

Integral

→ Define محدد
→ Indefine غير محدد

$$\int f(x) \cdot dx = g(x) + C$$

$$\int_a^b f(x) \cdot dx = [g(x)]_a^b = g(b) - g(a) = \text{Value}$$

Rules

$$1- \int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$2- \int \underbrace{(ax+b)^n}_{\text{linear}} dx = \frac{(ax+b)^{n+1}}{(n+1) \cdot a} \rightarrow \text{X, value}$$

$$3- \int \underbrace{k}_{\text{constant}} dx = k \int dx = kx + C$$

$$4- \int \underbrace{k^{(ax+b)}}_{e^x \text{ also}} dx = \frac{k^{ax+b}}{a \cdot \ln(k)} + C$$

linear ليس يكون

$$5- \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

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$$6- \int \frac{\overset{\text{const}}{N}}{\underset{\text{خط ذو ميل}}{\text{linear}}} dx = \frac{N}{\text{الميل}} * \ln |\text{linear}| + C$$

$$\text{Ex: } \int \frac{3}{x-4} dx$$

$$= \frac{3}{1} * \ln |x-4| + C$$

Ex: Evaluate the following integrals

$$1- \int 5x^3 + \frac{3}{x^2} - \frac{4}{x} + 2 dx$$

$$= \int 5x^3 - 3x^{-2} - 4/x^{+1} + 2 dx$$

$$= \frac{5}{\cancel{4}} x^4 - 3 \frac{x^{-1}}{-1} - 4 \ln x + 2x + C$$

$$2- \int e^{2x+1} + 3^x dx$$

$$= \frac{e^{2x+1}}{2 \ln e} + \frac{3^x}{(1) \ln 3} + C$$

$$= \frac{1}{2} e^{2x+1} + \frac{1}{\ln 3} 3^x + C$$

* لازم نتأكد من القوة
شروط تكون linear

$$\text{Ex: } \int \frac{2x}{3+x^2} dx$$

$$= \ln |3+x^2| + C$$

$$\text{Ex: } \int \frac{x}{5-x^2}$$

$$= \frac{1}{-2} \int \frac{(-2)x}{5-x^2} dx$$

$$= -\frac{1}{2} \ln |5-x^2| + C$$

Trigonometric functions:

$$1- \int \sin x \, dx = \frac{-\cos x}{(1)} + C \Rightarrow -\cos x + C$$

← مشتق الزاوية

$$2- \int \cos x \, dx = \frac{\sin x}{(1)} + C$$

$$3- \int \tan x \, dx = -\int \frac{\sin(x)}{\cos(x)} dx = -\ln |\cos(x)| + C$$

or $\ln |\sec(x)| + C$

$$4- \int \cot x \, dx = \int \frac{\cos x}{\sin x} dx = \ln |\sin(x)| + C$$

$$5- \int \sec(x) \, dx = \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)} = \ln |\sec(x) + \tan(x)| + C$$

$$6 - \int \csc(x) dx = \ln |\csc(x) - \cot(x)| + C$$

$$7 - \int \sec^2(x) dx = \tan(x) + C$$

$$8 - \int \csc^2(x) dx = -\cot(x) + C$$

$$9 - \int \tan^2(x) dx = \tan(x) - x + C$$

$$10 - \int \sin^2(x) dx = \int \frac{1}{2} (1 - \cos(2x)) dx$$

قانون نصف الزاوية

$$= \int \frac{1}{2} \left(x - \frac{\sin(2x)}{2} \right) + C$$

$$11 - \int \cos^2(x) dx = \int \frac{1}{2} (1 + \cos(2x)) dx$$

$$= \frac{1}{2} \left(x + \frac{\sin(2x)}{2} \right) + C$$

$$12 - \int \sec(x) \cdot \tan(x) dx = \sec(x) + C$$

$$13 - \int \csc(x) \cdot \cot(x) dx = -\csc(x) + C$$

Ex: Find the following integral

$$1- \int \frac{1}{\cos^2(x)} dx = \int \sec^2(x) dx \\ = \tan(x) + c$$

$$2- \int 2 \sin(3x) dx = 2 \int \sin(3x) dx \\ = 2 \frac{-\cos(3x)}{3} + c$$

$$3- \int (5 + 5 \cot^2(x)) dx = 5 \int (1 + \cot^2(x)) dx \\ = 5 \int \csc^2(x) dx \\ = -5 \cot(x) + c$$

$$4- \int \sec(2x) \cdot (\sec(2x) - 3 \tan(2x) + 4 \cos(2x)) dx \\ = \int \sec^2(2x) - 3 \sec(2x) \tan(2x) + 4 \underbrace{\sec(2x) \cos(2x)}_{\frac{1}{\cos(2x)}} dx \\ = \frac{\tan(2x)}{2} - \frac{3 \sec(2x)}{2} + 4x + c$$

* يجب ان تكون جميع الزوايا خطية للدوال المتثلية

Inverse Trigonometric functions

$$1 - \int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$$

$$* \int \frac{b}{a^2 + k^2 x^2} dx = \frac{b}{\underbrace{a}_{\text{constant}}} \cdot \tan^{-1} \left(\frac{kx}{a} \right) + C$$

$$2 - \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$$

$$* \int \frac{b}{\sqrt{a^2 - k^2 x^2}} dx = \frac{b}{k} \sin^{-1} \left(\frac{kx}{a} \right) + C$$

$$3 - \int \sinh(x) dx = \cosh(x) + C$$

$$- \int \cosh(x) dx = \sinh(x) + C$$

$$\begin{aligned} - \int \tanh(x) dx &= \int \frac{\sinh(x)}{\cosh(x)} dx \\ &= \ln |\cosh(x)| + C \end{aligned}$$

$$6 - \int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1}(x) + C$$

$$7 - \int \frac{-1}{\sqrt{1+x^2}} dx = \cot^{-1}(x) + C$$

$$8 - \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}(x) + C$$

$$9 - \int \frac{-1}{x\sqrt{x^2-1}} dx = \csc^{-1}(x) + C$$

Ex:

$$1 - \int \frac{x-5}{1+x^2} dx$$

∴ ہم توزیع البسط علی المقام

$$= \frac{1}{2} \int \frac{(2)x}{1+x^2} dx - \int \frac{5}{1+x^2} dx$$

$$= \frac{1}{2} \ln |1+x^2| - 5 \int \frac{1}{1+x^2} dx$$

$$= \frac{1}{2} \ln |1+x^2| - 5 \tan^{-1}(x) + C$$

$$2 - \int \frac{dx}{9 + 4x^2}$$

$$* \int \frac{b}{a^2 + k^2 x^2} dx = \frac{b}{a \cdot k} \tan^{-1} \left(\frac{kx}{a} \right) + C$$

$$= \frac{1}{3 \cdot (2)} \tan^{-1} \left(\frac{2x}{3} \right) + C$$

$$= \frac{1}{6} \tan^{-1} \left(\frac{2x}{3} \right) + C$$

$$3 - \int \frac{4}{\sqrt{1-x^2}} dx$$

$$= 4 \int \frac{1}{\sqrt{1-x^2}} dx$$

$$= 4 \sin^{-1}(x) + C$$

$$3 - \int \frac{1-x^2}{1-x^4} dx$$

$$= \int \frac{\cancel{(1-x^2)}}{(\cancel{1-x^2})(1+x^2)} dx$$

$$= \int \frac{1}{1+x^2} dx$$

$$= \tan^{-1}(x) + C$$