Theory of structure

Deflection

L12

Assistant Lecturer

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member	origin	limit	EI	M	m_1	m_2	m_3
ad	d	$0 \to 2$	2	$-50X-60$	-1	$\frac{X+2}{6}$	$-(X+2)$
db	b	$0 \to 2$	2	$-30X$	-1	$\frac{X}{6}$	$-X$
bc	b	$0 \to 6$	1	$30X-5X^2$	0	$1-\frac{X}{6}$	0

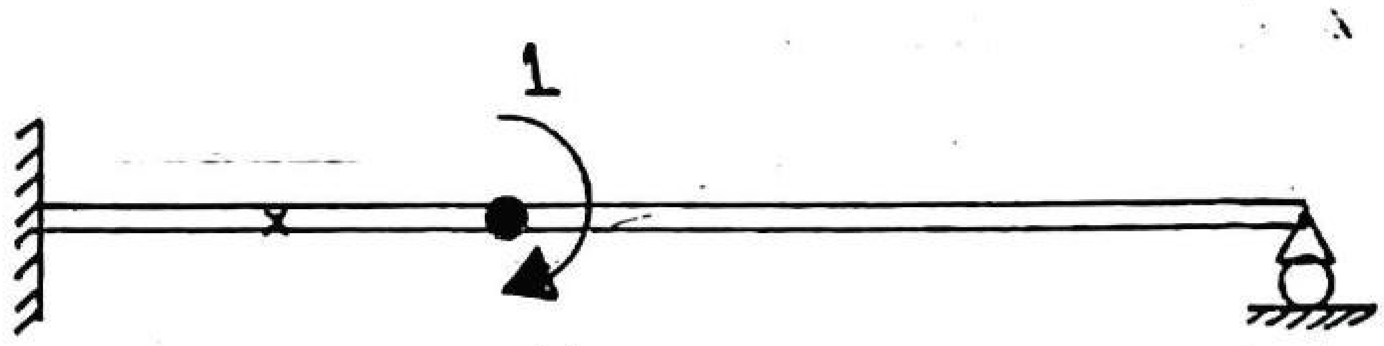
$$\theta_{ba} = \Sigma \int \frac{M m_1 \, dx}{EI}$$

$$= \int_0^2 \frac{(-50X-60)(-1)\,dx}{2EI} + \int_0^2 \frac{(-30X)(-1)\,dx}{2EI} + 0$$

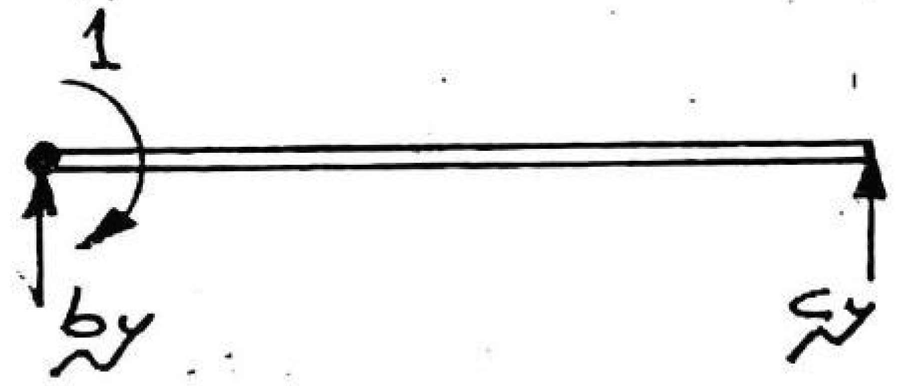
$$= \frac{1}{2EI}\left[\int_0^2 (50X+60)\,dx + \int_0^2 30X\,dx\right]$$

$$= \frac{1}{2EI}\left[\left[25X^2+60X\right]_0^2 + \left[15X^2\right]_0^2\right]$$

$$\theta_{ba} = \frac{140}{EI} \text{ rad}$$



* To find (θ_{bc}) M_2 :-
* To find reaction:-
* for part bc :-

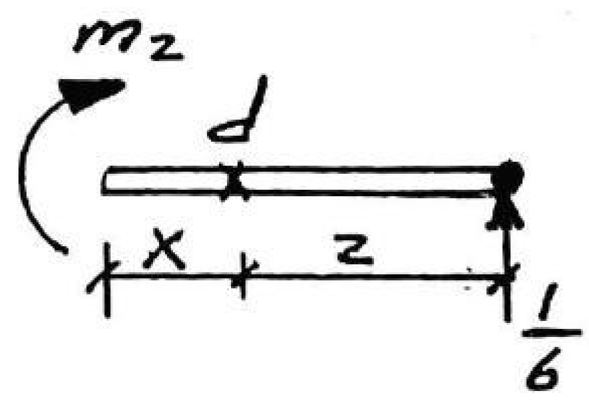


$\Sigma M_c = 0$

$1 + by * 6 = 0$

$by = \frac{1}{6} \downarrow$

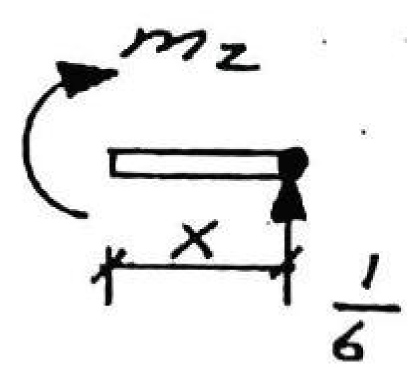
① for part ad:-



$m_2 = \frac{1}{6} * (X+2)$

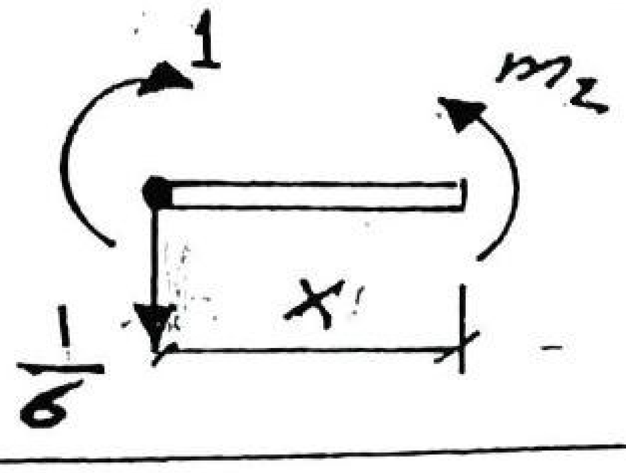
$m_2 = \frac{X+2}{6}$

② for part db :-



$m_2 = \frac{X}{6}$

③ for part bc :-



$m_2 = 1 - \frac{X}{6}$

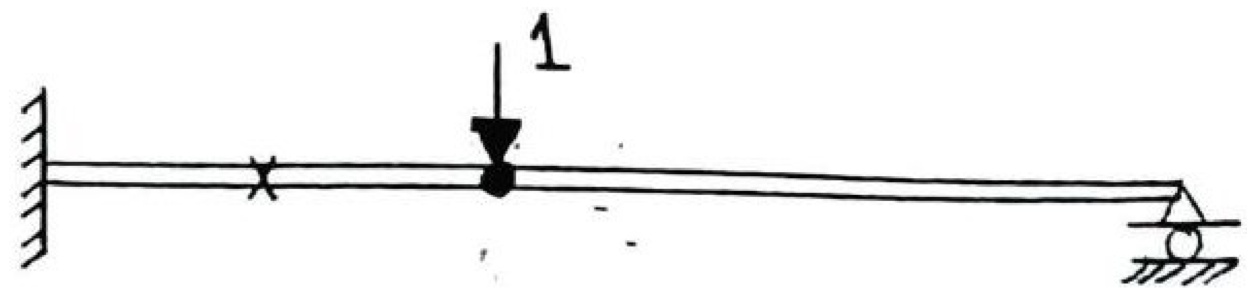
$$\therefore \theta_{bc} = \sum \int \frac{M m_2\, dx}{EI}$$

$$= \int_0^2 \frac{(-50x - 60)\left(\frac{x+2}{6}\right) dx}{2EI} + \int_0^2 \frac{(-30x)\left(\frac{x}{6}\right) dx}{2EI} + \int_0^6 \frac{(30x - 5x^2)\left(1 - \frac{x}{6}\right) dx}{EI}$$

$$= \frac{1}{6EI}\left[\int_0^2 \frac{(-50x^2 - 160x - 120)dx}{2} + \int_0^2 (-15x)dx + \int_0^6 (180x - 60x^2 + 5x^3)dx\right]$$

$$= \frac{1}{6EI}\left[\left[-\frac{25}{3} - 40x^2 - 60x\right]_0^2 + \left[-5x^3\right]_0^2 + \left[90x^2 - 20x^3 + 1.25x^4\right]_0^6\right]$$

$$\theta_{bc} = \frac{25.56}{EI} \text{ rad}$$



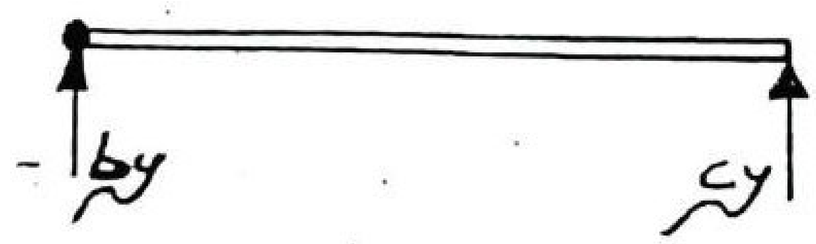
* To find M_3 (Δb):-

* To find reaction:-

* For part bc:-

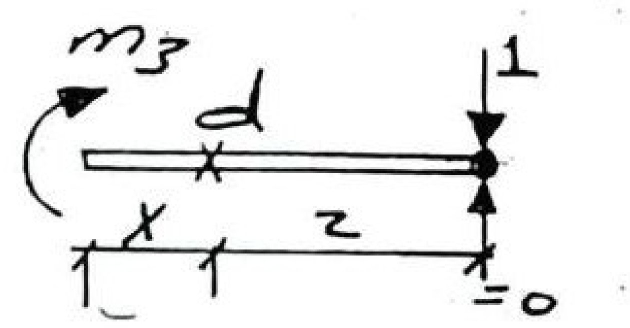
$\sum M_c = 0$

$by = 0$



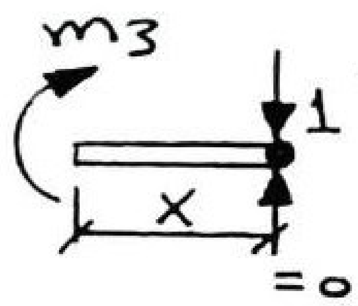
① for part ad:-

$m_3 = -(x + 2)$



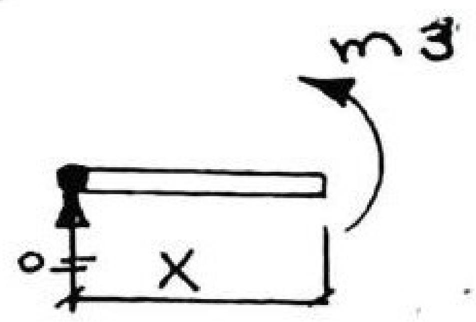
② for part db:-

$m_3 = -x$



③ for part bc:-

$m_3 = 0$



$$\therefore \Delta b = \sum \int \frac{M m_3\, dx}{EI}$$

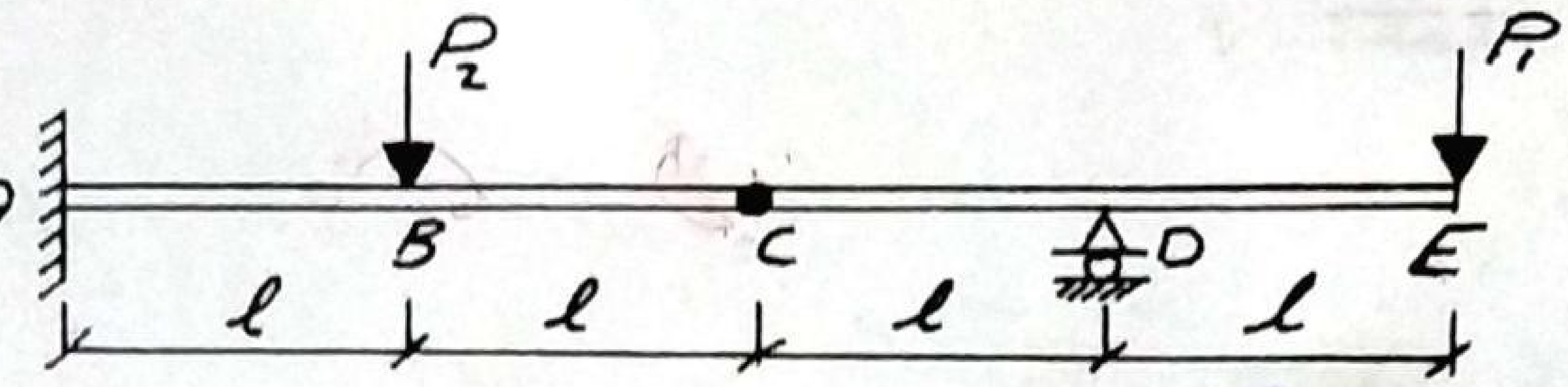
$$= \int_0^2 \frac{(-50x - 60)(-(x+2))dx}{2EI} + \int_0^2 \frac{(-30x)(-x)dx}{2EI} + 0$$

$$= \frac{1}{2EI}\left[\int_0^2 (50x^2 + 160x + 120)dx + \int_0^2 (30x^2)dx\right]$$

$$= \frac{1}{2EI}\left[\left[\frac{50}{3}x^3 + 80x + 120x\right]_0^2 + \left[10x^3\right]_0^2\right]$$

$$\Delta b = \frac{386.67}{EI}$$

Ex:- For the beam shown find (P_1/P_2) to make the deflection at (c) equal to zero. Assume $EI = Const$



H.W/ ملزمة ٦

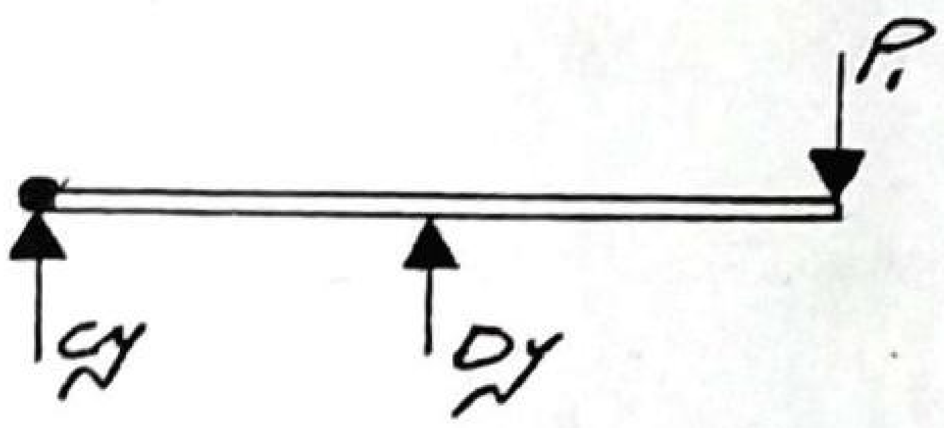
Sol.

* في هذا النوع من المسائل الذي تكون فيه مثلاً أحدى القوى مجهولة أو أحدى المسافات مجهولة ويتم اعطاء قيمة التشوه في نقطة معينة فسوف يتم اعتبار ان هذا التشوه هو المطلوب ومن ثم حل السؤال بصورة طبيعية وكتابة معادلة التشوه المعلوم وبعد كتابة معادلة التشوه سوف يكون طرفي التكامل المجهول ويتم تعويض قيمة التشوه لاستخراج هذا المجهول .

* To find M:-

* To find reaction:-

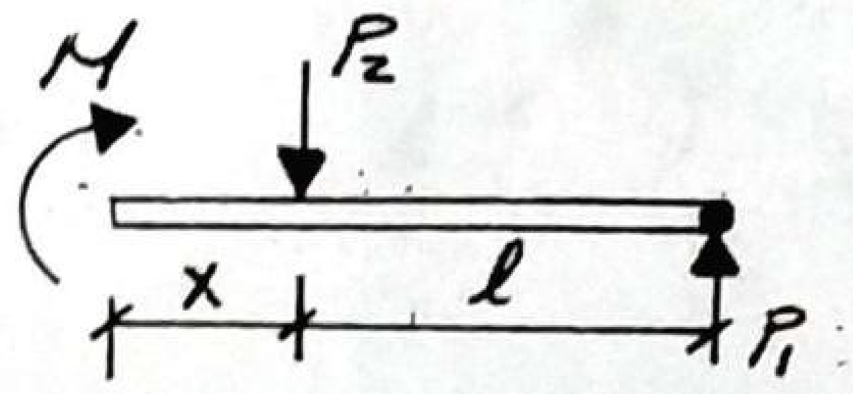
* for part CDE



$\circlearrowleft \sum M_D = 0$

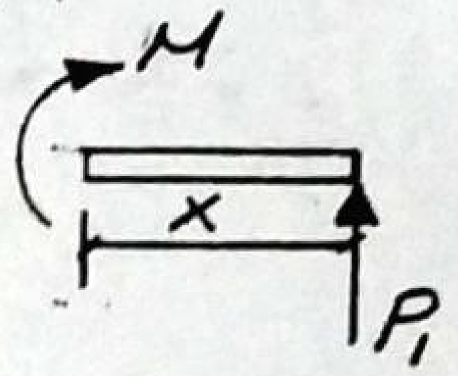
$C_y \cdot l + P_1 \cdot l = 0 \Rightarrow C_y = P_1 \downarrow$

① for part AB :-



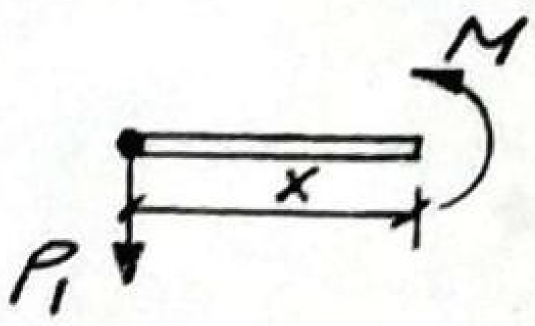
$M = P_1(l + x) - P_2(x)$

② for part BC :-



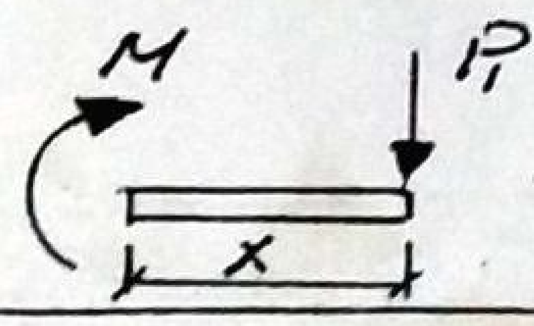
$M = P_1(x)$

③ for part CD :-



$M = -P_1(x)$

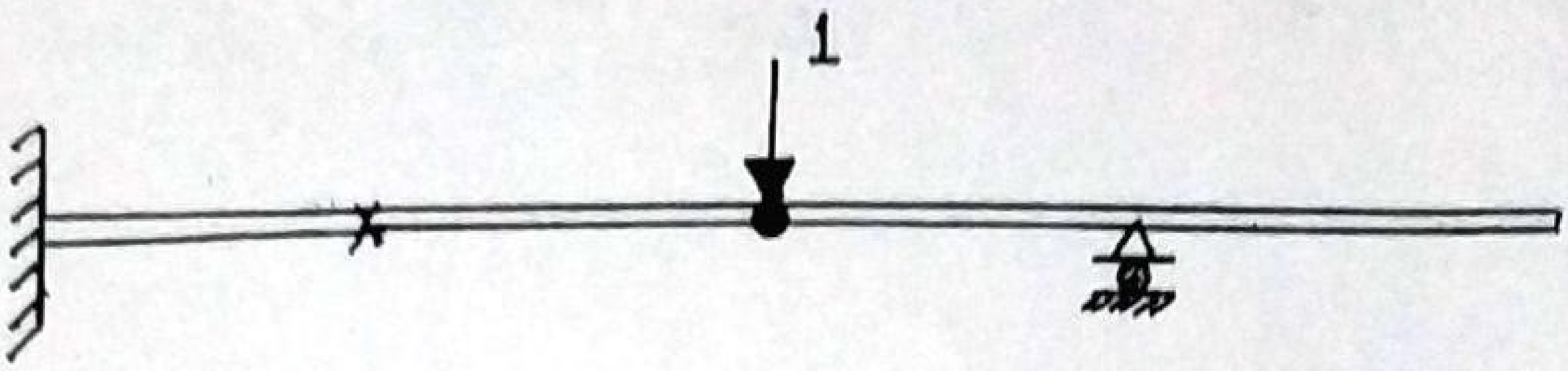
④ for part DE :-



$M = -P_1(x)$

* To find m (Δc):-

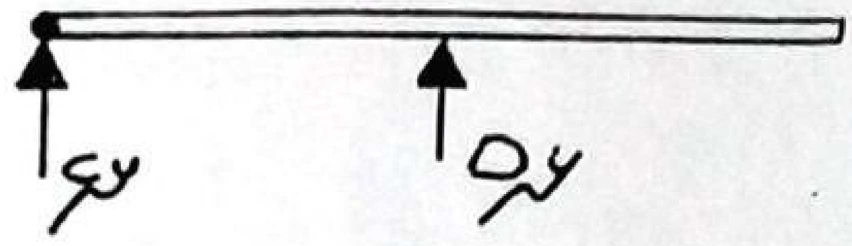
* To find reaction:-



* for part CDE :-

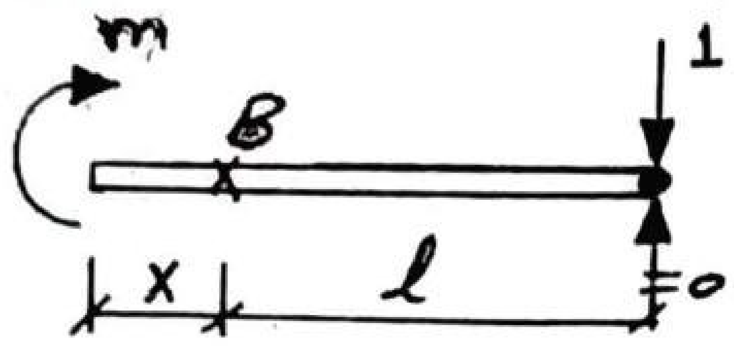
$\sum M_D = 0$

$C_y = 0$



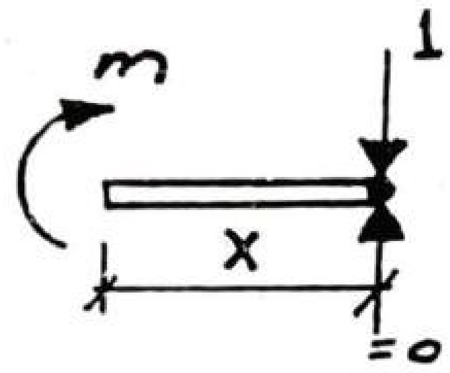
① for part AB:-

$m = -(l + x)$



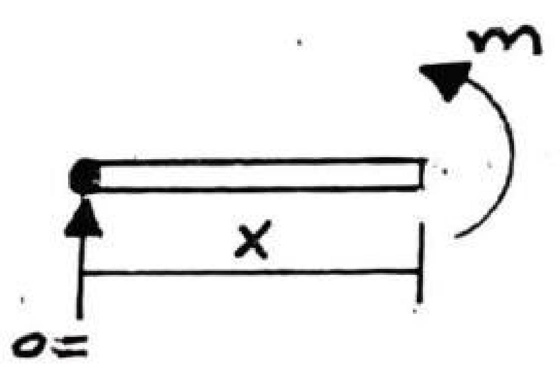
② for part Bc :-

$m = -x$



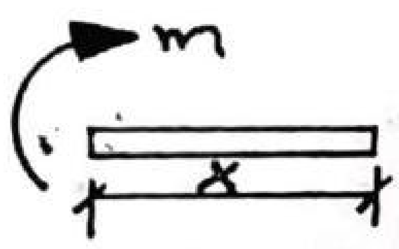
③ for part CD :-

$m = 0$



④ for part DE :-

$m = 0$



member	origin	Limit	EI	M	m
AB	B	$0 \rightarrow l$	1	$P_1(l+x) - P_2 x$	$-(l+x)$
Bc	C	$0 \rightarrow l$	1	$P_1 x$	$-x$
CD	C	$0 \rightarrow l$	1	$-P_1 x$	0
DE	E	$0 \rightarrow l$	1	$-P_1 x$	0

$$\therefore \Delta c = \Sigma \int \frac{M m\, dx}{EI}$$

$$= \int_0^l \frac{(P_1(l+x) - P_2 X) + (-l-x)\, dx}{EI} + \int_0^l \frac{(P_1 X)(-X)\, dx}{EI} + 0 + 0$$

$$= \frac{1}{EI}\left[\int_0^l [P_1(-l^2 - 2xl - x^2) + P_2(lx - x^2)\, dx + \int_0^l (-P_1 x^2)\, dx\right]$$

$$= \frac{1}{EI}\left[P_1\left[-l^2 x - l x^2 - \frac{x^3}{3}\right]_0^l + P_2\left[\frac{l x^2}{2} + \frac{x^3}{3}\right]_0^l + \left[\frac{P_1 X^3}{3}\right]_0^l\right]$$

$$\Delta c = \frac{1}{EI}\left[P_1\left(-l^3 - l^3 - \frac{l^3}{3}\right) + P_2\left(\frac{l^3}{2} + \frac{l^3}{3}\right) - \frac{P_1 l^3}{3}\right]$$

$\therefore \Delta c = 0 \longrightarrow$ معطاة في السؤال

$$\therefore 0 = \frac{1}{EI}\left[-\frac{7}{3} P_1 l^3 + \frac{5}{6} P_2 l^3 - \frac{P_1 l^3}{3}\right]$$

$$0 = \frac{l^3}{EI}\left[-\frac{8}{3} P_1 + \frac{5}{6} P_2\right]$$

$$\therefore -\frac{8}{3} P_1 + \frac{5}{6} P_2 = 0 \qquad \div P_2$$

$$-\frac{8}{3} \times \frac{P_1}{P_2} + \frac{5}{6} = 0 \quad \Rightarrow \quad \frac{P_1}{P_2} = \frac{5}{16}$$