PHYSICS

Engineering Mechanics

Lecture 1

The Moment of The Couple

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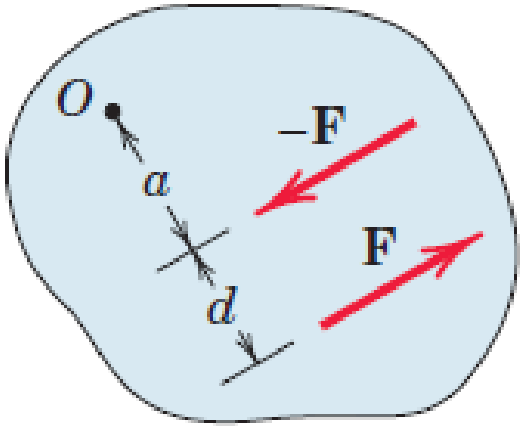
THE COUPLE

- The moment produced by two equal, opposite, and non-collinear forces is called a ***couple***.
- Consider the action of two equal and opposite forces **F** and -**F** a distance ***d*** apart, as shown in Fig *a*.
- These two forces cannot be **combined** into a single force because their sum in every direction is **zero**.



(*a*)

- Their only effect is to produce a tendency of rotation.

- The combined moment of the two forces about an axis normal to their plane and passing through point O is the couple M.

$$M = F\,(a + d) - Fa$$

Or

$$M = Fd$$

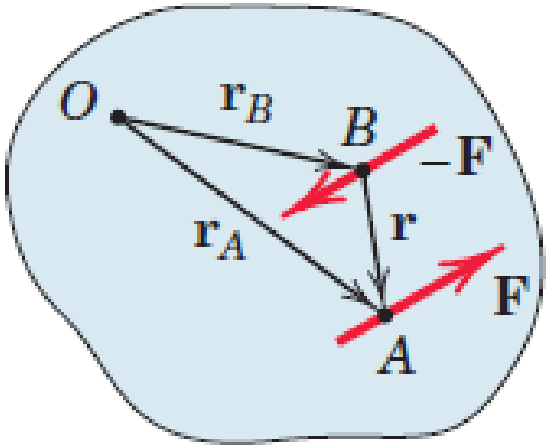
VECTOR ALGEBRA METHOD

- The combined moment about point ***O*** of the forces forming the couple of Fig *b* is:

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

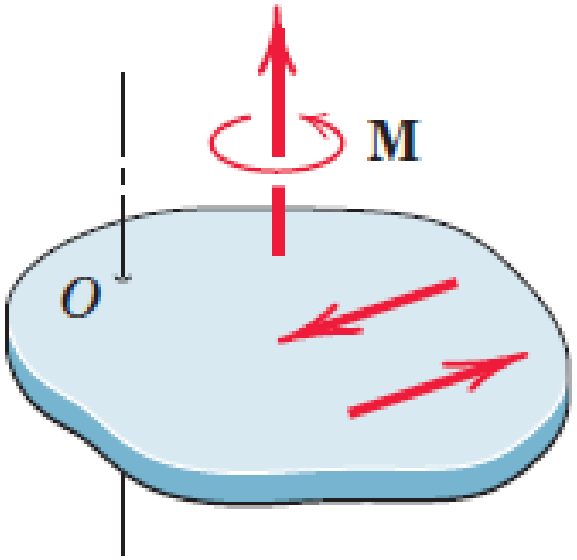
Because $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$



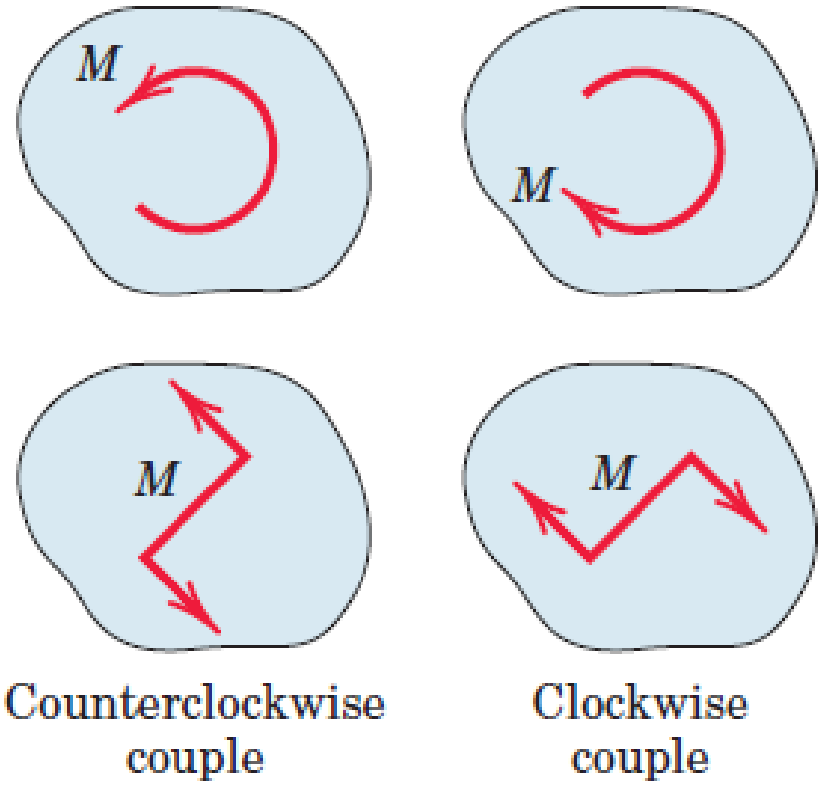
(*b*)

- we may represent **M** by a free vector, as show in Fig *c*, where the direction of **M** is normal to the plane of the couple and the sense of **M** is established by the right-hand rule.



(*c*)

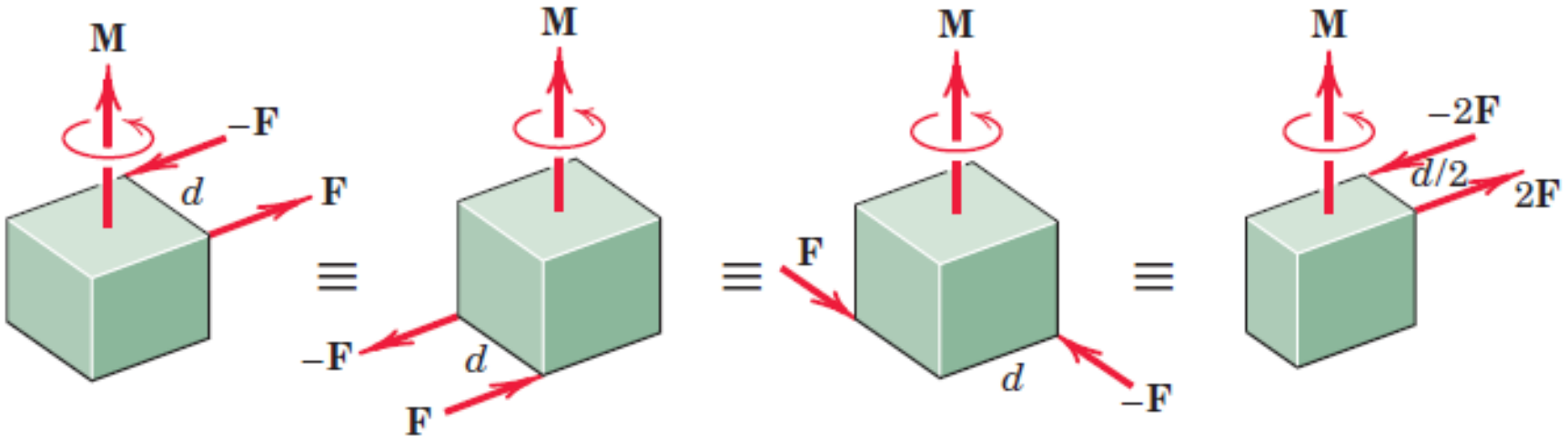
- The sense of a couple vector as clockwise or counterclockwise by one of the conventions shown in Fig *d*.



(*d*)

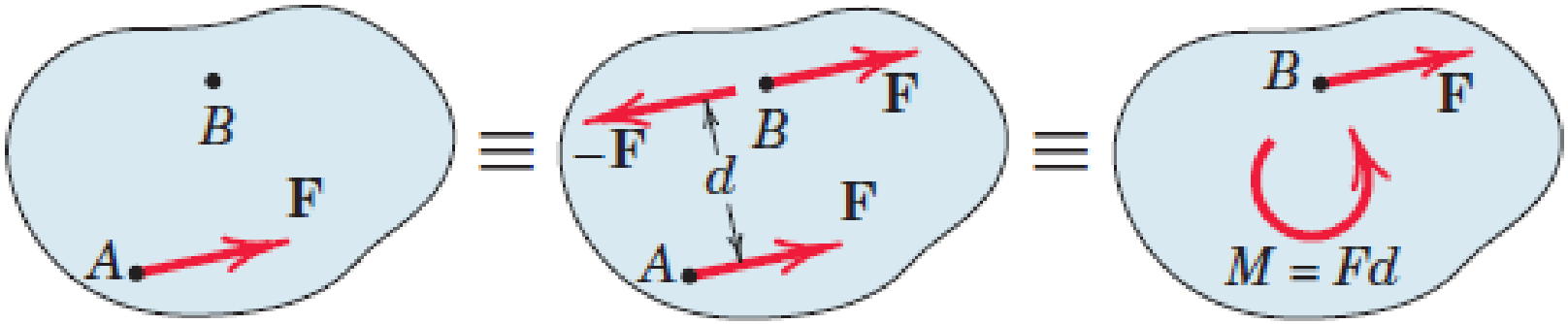
EQUIVALENT COUPLES

- **The figure below shows four different configurations of the same couple M.**

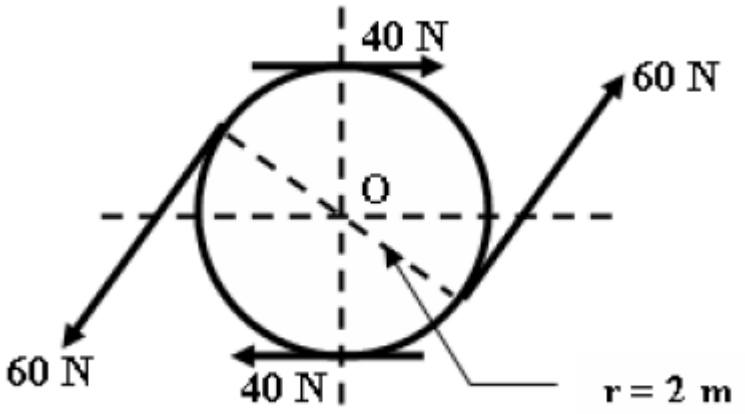


FORCE–COUPLE SYSTEMS

- We can represent the effect of a force more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.



Ex1:Compute the magnitude and direction of the resultant couples action on the body shown.



Solution:

Mc = 60 * 4 – 40 * 4

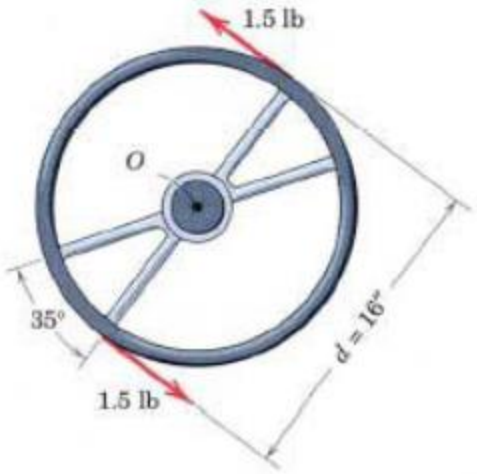
= 240 – 160 = 80 N.m

Ex2:Determine the moment associated with the forces shown in fig.

Solution:

Mc = F * d

= 1.5 * 16= 24 Ib.In

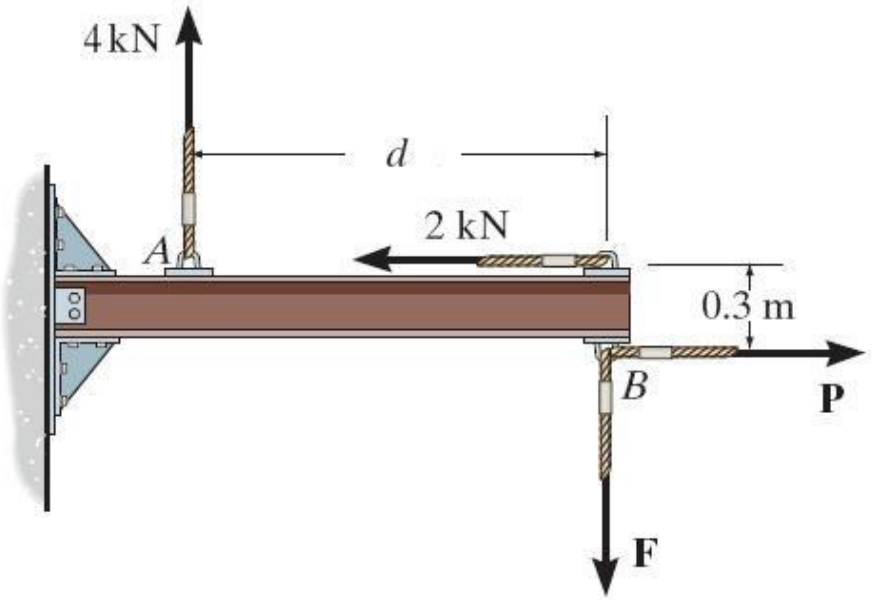


Example: Two couples act on the beam. The resultant couple is zero. Find the magnitudes of the forces P and F and the distance d.

1) Use definition of a scalar couple to find P and F.

2) Determine the net moment (couple).

3) Equate the net moment to zero to find d.



From the definition of a couple:

P = 2 kN

F = 4 kN

Determine the net moment

$\Sigma M = (2)(0.3) - (4)(d)$

It was given that the net moment equals zero. So

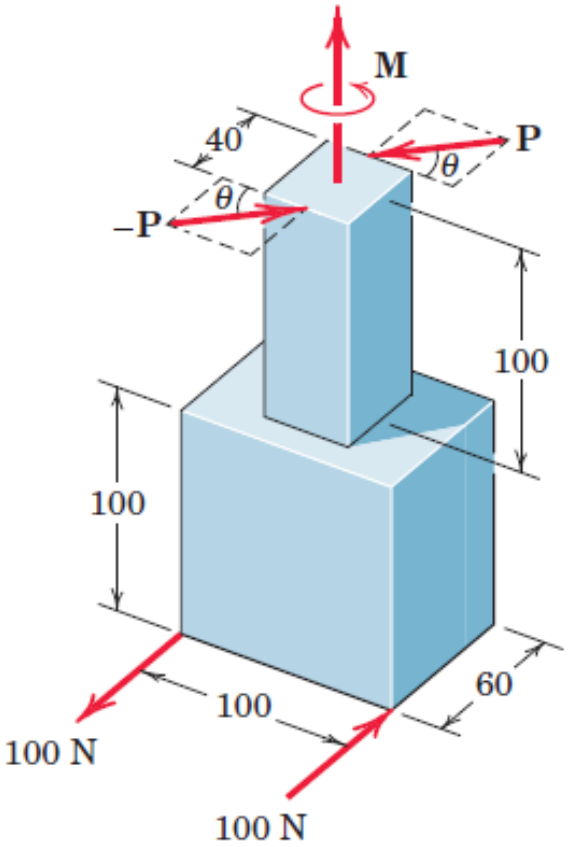
$\Sigma M = (2)(0.3) - (4)(d) = 0$

Now solve this equation for d.

d = (0.6) N·m / (4) N = 0.15 m

Sample Problem 2/7

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and **−P**, each of which has a magnitude of 400 N. Determine the proper angle θ.



Dimensions in millimeters

Solution. The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $$M = 100(0.1) = 10\ \text{N}\cdot\text{m}$$

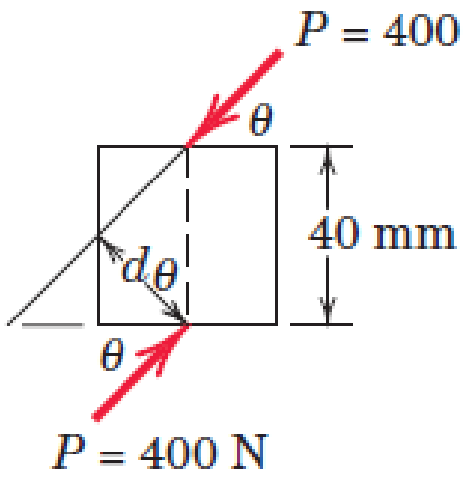
The forces $\mathbf{P}$ and $-\mathbf{P}$ produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

① Equating the two expressions gives

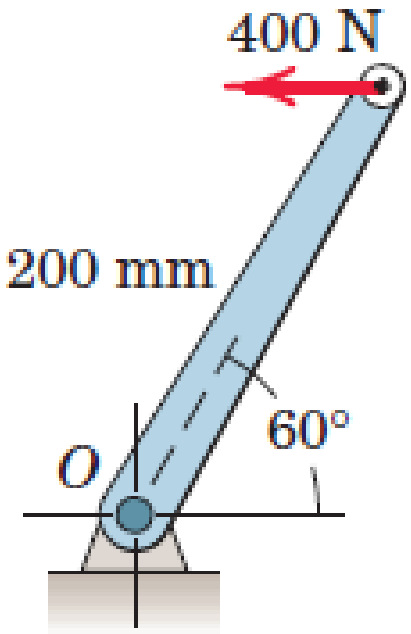
$$10 = (400)(0.040)\cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3^\circ \qquad \textit{Ans.}$$



Sample Problem 2/8

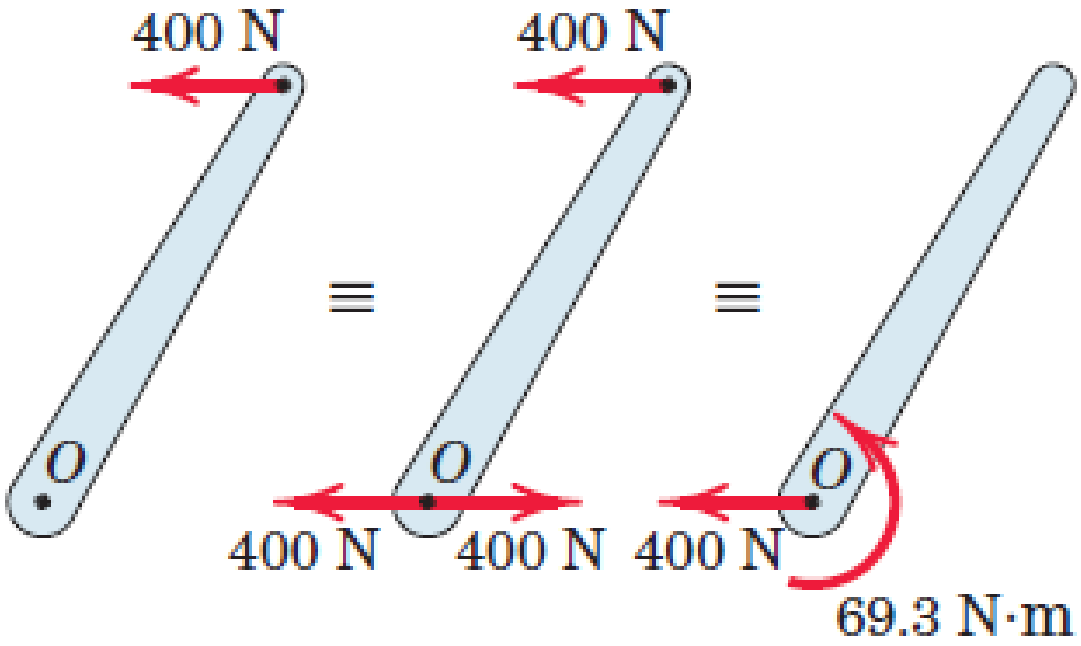
Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at O and a couple.



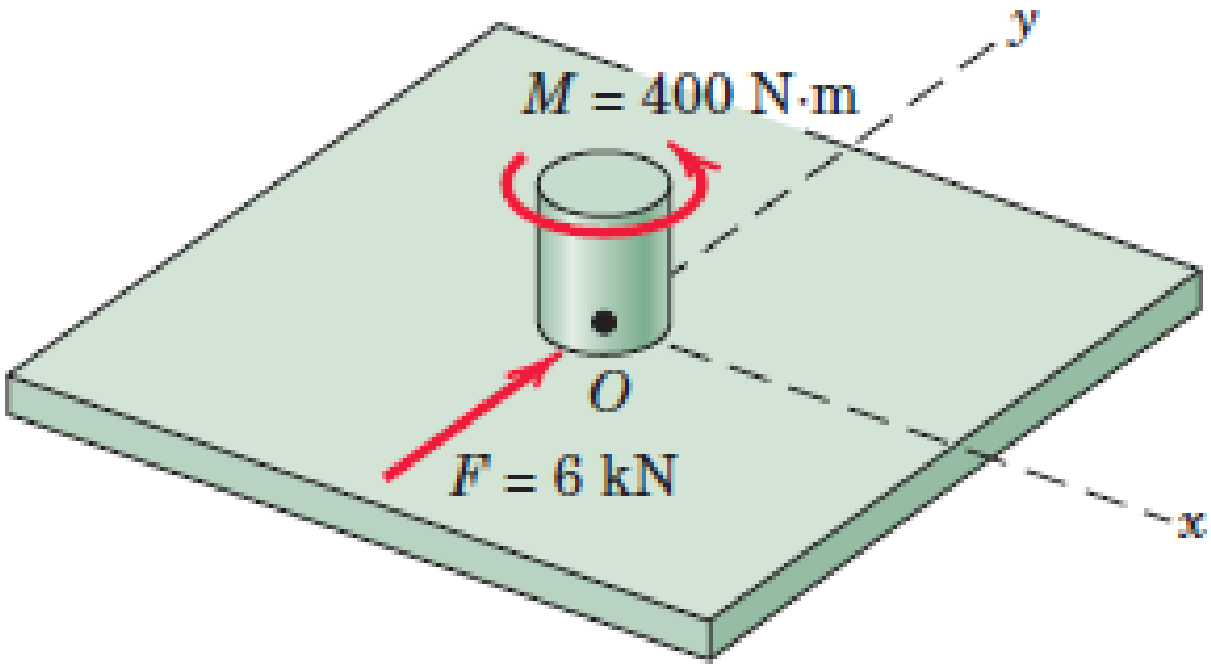
Solution. We apply two equal and opposite 400-N forces at O and identify the counterclockwise couple

$[M = Fd]$ $\quad M = 400(0.200 \sin 60°) = 69.3 \text{ N}\cdot\text{m}$ *Ans.*

① Thus, the original force is equivalent to the 400-N force at O and the 69.3-N·m couple as shown in the third of the three equivalent figures.



2/60 The indicated force–couple system is applied to a small shaft at the center of the plate. Replace this system by a single force and specify the coordinate of the point on the x-axis through which the line of action of this resultalt force passes.



Problem 2/60

2/60

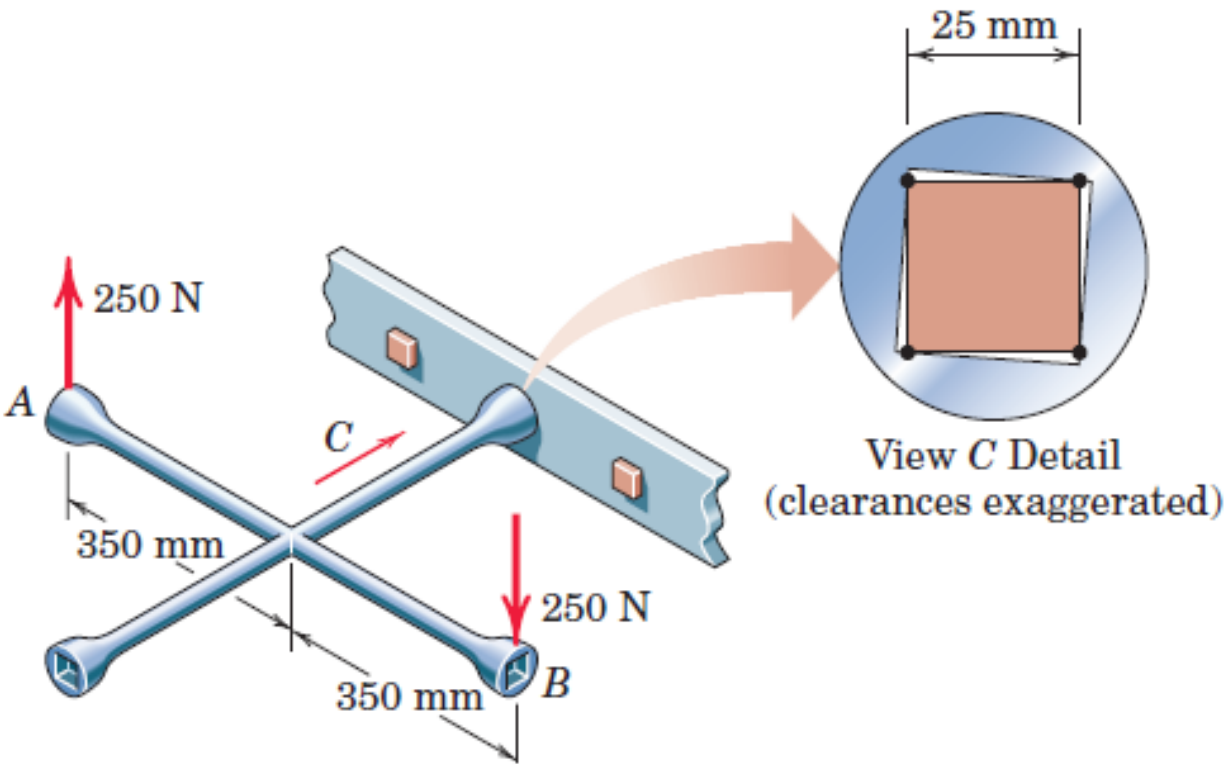
$$R = 6j \text{ KN}$$

$$X = \frac{400}{6000} = 0.0667 \text{ m}$$

$$X = 66.7 \text{ mm}$$

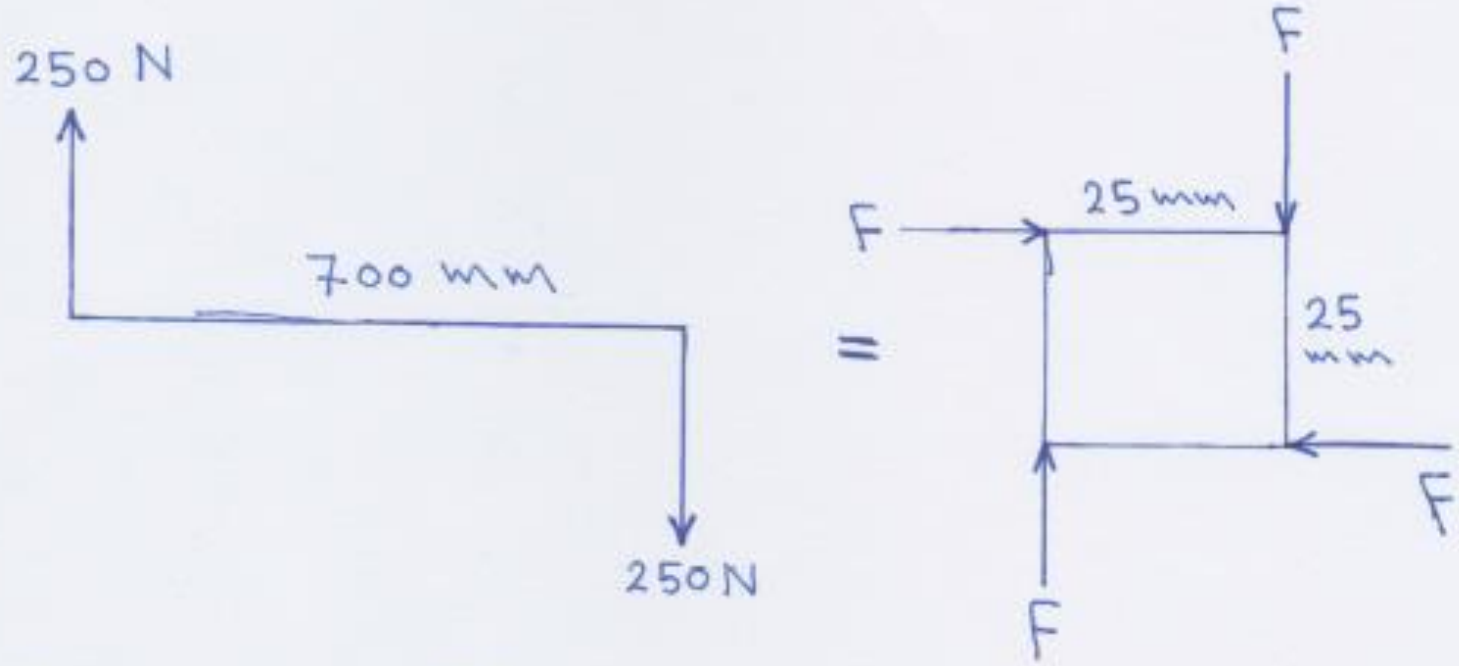
2/65 A lug wrench is used to tighten a square-head bolt. If 250-N forces are applied to the wrench as shown, determine the magnitude F of the equal forces exerted on the four contact points on the 25-mm bolt head so that their external effect on the bolt is equivalent to that of the two 250-N forces. Assume that the forces are perpendicular to the flats of the bolt head.

Ans. $F = 3500$ N



Problem 2/65

2 / 65

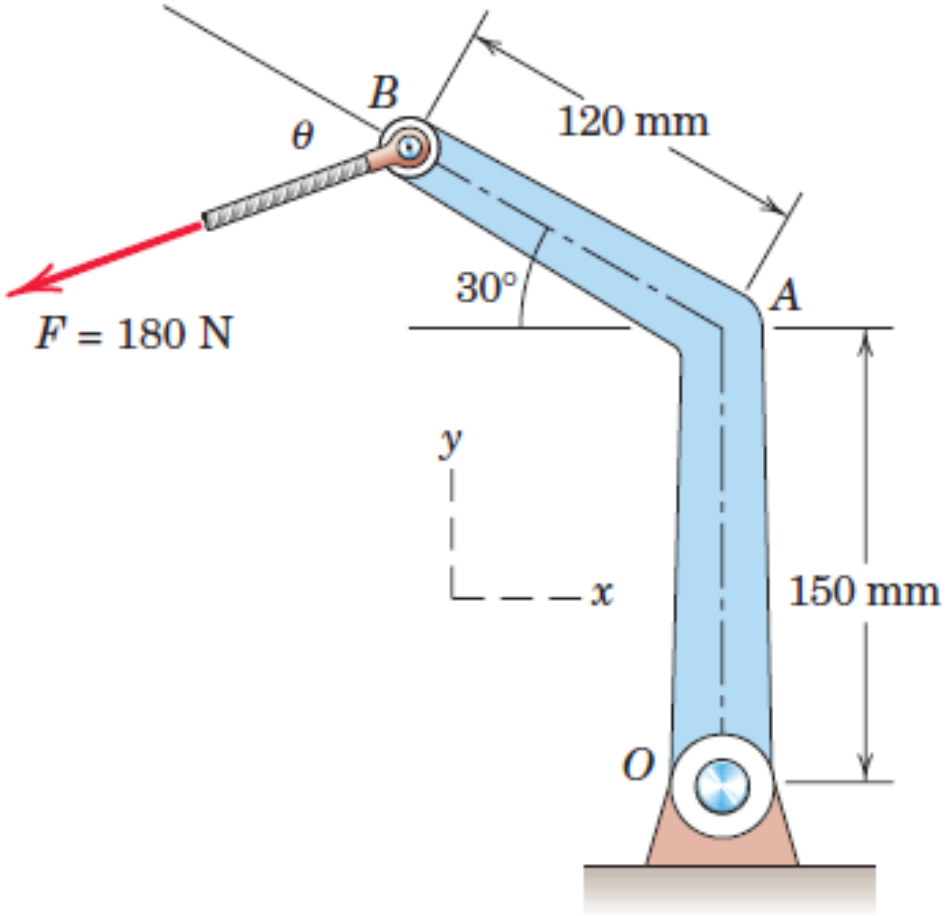


$$M = 250\,(0.7) = 2F\,(0.25)$$

$$F = 350\ N$$

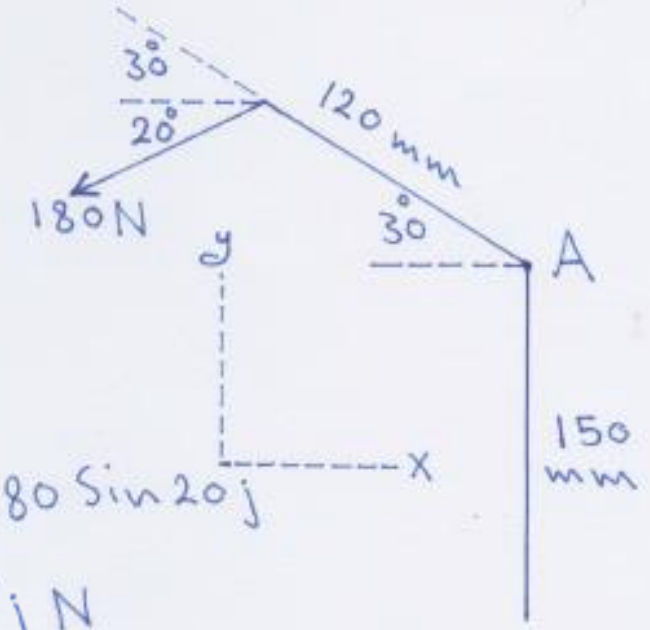
2/67 The 180-N force is applied to the end of body OAB. If $\theta = 50°$, determine the equivalent force–couple system at the shaft axis O.

Ans. $\mathbf{F} = -169.1\mathbf{i} - 61.6\mathbf{j}$ N, $M_O = 41.9$ N·m CCW



Problem 2/67

2/67



$$F = F_x i + F_y j$$

$$= -180 \cos 20 i - 180 \sin 20 j$$

$$= -169.1 i - 61.6 j \text{ N}$$

$$\circlearrowleft_+ M_o = F_x (150 + 120 \sin 30) + F_y (120 \cos 30)$$

$$= 169.1(150 + 120 \sin 30) + 61.6 (120 \cos 30)$$

$$= 41900 \text{ N.mm} \quad \text{or} \quad 41.9 \text{ N.m CCW}$$