Al-Mustaqbal university

Engineering technical college

Department of Building
&Construction Engineering

Mathematics

First class

Lecture No.8

Assist. Lecture

Alaa Hussein AbdUlameer

2. Inverse trigonometric functions

Inverse trigonometric functions are simply defined as the inverse functions of the basic trigonometric functions which are sine, cosine, tangent, cotangent, secant, and cosecant functions. They are also termed as arcus functions, antitrigonometric functions or cyclometric functions. These inverse functions in trigonometry are used to get the angle with any of the trigonometry ratios. The inverse trigonometry functions have major applications in the field of engineering, physics, geometry and navigation.

What are Inverse Trigonometric Functions?

Inverse trigonometric functions are also called "**Arc Functions**" since, for a given value of trigonometric functions, they produce the length of arc needed to obtain that particular value. The inverse trigonometric functions actually performs the opposite operation of the trigonometric functions such as sine, cosine, tangent, cosecant, secant, and cotangent. We know that, trig functions are specially applicable to the right angle triangle. These six important functions are used to find the angle measure in a right triangle when two sides of the triangle measures are known.

Formulas

The basic inverse trigonometric formulas are as follows:

Inverse Trig Functions	**Formulas**
Arcsine	$\sin^{-1}(-x) = -\sin^{-1}(x), x \in [-1, 1]$
Arccosine	$\cos^{-1}(-x) = \pi - \cos^{-1}(x), x \in [-1, 1]$
Arctangent	$\tan^{-1}(-x) = -\tan^{-1}(x), x \in R$
Arccotangent	$\cot^{-1}(-x) = \pi - \cot^{-1}(x), x \in R$
Arcsecant	$\sec^{-1}(-x) = \pi - \sec^{-1}(x), \lvert x \rvert \geq 1$
Arccosecant	$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x), \lvert x \rvert \geq 1$

Inverse Trigonometric Functions Graphs

There are particularly six inverse trig functions for each trigonometry ratio. The inverse of six important trigonometric functions are:

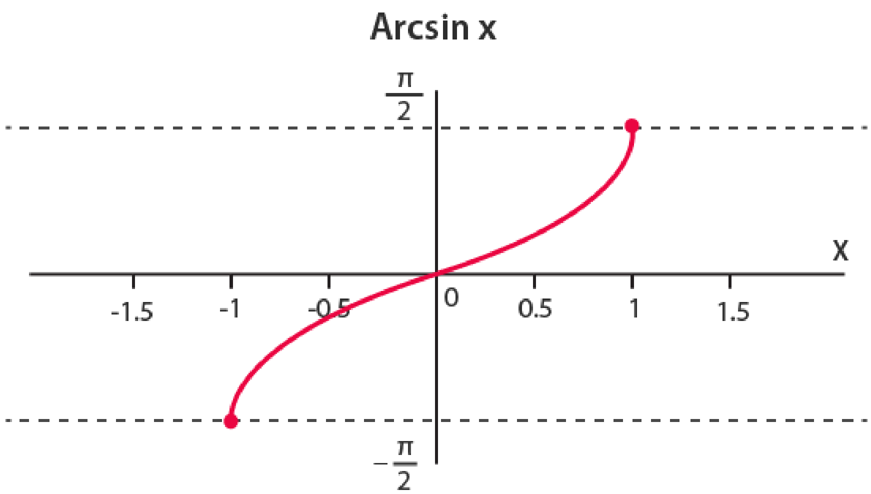
- Arcsine
- Arccosine
- Arctangent
- Arccotangent
- Arcsecant

- Arccosecant

Let us discuss all the six important types of inverse trigonometric functions along with its definition, formulas, graphs, properties and solved examples.

Arcsine Function

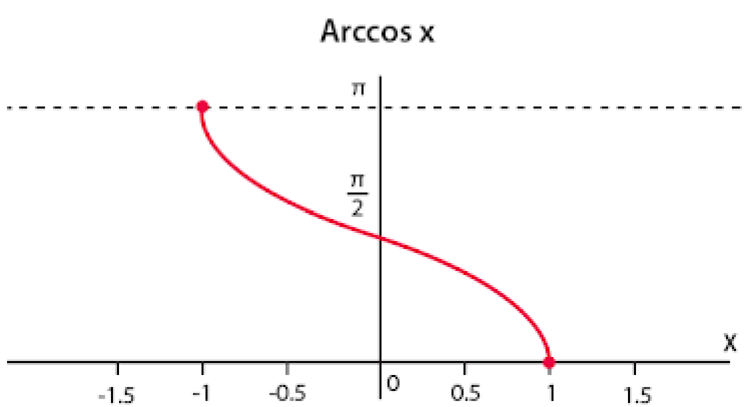
Arcsine function is an inverse of the sine function denoted by $\sin^{-1}x$. It is represented in the graph as shown below:



Domain	$-1 \leq x \leq 1$
Range	$-\pi/2 \leq y \leq \pi/2$

Arccosine Function

Arccosine function is the inverse of the cosine function denoted by $\cos^{-1}x$. It is represented in the graph as shown below:

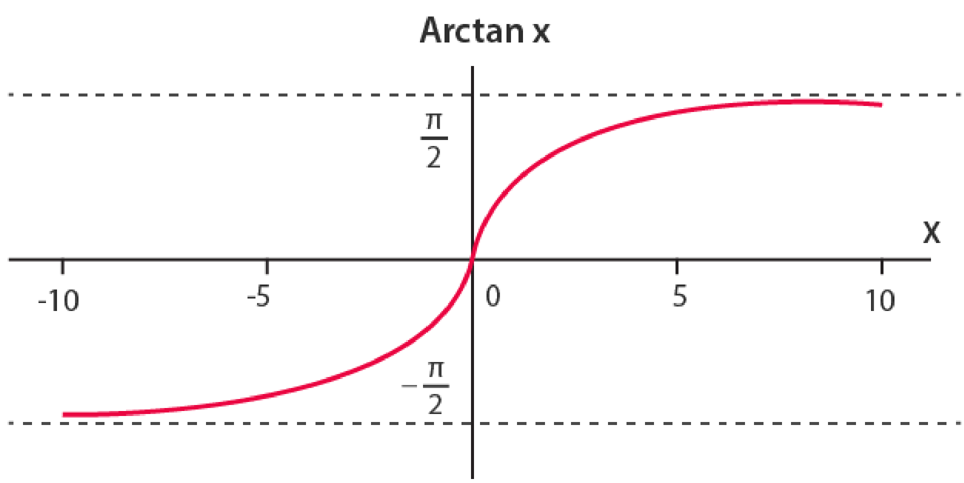


Therefore, the inverse of cos function can be expressed as; **y = cos^{-1}x (arccosine *x*)Domain & Range of arcsine function:**

Domain	$-1 \leq x \leq 1$
Range	$0 \leq y \leq \pi$

Arctangent Function

Arctangent function is the inverse of the tangent function denoted by tan^{-1}*x*. It is represented in the graph as shown below:

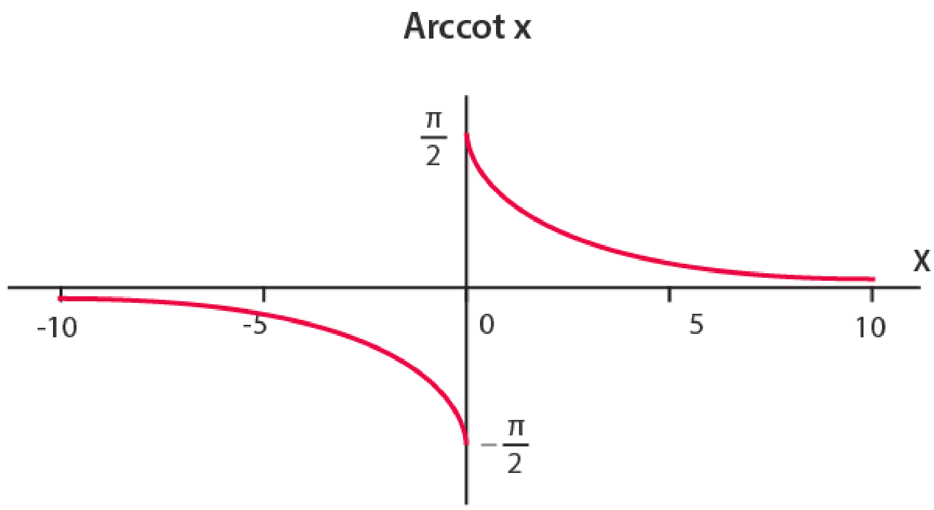


Therefore, the inverse of tangent function can be expressed as; **y = tan^{-1}x (arctangent *x*)Domain & Range of Arctangent:**

Domain	$-\infty < x < \infty$
Range	$-\pi/2 < y < \pi/2$

Arccotangent (Arccot) Function

Arccotangent function is the inverse of the cotangent function denoted by cot^{-1}*x*. It is represented in the graph as shown below:

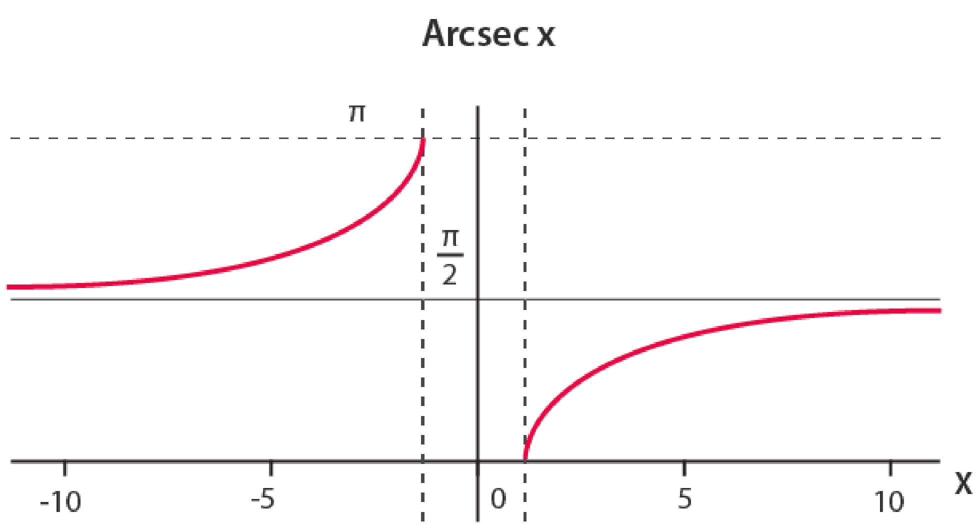


Therefore, the inverse of cotangent function can be expressed as; **y = cot^{-1}x (arccotangent *x*)** Domain & Range of Arccotangent:

Domain	-∞ < x < ∞
Range	0 < y < π

Arcsecant Function

What is arcsecant (arcsec)function? Arcsecant function is the inverse of the secant function denoted by sec^{-1}*x*. It is represented in the graph as shown below:

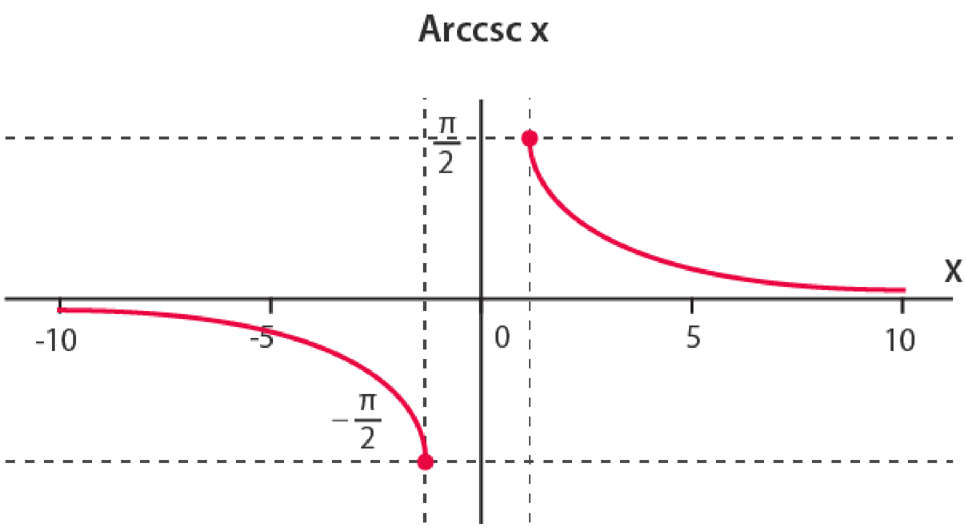


Therefore, the inverse of secant function can be expressed as; **y = sec^{-1}x (arcsecant *x*)Domain & Range of Arcsecant:**

Domain	-∞ ≤ x ≤ -1 or 1 ≤ x ≤ ∞
Range	-π/2 < y < π/2 ; y≠0

Arccosecant Function

What is arccosecant (arccsc *x*) function? Arccosecant function is the inverse of the cosecant function denoted by cosec^{-1}*x*. It is represented in the graph as shown below:



Therefore, the inverse of cosecant function can be expressed as; **y = cosec^{-1}x (arccosecant *x*)Domain & Range of Arccosecant**

Domain	-∞ ≤ x ≤ -1 or 1 ≤ x ≤ ∞
Range	-π/2 < y < π/2 ; y≠0

Inverse Trigonometric Functions Table

Let us rewrite here all the inverse trigonometric functions with their notation, definition, domain and range.

Function Name	**Notation**	**Definition**	**Domain of x**	**Range**
Arcsine or inverse sine	y = sin^{-1}(x)	x=sin y	−1 ≤ x ≤ 1	• − π/2 ≤ y ≤ π/2 • -90°≤ y ≤ 90°

Arccosine or inverse cosine	$y=\cos^{-1}(x)$	$x=\cos y$	$-1 \leq x \leq 1$	• $0 \leq y \leq \pi$ • $0° \leq y \leq 180°$
Arctangent or Inverse tangent	$y=\tan^{-1}(x)$	$x=\tan y$	For all real numbers	• $-\pi/2 < y < \pi/2$ • $-90° < y < 90°$
Arccotangent or Inverse Cot	$y=\cot^{-1}(x)$	$x=\cot y$	For all real numbers	• $0 < y < \pi$ • $0° < y < 180°$
Arcsecant or Inverse Secant	$y = \sec^{-1}(x)$	$x=\sec y$	$x \leq -1$ or $1 \leq x$	• $0\leq y<\pi/2$ or $\pi/2<y\leq\pi$ • $0°\leq y<90°$ or $90°<y\leq 180°$
Arccosecant	$y=\csc^{-1}(x)$	$x=\csc y$	$x \leq -1$ or $1 \leq x$	• $-\pi/2\leq y<0$ or $0<y\leq\pi/2$ • $-90°\leq y<0°$ or $0°<y\leq 90°$

Inverse Trigonometric Functions Derivatives

The derivatives of inverse trigonometric functions are first-order derivatives. Let us check here the derivatives of all the six inverse functions.

Inverse Trig Function	**dy/dx**
$\sin^{-1}(x)$	$1/\sqrt{(1-x^2)}$
$\cos^{-1}(x)$	$-1/\sqrt{(1-x^2)}$
$\tan^{-1}(x)$	$1/(1+x^2)$
$\cot^{-1}(x)$	$-1/(1+x^2)$
$\sec^{-1}(x)$	$1/[\lvert x\rvert\sqrt{(x^2-1)}]$
$\csc^{-1}(x)$	$-1/[\lvert x\rvert\sqrt{(x^2-1)}]$

Inverse Trigonometric Functions Properties

The inverse trigonometric functions are also known as Arc functions. Inverse Trigonometric Functions are defined in a certain interval (under restricted domains).

Trigonometry Basics

Trigonometry basics include the basic trigonometry and trigonometric ratios such as sin x, cos x, tan x, cosec x, sec x and cot x. The following article from BYJU'S discusses the basic definition of another tool of trigonometry – Inverse Trigonometric Functions.

Inverse Trigonometric Functions Problems

Example 1: Find the value of x, for sin(x) = 2.

Solution: Given: sinx = 2 x =$\sin^{-1}$(2), which is not possible.

Hence, there is no value of x for which sin x = 2; since the domain of $\sin^{-1}$x is -1 to 1 for the values of x.

Example 2: Find the value of $\sin^{-1}$(sin (π/6)).

Solution:

$\sin^{-1}(\sin(\pi/6)) = \pi/6$ (Using identity $\sin^{-1}(\sin(x)) = x$)

Example 1:

$$\sin^{-1}\left(sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Example 2:

$$\sin^{-1}\left(sin\frac{2\pi}{3}\right)$$

$$\pi - \theta$$

$$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\sin^{-1}\left(sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Example 3:

$$\cos\left(\cos^{-1}(3)\right) = undefined$$

Example 4:

$$\sin^{-1}(\cos 60)$$

$$\sin^{-1} = \frac{1}{2} = 30 = \frac{\pi}{6}$$

Example 5:

$$\tan\left(\tan^{-1}\frac{2\pi}{6}\right) = \frac{2\pi}{6}$$

Example 6:

$$\cos^{-1}\left(cos\frac{5\pi}{4}\right)$$

$$2\pi - \theta$$

$$2\pi - \frac{5\pi}{4} = \frac{3\pi}{4}$$

Example 7:

$$f(x) = \sin x + 5 \; find\ the\ domain\ and\ range\ for\ f(x)^{-1}$$

$$y = sinx + 5$$

$$y - 5 = sinx$$

$$\sin^{-1}(y-5) = \sin^{-1} sinx$$

$$\sin^{-1}(y-5) = x$$

$$f(x)^{-1} = y = \sin^{-1}(x-5)$$

Domain:

$$-1 \leq (x-5) \leq 1$$

$$-1 + 5 \leq (x) \leq 1 + 5$$

$$4 \leq (x) \leq 6$$

Range

$$\frac{-\pi}{2} \leq \sin^{-1}(x-5) \leq \frac{\pi}{2}$$

Example 8: $If\ f(x) = 4((\sin \pi - 2x) - 1)$

$find\ the\ domain\ and\ range\ for\ f(x)^{-1}$

$$y = 4((\sin \pi - 2x) - 1) \div 4$$

$$\frac{y}{4} = (\sin \pi - 2x) - 1$$

$$\frac{y}{4} + 1 = (\sin \pi - 2x) * \sin^{-1}$$

$$\sin^{-1}(\frac{y}{4} + 1) = \sin^{-1}(\sin \pi - 2x)$$

$$\sin^{-1}(\frac{y}{4} + 1) = (\pi - 2x)$$

$$[\sin^{-1}(\frac{y}{4} + 1) - \pi = -2x] * -1$$

$$2 \div [2x = \pi - \sin^{-1}(\frac{y}{4} + 1)]$$

$$x = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{y}{4} + 1\right)$$

$$f(x)^{-1} = y = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{x}{4} + 1\right)$$

$Domain:$

$$-1 \le x\left(\frac{X}{4} + 1\right) \le 1$$

$$-2 \le x\,\frac{X}{4} \le 0$$

$$-8 \le X \le 0$$

Range:

$$\frac{-\pi}{2} \le \sin^{-1} y \le \frac{\pi}{2}$$

$$\frac{-\pi}{2} \le \sin^{-1}\left(\frac{X}{4} + 1\right) \le \frac{\pi}{2}$$

$$\frac{-\pi}{4} \le \frac{1}{2}\sin^{-1}\left(\frac{X}{4} + 1\right) \le \frac{\pi}{4}$$

$$\frac{\pi}{4} \ge \frac{-1}{2}\sin^{-1}\left(\frac{X}{4} + 1\right) \ge \frac{-\pi}{4}$$

$$\frac{3\pi}{4} \ge \frac{\pi}{2}\frac{-1}{2}\sin^{-1}\left(\frac{X}{4} + 1\right) \ge \frac{\pi}{4}$$

H.W find domain of function

1. If $f(x) = \frac{1}{\sin^{-1}(3x+3)}$

2. $f(x) = 2 + \sin^{-1}(3x + 5)$
   $find\ domain\ and\ range\ of\ the\ function$
3. $f(x) = \pi + 7\cos^{-1}(2x - 1)$
   $find\ domain\ and\ range\ of\ the\ function$
4. $\tan^{-1}(\tan \frac{3\pi}{2})$
5. $\sin^{-1}(\sin \frac{13\pi}{6})$