PHYSICS
Engineering Mechanics
Lecture 3

Moment of the Force

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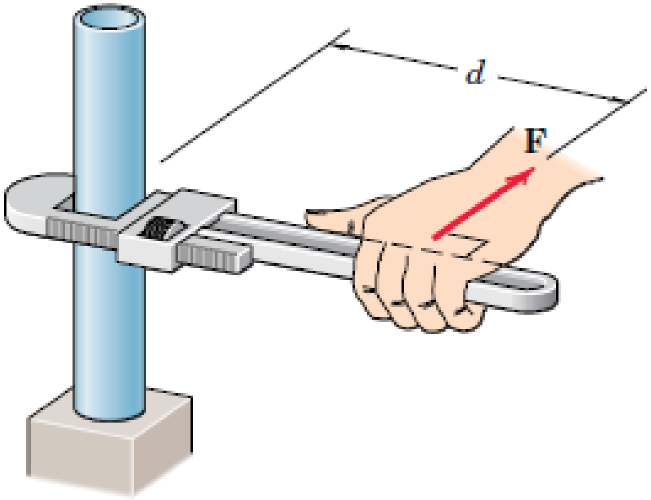
- **The moment M**: is a force that tends to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action

- Moment is also referred to as ***torque***.



MOMENT ABOUT A POINT

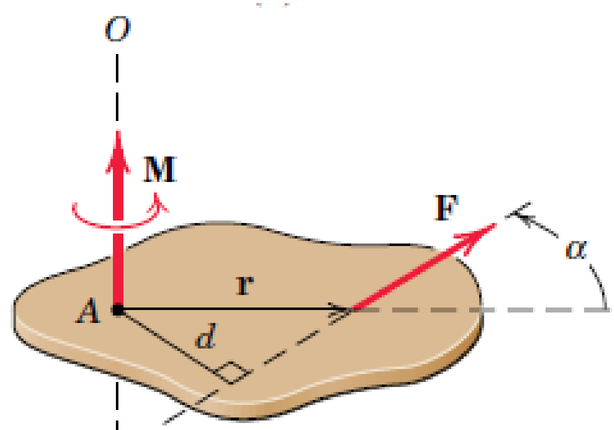
- The magnitude of the ***moment*** or ***tendency*** of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the ***force*** and to the ***moment arm d***, which is the perpendicular distance from the axis to the line of action of the force.

M=F.d

M = the moment (N.M)

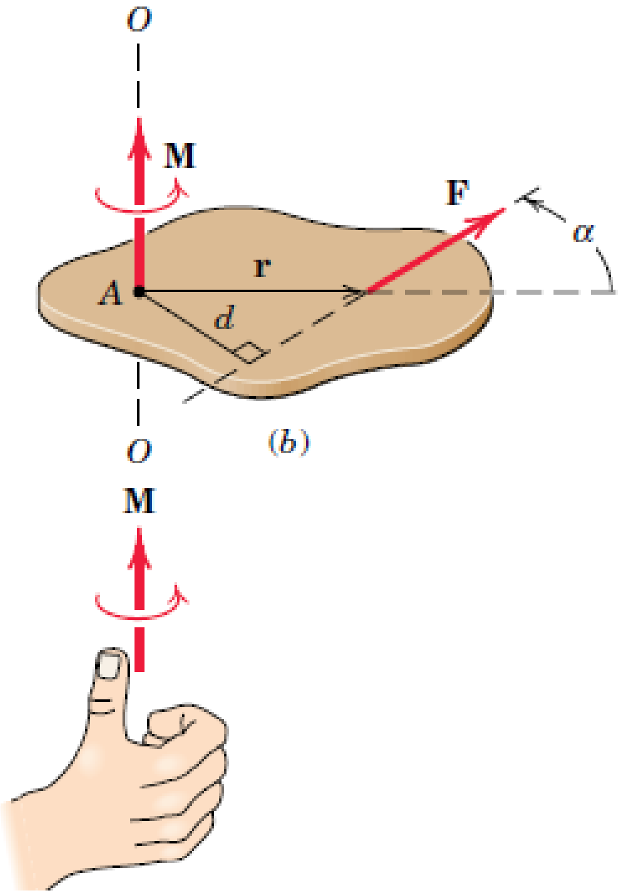
F = applied force (N).

d = moment arm (m).



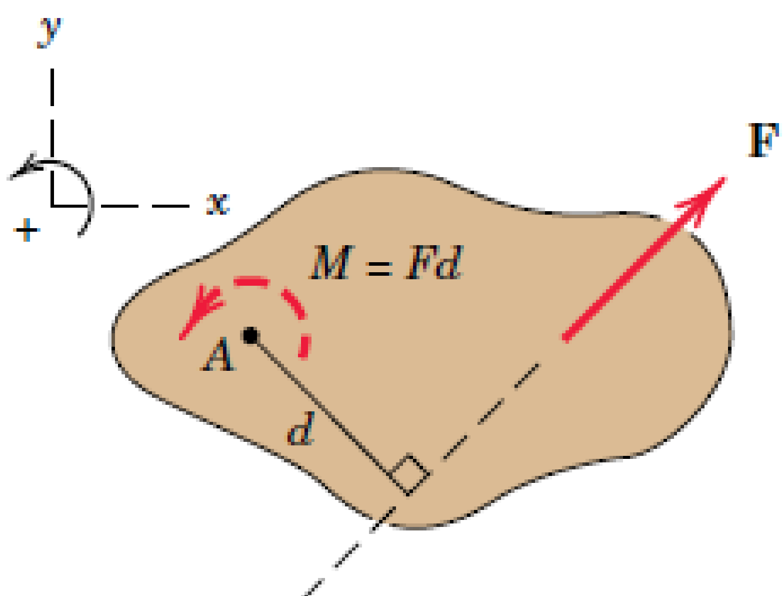
(b)

- The moment is a vector **M** perpendicular to the plane of the body.
- The sense of **M** depends on the direction in which **F** tends to rotate the body.
- The right-hand rule is used to identify this sense.



- The moment of force **F** about point *A* in **Fig *d*** has the magnitude $M = Fd$ and is counterclockwise.

- **Plus sign (+) for counterclockwise moments and a minus sign (-) for clockwise moments.**

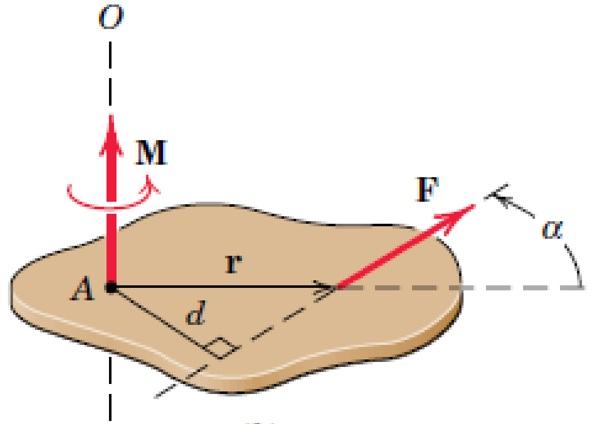


(d)

THE CROSS PRODUCT

- The moment of **F** about point ***A*** of **Fig *b*** may be represented by the cross-product expression:

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$



(b)

- Where **r** is a position vector which runs from the moment reference point ***A*** to ***any*** point on the line of action of **F**.

VARIGNON'S THEOREM

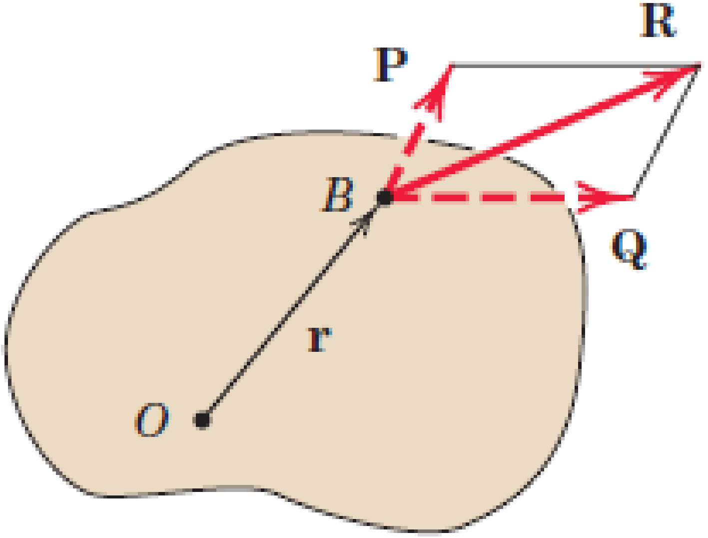
- One of the most useful principles of mechanics is ***Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.**

$M_o = \mathbf{r} \times R$

Because, R = P + Q

$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$

$M_o = \mathbf{r} \times R = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$

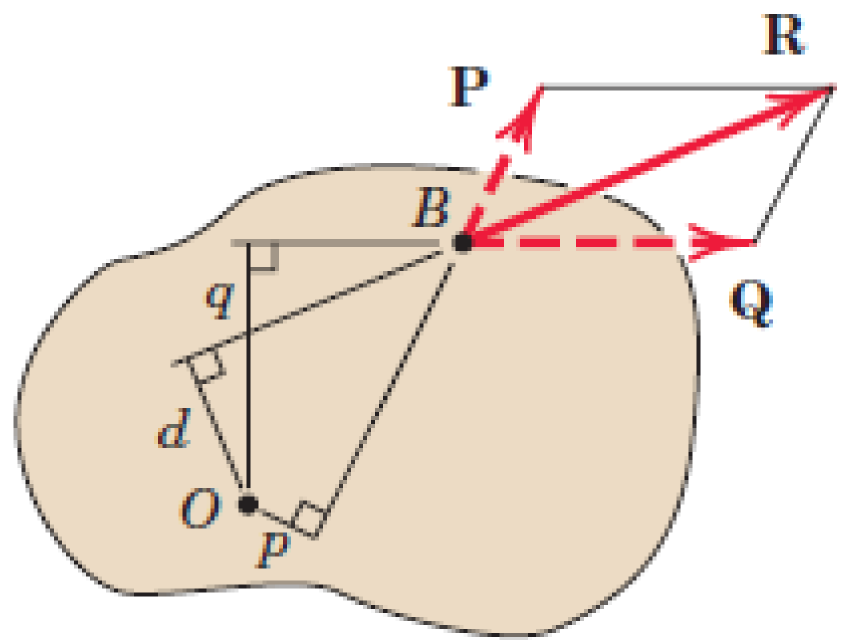


(a)

Figure *b* illustrates the usefulness of Varignon's theorem.

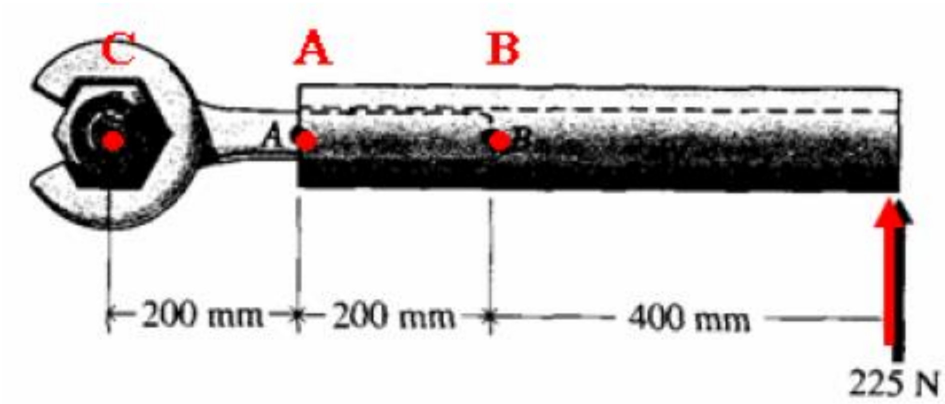
The moment of R about point *O* is *Rd*.

$$M_{\circ} = R \cdot d = P \cdot p + Q \cdot q$$



(*b*)

Ex:Determine the moment of the force 225 N about the Points A , B , and C.

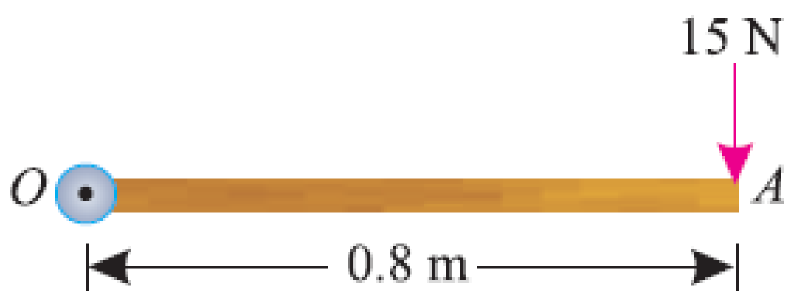


MA=|F|dA=225*0.6 = 135 Nm

MB=|F| dB=225*0.4 = 90 Nm

MC=|F| dC=225*0.8 = 180 Nm

Example. A force of 15 N is applied perpendicular to the edge of a door 0.8 m wide as shown in Fig. (a). Find the moment of the force about the hinge.



(a)

Solution. Given : Force applied (P) = 15 N and width of the door (l) = 0.8 m

Moment when the force acts perpendicular to the door.

We know that the moment of the force about the hinge,

$M = P * l = 15 * 0.8 = 12.0$ N.m **Ans.**

Example: Determine the moment of the force 500 N about the point A and B .

Solution:

$\cos(60) = 200/L$

$\cos(60) = dac/(L-160)$

$L = 200/\cos(60)$

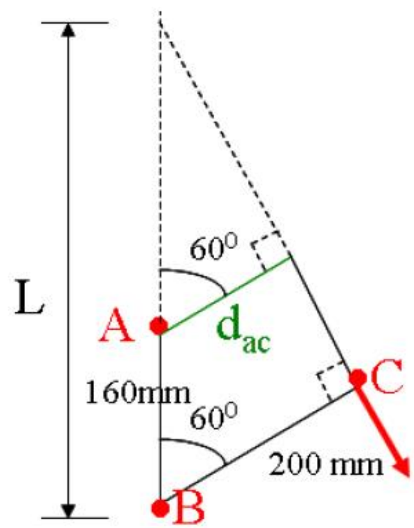
$L = 160 + dac/\cos(60)$

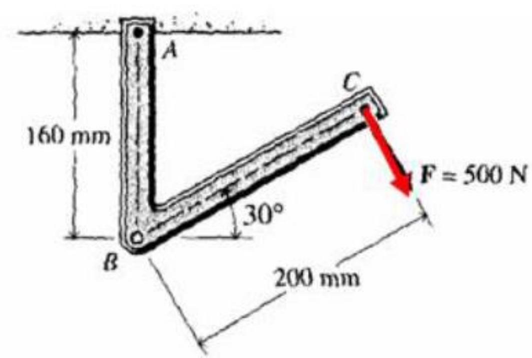
$dac = 200 - 160\cos(60) = 120\ mm$

$dac = 120\ mm$

$MA = |F| dAC = 500 * 0.12 = 60\ Nm$

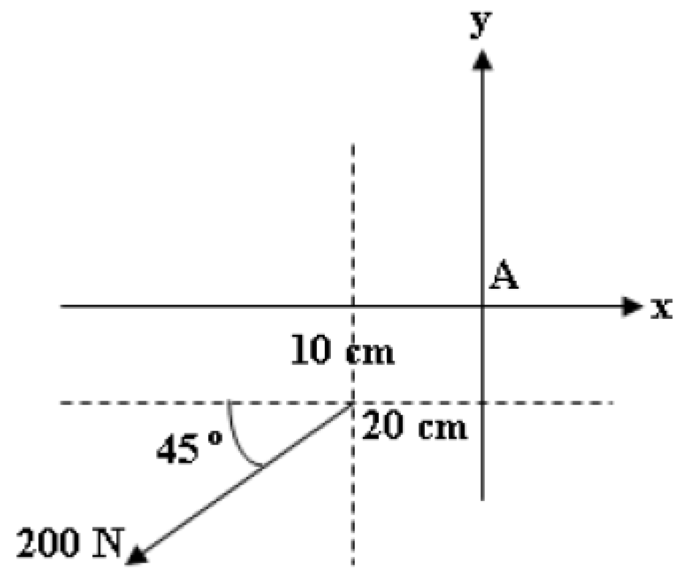
$MB = |F|\ dB = 500 * 0.2 = 100\ Nm$





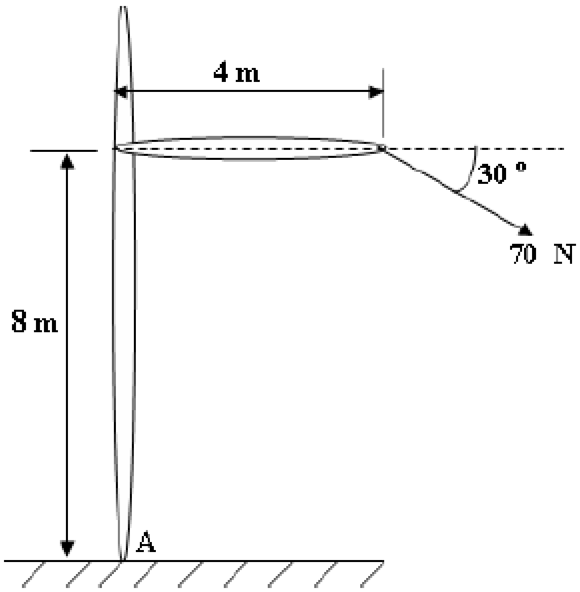
Ex3:Find the moment of the force 200 N About the point(A) shown in fig.
Solution:

$F_x = F \cdot \cos\theta = 200 \cos 45$
$= 200 * 0.707 = 141.42$ N
$F_y = F \cdot \sin\theta = 200 \sin 45$
$= 200 * 0.707 = 141.42$ N
$M1 = F_x * d = 141.42 * 10 = 1414.2$ N . cm
$M2 = F_y * d = 141.42 * 20 = 2828.4$ N .cm
$M(A) = M1 - M2 = -1414.2$ N .cm



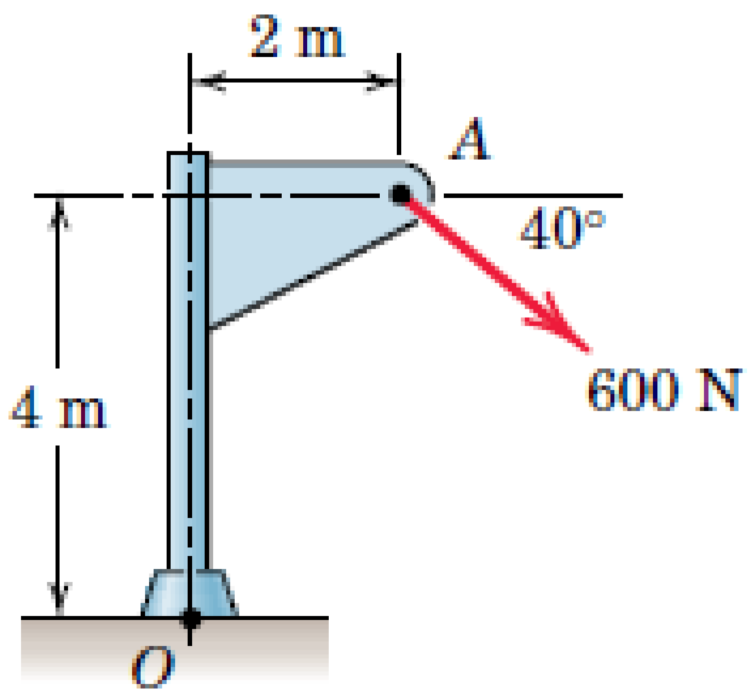
Ex4:Determine the moment of the force(70 N) shown in fig. about the Point (A) .
Solution:

Fx = F . cos θ = 70 cos 30
= 70 * 0.866 = 60.62 N
Fy = F . sin θ = 70 sin 30
= 70 * 0.5 = 35 N
M1 = Fx * d = 60.62 * 8 = 484.97 N . m
M2 = Fy * d = 35 * 4 = 140 N . m
M (A) = M1 + M2 = 484.97 N + 140 = 624.97 N . m



Sample problem 2/5

Calculate the magnitude of the moment about the base point O of the 600 N force.

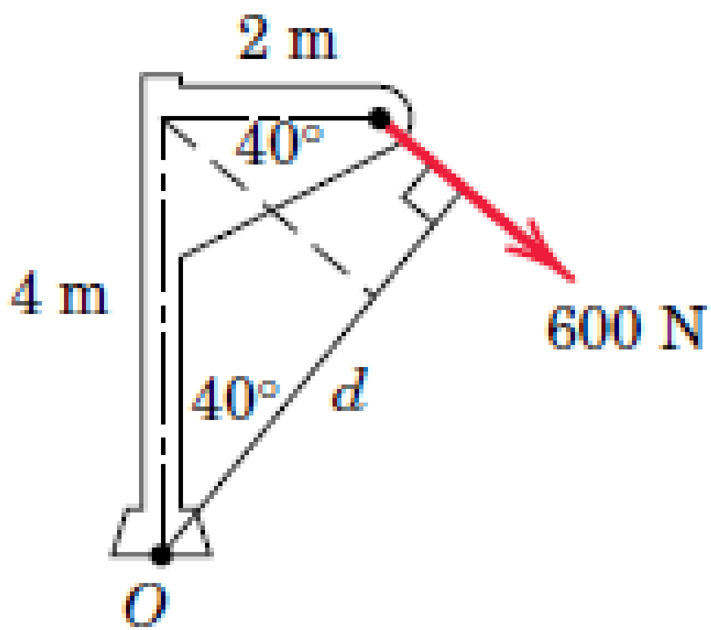


Solution. *(I)* The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

① By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N·m}$$

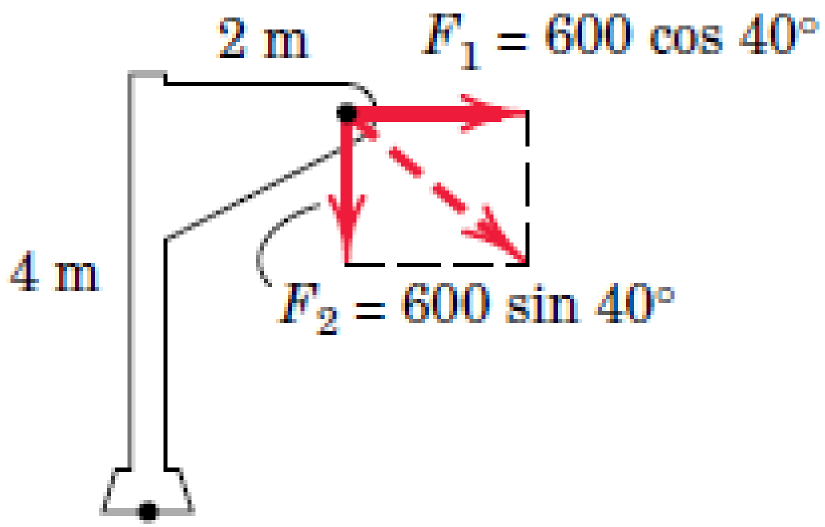


(II) Replace the force by its rectangular components at A

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

② $$M_O = 460(4) + 386(2) = 2610 \text{ N·m}$$



(V) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

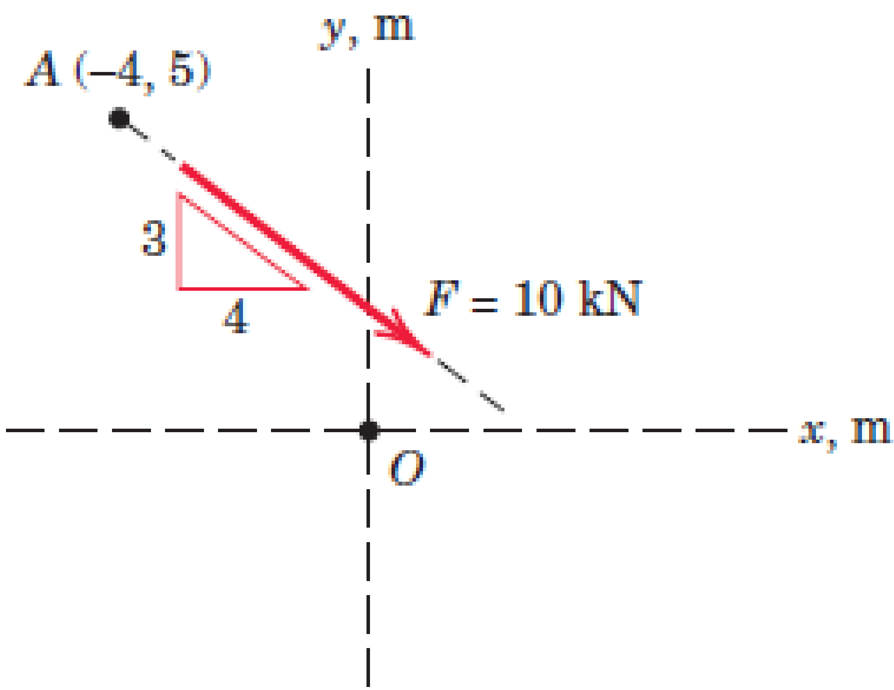
④
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40^\circ - \mathbf{j} \sin 40^\circ)$$
$$= -2610\mathbf{k}\ \text{N}\cdot\text{m}$$

The minus sign indicates that the vector is in the negative *z*-direction. The magnitude of the vector expression is

$$M_O = 2610\ \text{N}\cdot\text{m}$$ *Ans.*

2/29 The 10-kN force is applied at point A. Determine the moment of $\mathbf{F}$ about point O. Determine the points on the x- and y-axes about which the moment of $\mathbf{F}$ is zero.

Ans. $M_O = 16\ \text{kN}\cdot\text{m CW}$
$(x, y) = (2.67, 0)$ m and $(0, 2)$ m

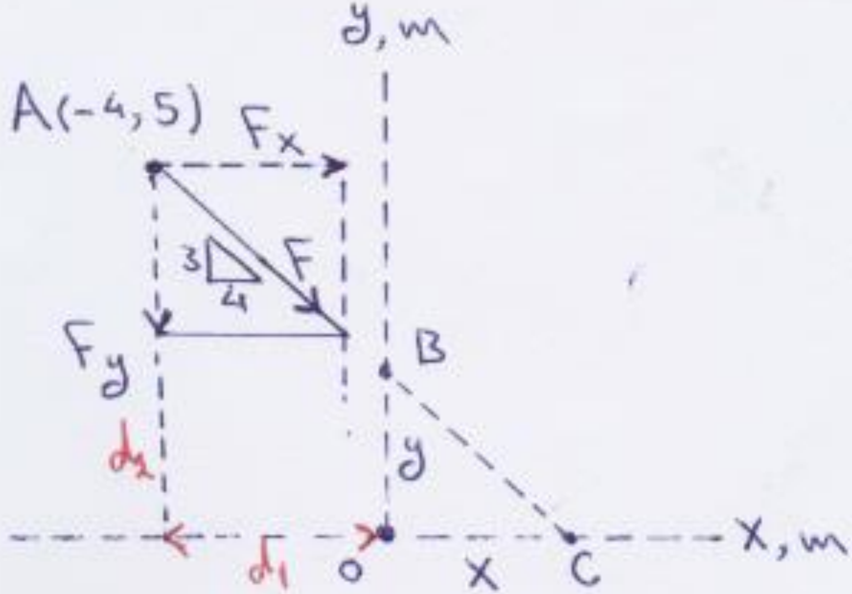


Problem 2/29

2/29

$$F_x = F\cos\theta$$

$$F_x = 10\left(\frac{4}{5}\right) = 8\ KN$$



$$|F_y| = F\sin\theta$$

$$= 10\left(\frac{3}{5}\right) = 6\ KN$$

$$\circlearrowright + \quad M_o = F_x(d_2) - F_y(d_1)$$

$$= 8(5) - 6(4) = 16\ KN.m\ \ CW$$

$$\tan\theta = \frac{3}{4} = \frac{5}{4+X} \Rightarrow X = 2.67 \text{ m}$$

$$\tan\theta = \frac{y}{X} = \frac{3}{4}$$

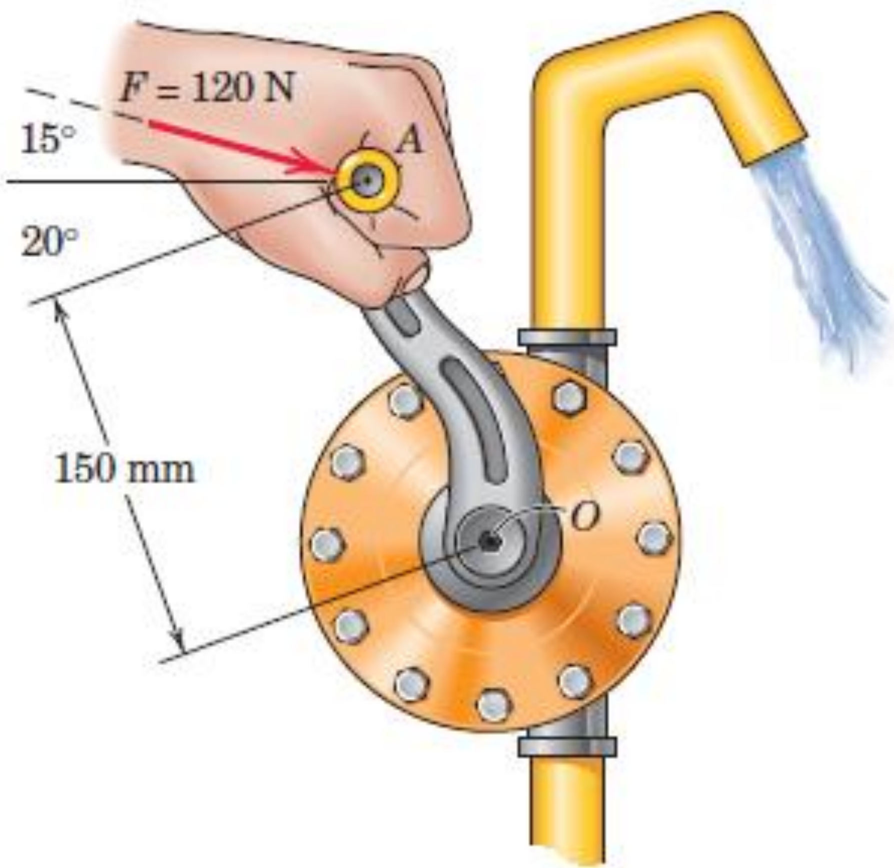
$$y = \frac{3X}{4} = \frac{3(2.67)}{4} = 2 \text{ m}$$

$\therefore$ The intercept points are :

$(2.67, 0)$ m and $(0, 2)$ m

2/33 In steadily turning the water pump, a person exerts the 120-N force on the handle as shown. Determine the moment of this force about point O.

Ans. $M_O = 14.74\ \text{N}\cdot\text{m CW}$



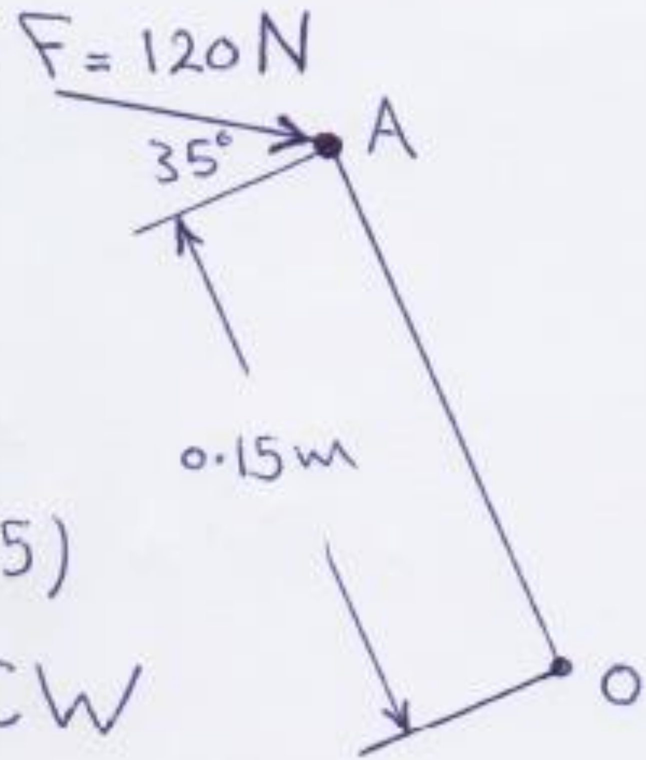
Problem 2/33

2/33

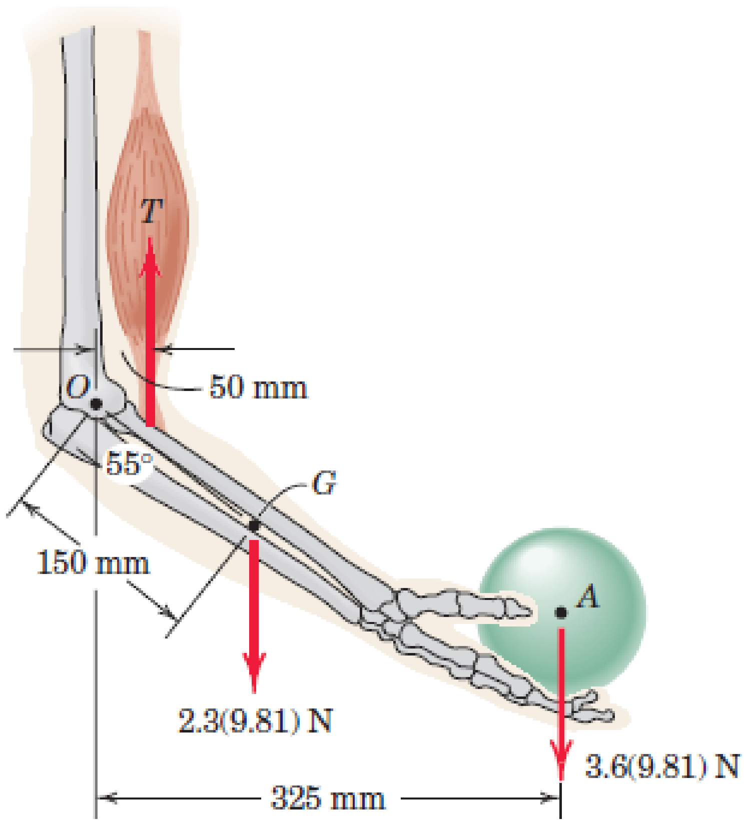
$$\overset{+}{\curvearrowright} M_o = F_x\,(d)$$

$$= 120 \cos 35\,(0.15)$$

$$= 14.74 \text{ N.m CW}$$

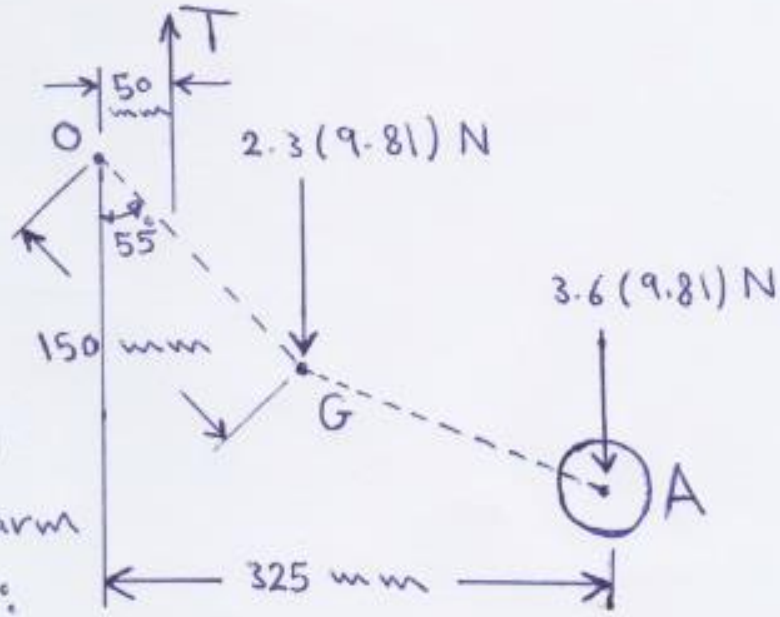


2/40 Elements of the lower arm are shown in the figure. The mass of the forearm is 2.3 kg with mass center at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the 3.6-kg homogeneous sphere. What must the biceps tension force be so that the overall moment about O is zero?



Problem 2/40

2/40



The combined moment about the elbow pivot O of the weights of the forearm and the 3.6 Kg sphere is:

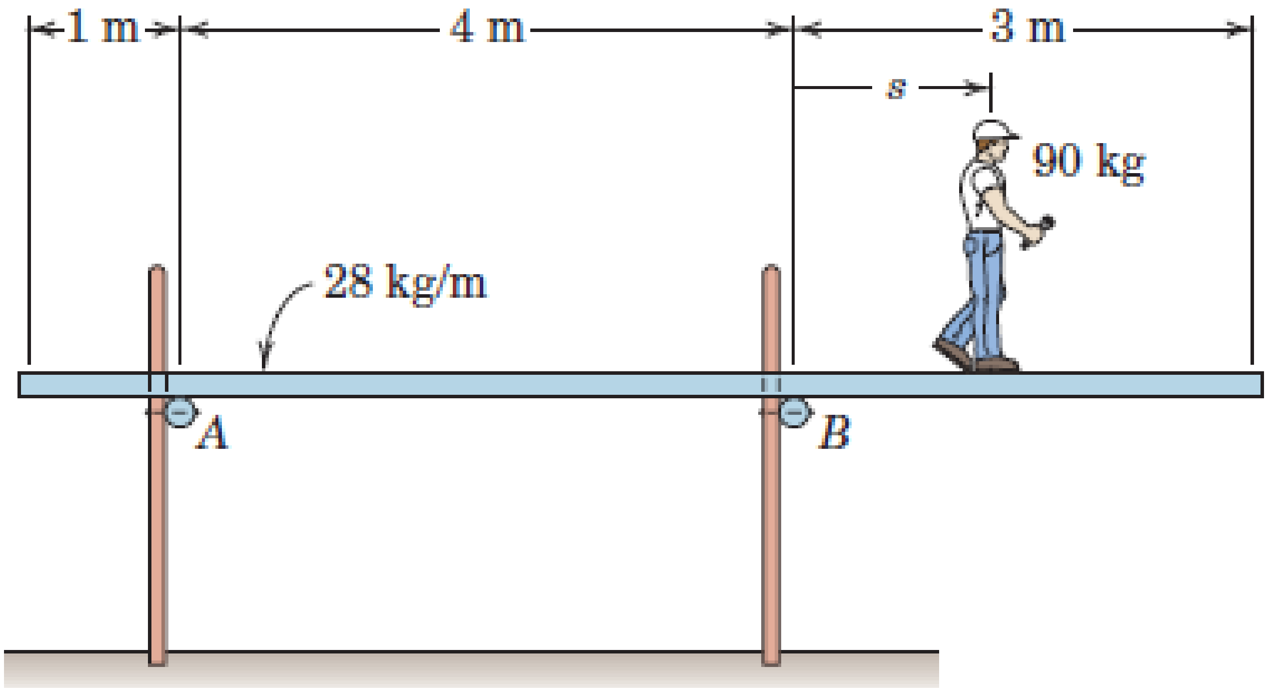
$$\overset{+}{\curvearrowright} M_o = 2.3(9.81)(0.150 \sin 55) + 3.6(9.81)(0.325)$$

$$= 14.25 \text{ N.m} \quad CW$$

$$\overset{+}{\curvearrowright} \Sigma M_o = 0 \Rightarrow -T(0.5) + 14.25 = 0$$

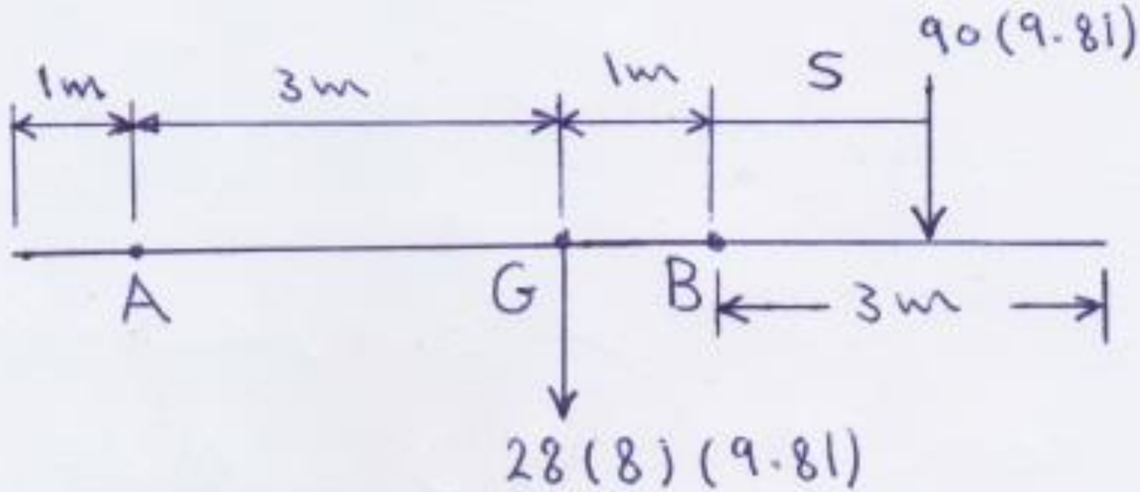
$$\therefore T = 28.5 \text{ N}$$

2/44 The uniform work platform, which has a mass per unit length of 28 kg/m, is simply supported by cross rods *A* and *B*. The 90-kg construction worker starts from point *B* and walks to the right. At what location *s* will the combined moment of the weights of the man and platform about point *B* be zero?



Problem 2/44

2/44

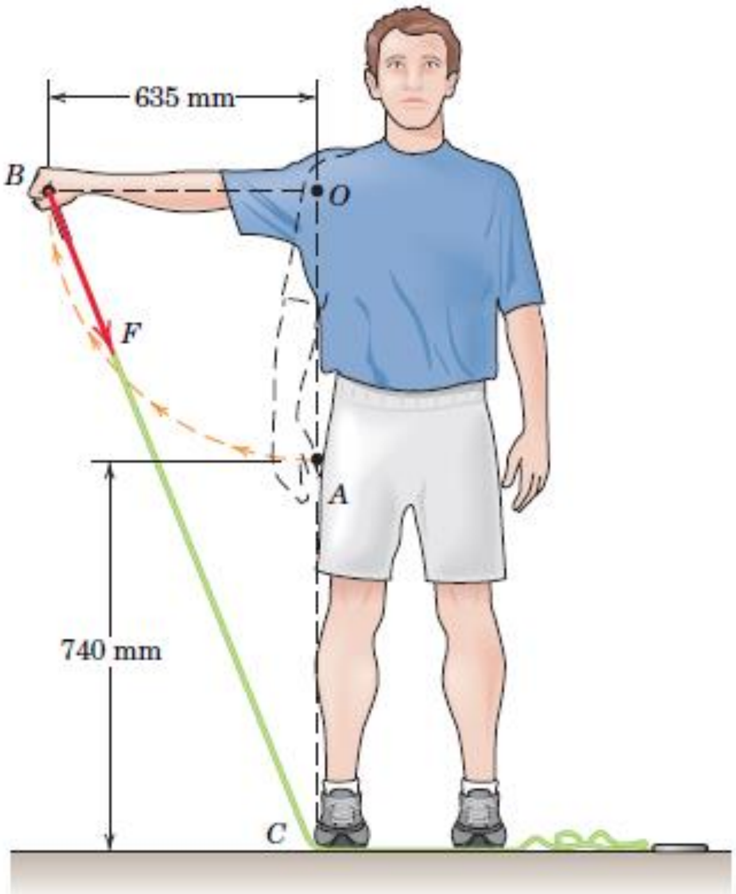


$$\circlearrowleft + \; M_B = 28(8)(9.81)(1) - 90(9.81)S = 0$$

$$S = 2.49 \text{ m}$$

2/49 An exerciser begins with his arm in the relaxed vertical position OA, at which the elastic band is unstretched. He then rotates his arm to the horizontal position OB. The elastic modulus of the band is k = 60 N/m—that is, 60 N of force is required to stretch the band each additional meter of elongation. Determine the moment about O of the force which the band exerts on the hand B.

Ans. $M_O = 26.8\ \text{N·m CCW}$



2/49

$$\overline{OC} = 635 + 740 = 1375 \text{ mm}$$

$$F = KX$$

$$= 60\left(\sqrt{0.635^2 + 1.375^2} - 0.740\right)$$

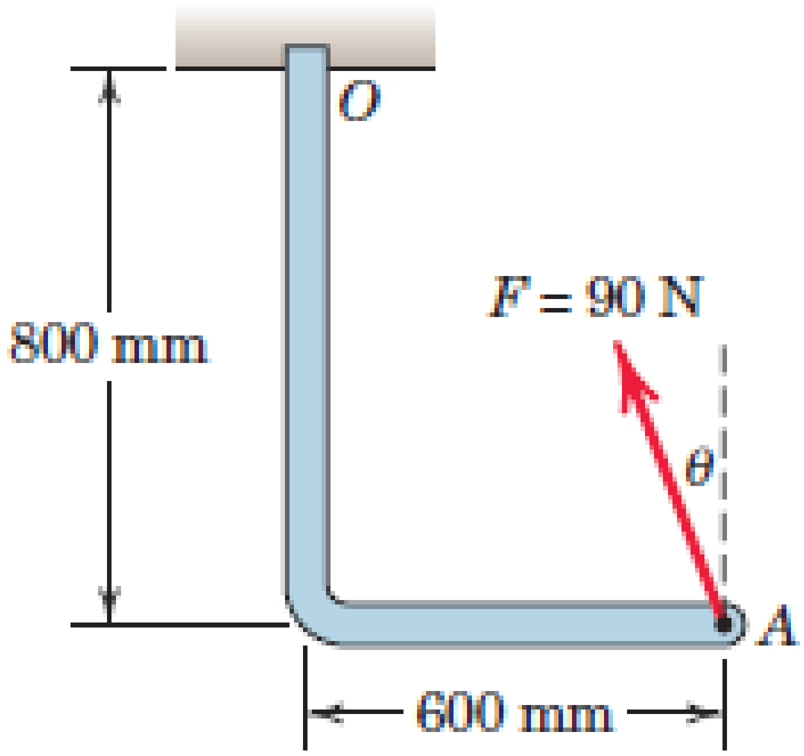
$$= 46.5 \text{ N}$$

$$\theta = \tan^{-1}\frac{1.375}{0.635} = 65.2^\circ$$

$$\circlearrowleft^{+} M_o = 46.5 \sin 65.2\,(0.635)$$

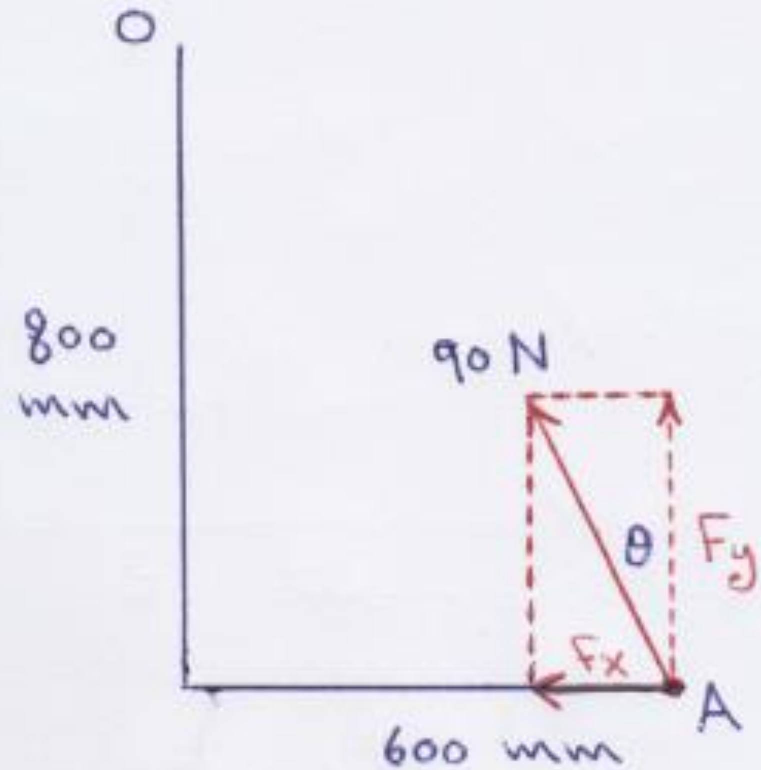
$$= 26.8 \text{ N.m} \quad \text{CCW}$$

2/50 (*a*) Calculate the moment of the 90-N force about point O for the condition $\theta = 15°$. Also, determine the value of θ for which the moment about O is (*b*) zero and (*c*) a maximum.



Problem 2/50

2/50

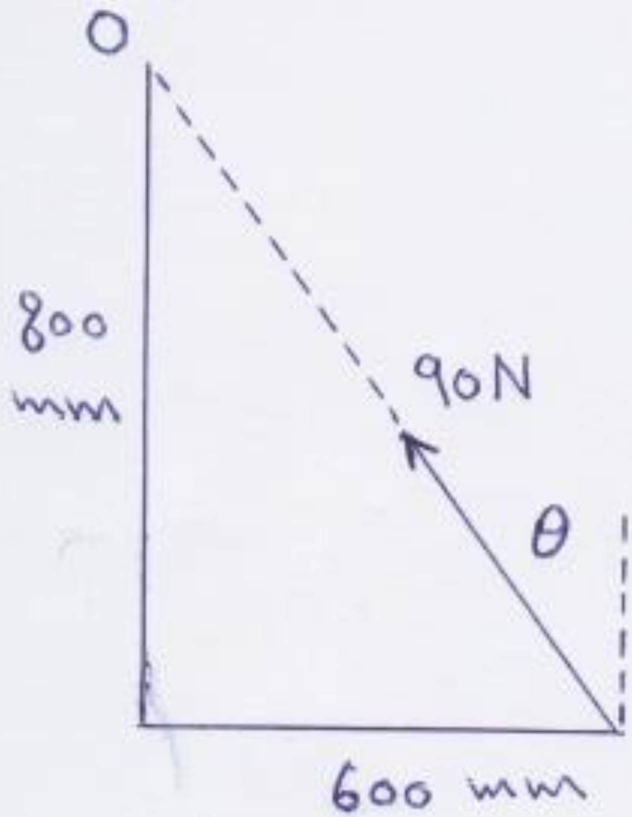


a)

$$\circlearrowleft + \quad M_o = F_y (0.6) - F_x (0.8)$$

$$M_o = 90 \cos 15 \, (0.6) - 90 \sin 15 \, (0.8)$$

$$= 33.5 \text{ N.m}$$



b)

$$\theta = \tan^{-1}\left(\frac{600}{800}\right) = 36.9^\circ$$

c)

$$\alpha = \tan^{-1}\left(\frac{600}{800}\right)$$

$$\alpha = 36.9^\circ$$

$$\theta = 90 + \alpha$$

$$= 90 + 36.9$$

$$= 126.9$$