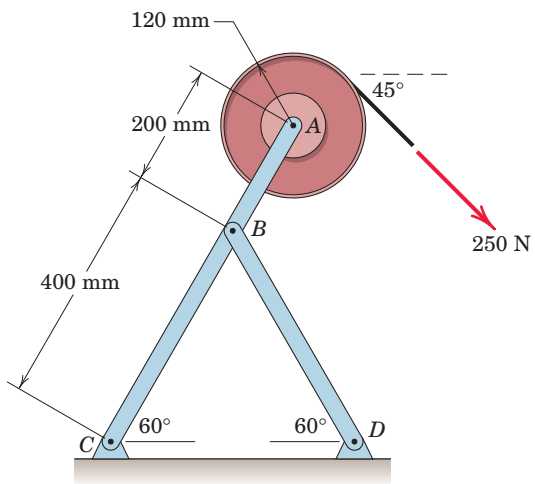
Problem 2/72

2/73 The tie-rod *AB* exerts the 250-N force on the steering knuckle *AO* as shown. Replace this force by an equivalent force–couple system at *O*.



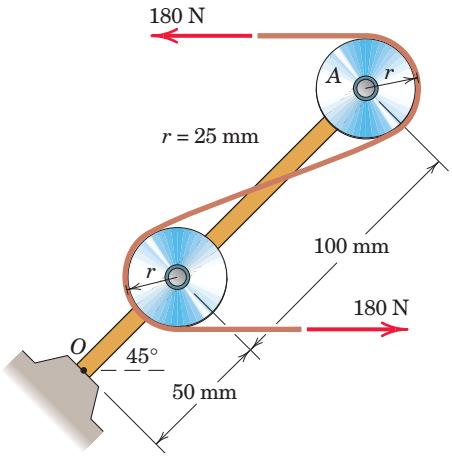
Problem 2/73

2/74 The 250-N tension is applied to a cord which is securely wrapped around the periphery of the disk. Determine the equivalent force–couple system at point *C*. Begin by finding the equivalent force–couple system at *A*.



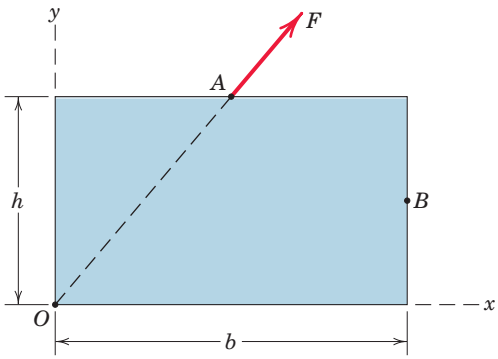
Problem 2/74

2/75 The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force–couple system at point O.



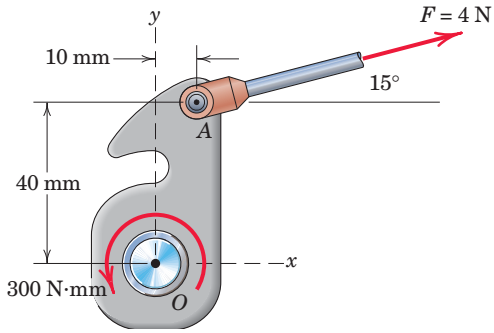
Problem 2/75

2/76 Points A and B are the midpoints of the sides of the rectangle. Replace the given force F acting at A by a force–couple system at B.



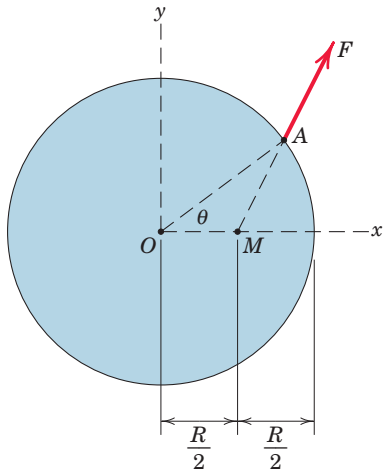
Problem 2/76

2/77 The device shown is a part of an automobile seat-back-release mechanism. The part is subjected to the 4-N force exerted at A and a 300-N·mm restoring moment exerted by a hidden torsional spring. Determine the y-intercept of the line of action of the single equivalent force.



Problem 2/77

2/78 The force F acts along line MA, where M is the midpoint of the radius along the x-axis. Determine the equivalent force–couple system at O if $\theta = 40°$.



Problem 2/78