Theory of structure

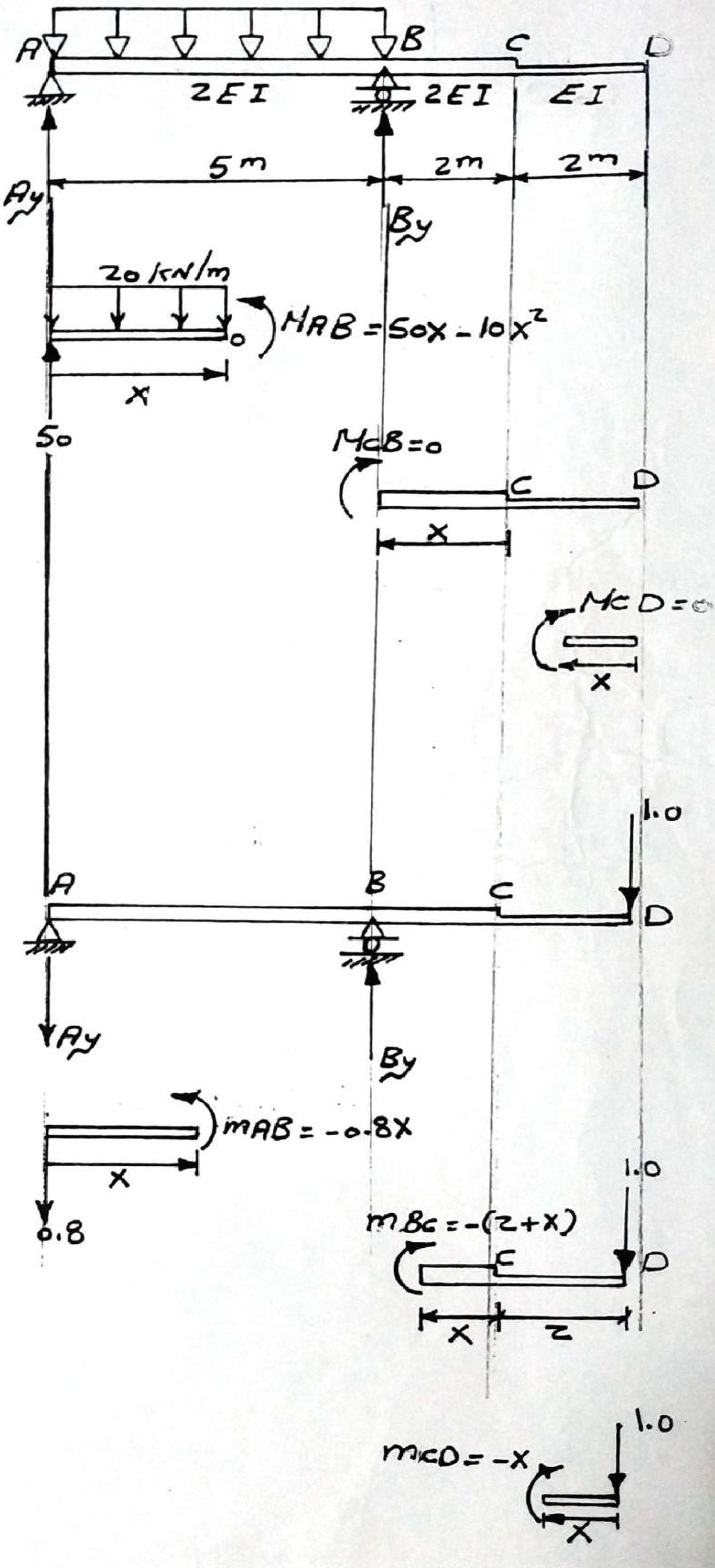
Deflection

L11

Assistant Lecturer

Maryam Mohammed Al-aarajy

<u>Ex</u> : By using unit-load method, Find the deflection in point D in the beam shown in fig.



<u>Sol.</u>

$\sum M_A = 0$ ⊕

$20*5*2.5 - B_y(5) = 0$

$B_y = 50 \uparrow$

$\sum F_y = 0 \uparrow$

$A_y - 20*5 + 50 = 0 \Rightarrow A_y = 50 \uparrow$

To Find (M):

For AB

For Bc

For CD

To Find (m):

$\sum M_A = 0$ ⊕

$1.0(9) - B_y(5) = 0$

$B_y = \frac{9}{5} \uparrow$

$\sum F_y = 0 \uparrow ⊕$

$-A_y + \frac{9}{5} - 1 = 0$

$A_y = 0.8$

For AB

For Bc

For CD

mem.	EI	origin	limit	M	m
AB	2EI	A	0 → 5	$50x - 10x^2$	$-0.8x$
BC	2EI	C	0 → 2	0	$-(2+x)$
CD	EI	D	0 → 2	0	$-x$

$$\Delta = \int \frac{M.m}{EI}\,dx$$

$$\Delta_D = \int_0^5 \frac{-0.8x\,(50x - 10x^2)}{2EI}\,dx + \int_0^2 \cancel{\frac{0 * -(2+x)}{2EI}}\,dx + \int_0^2 \cancel{\frac{0 * -x}{EI}}\,dx$$

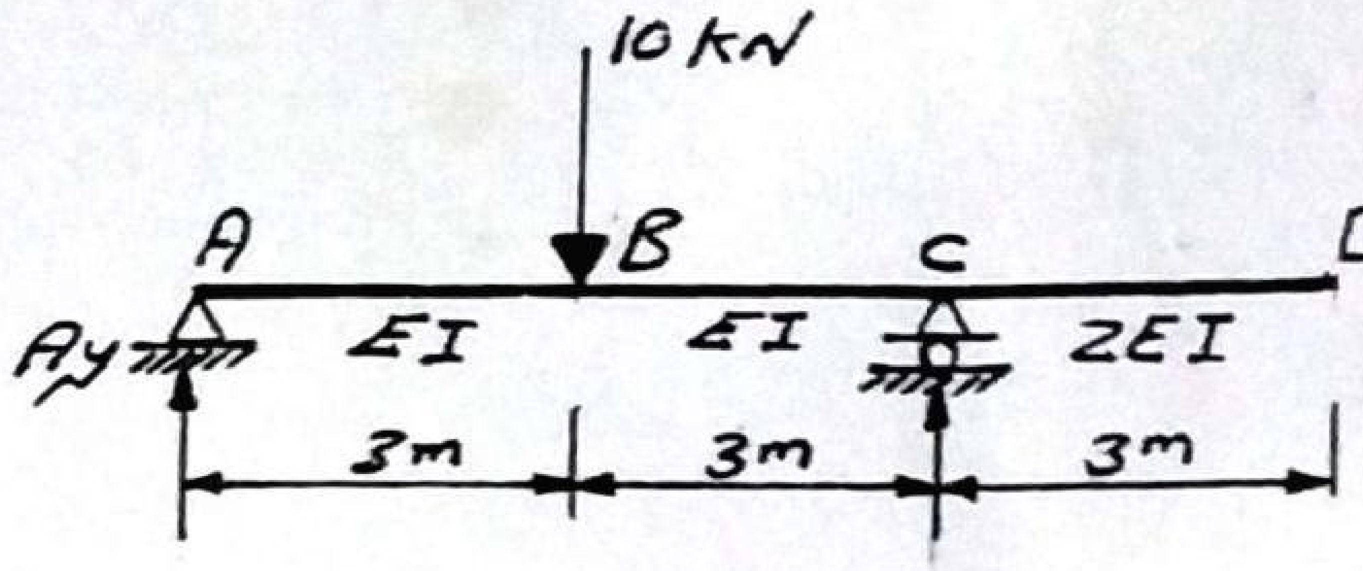
$$\Delta_D = \frac{-0.8}{2EI}\int_0^5 (50x^2 - 10x^3)\,dx$$

$$= \frac{-0.8}{2EI}\left[\frac{50}{3}x^3 - \frac{10}{4}x^4\right]_0^5$$

$$= \frac{-0.8}{2EI}\left[\left(\frac{50}{3}(5)^3 - \frac{10}{4}(5)^4\right) - (0)\right]$$

$$= \frac{208.33}{EI} = \frac{208.33}{EI} \uparrow$$

Ex:- By using unit-load method, Find the deflection in point D in the beam shown in fig.



Sol.

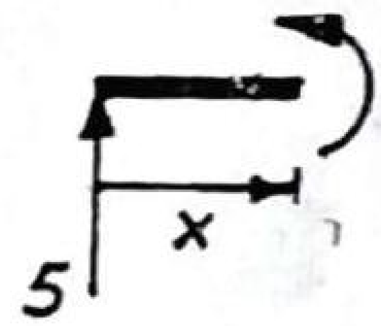
$\sum M_C = 0$ ⊕

$A_y(6) - 10(3) = 0$

$A_y = 5 \uparrow$

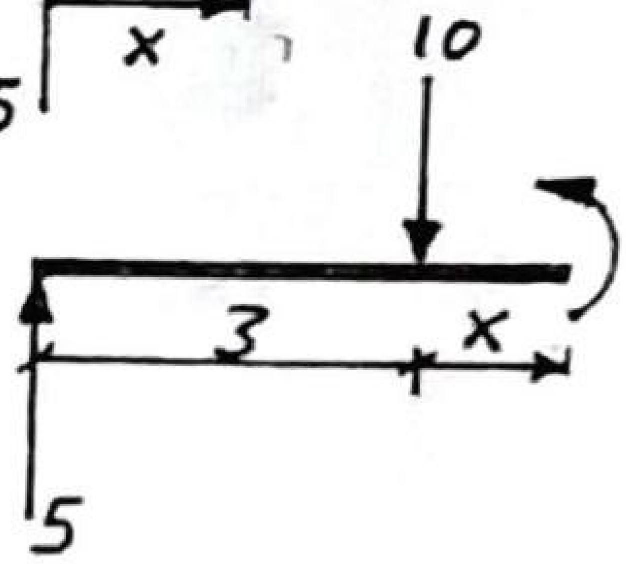
To Find (M):

For AB:



$M_{AB} = 5x$

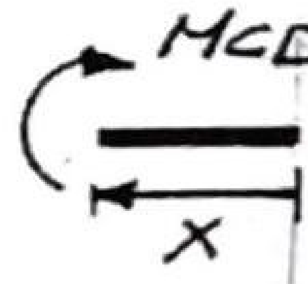
For BC:



$M_{BC} = 5(3+x) - 10x$

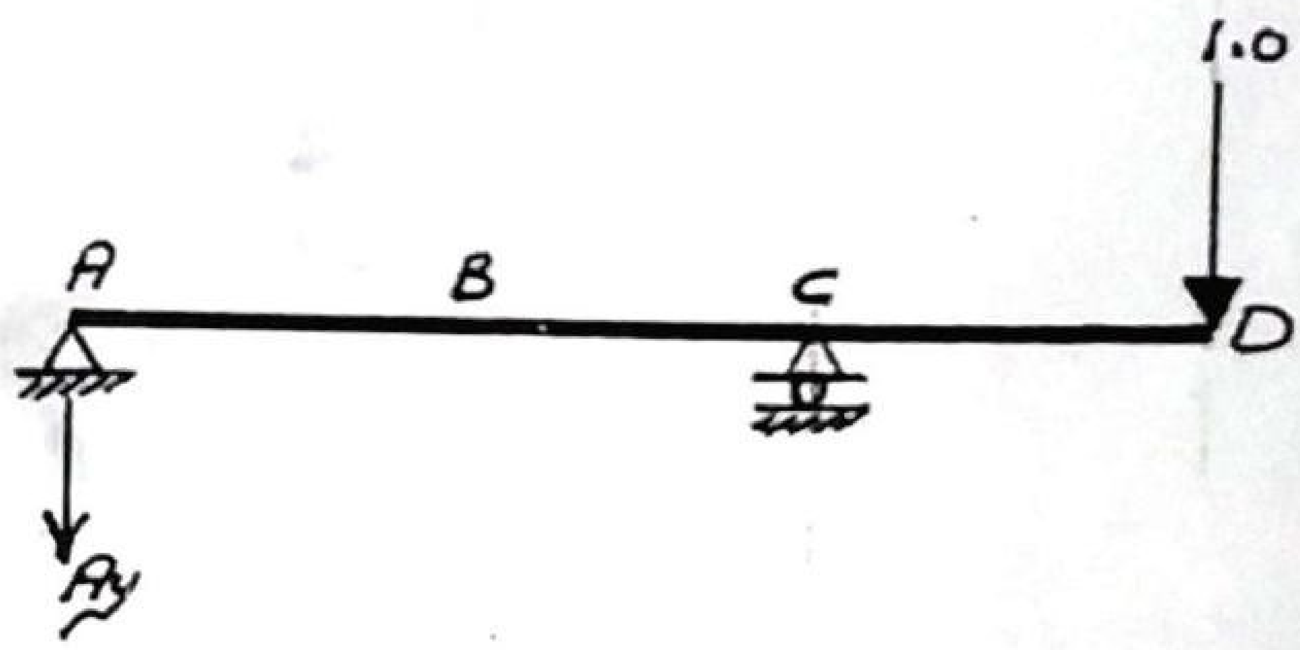
$= 15 - 5x$

For CD:



$M_{CD} = 0$

To Find (m):

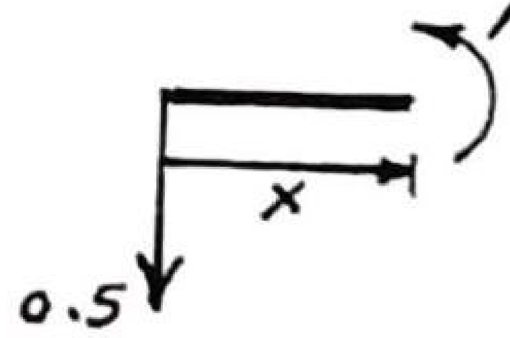


$\sum M_C = 0$ ⊕

$1 * 3 - A_y(6) = 0$

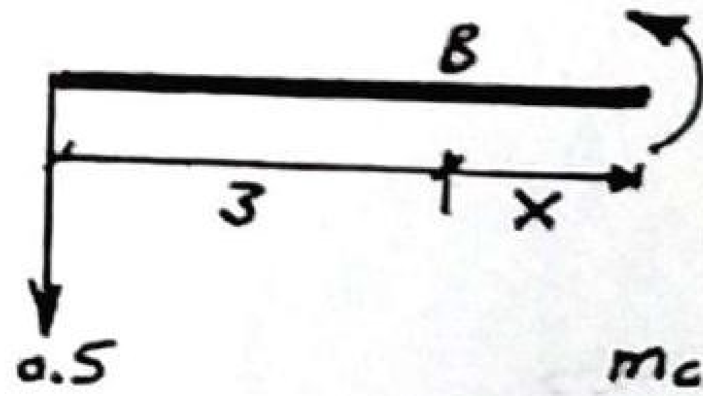
$A_y = 0.5 \downarrow$

For AB:



$m_{AB} = -0.5x$

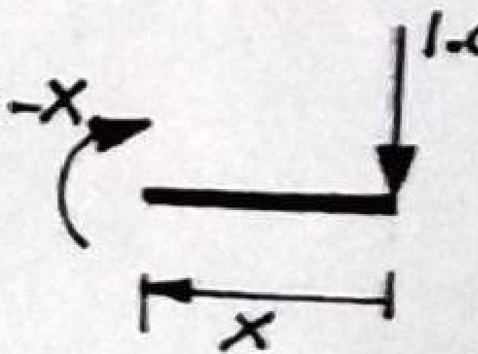
For BC:



$m_{BC} = -0.5(3+x)$

$= -(1.5 + 0.5x)$

For CD:



$m_{CD} = -x$

mem.	EI	origin	Limit	M	m
AB	EI	A	$0 \to 3$	$5x$	$-0.5x$
BC	EI	B	$0 \to 3$	$15-5x$	$-(1.5+0.5x)$
CD	2EI	D	$0 \to 3$	0	$-x$

$$\Delta = \int \frac{M.m}{EI} dx$$

$$\Delta D = \int_0^3 \frac{5x * -0.5x}{EI} dx + \int_0^3 \frac{(15-5x) * -(1.5+0.5x)}{EI} dx + \cancel{\int_0^3 \frac{0 * -x}{2EI} dx} \to 0$$

$$= -\frac{2.5}{EI}\int_0^3 x^2 dx - \frac{1}{EI}\int_0^3 (22.5 - 2.5x^2)\, dx$$

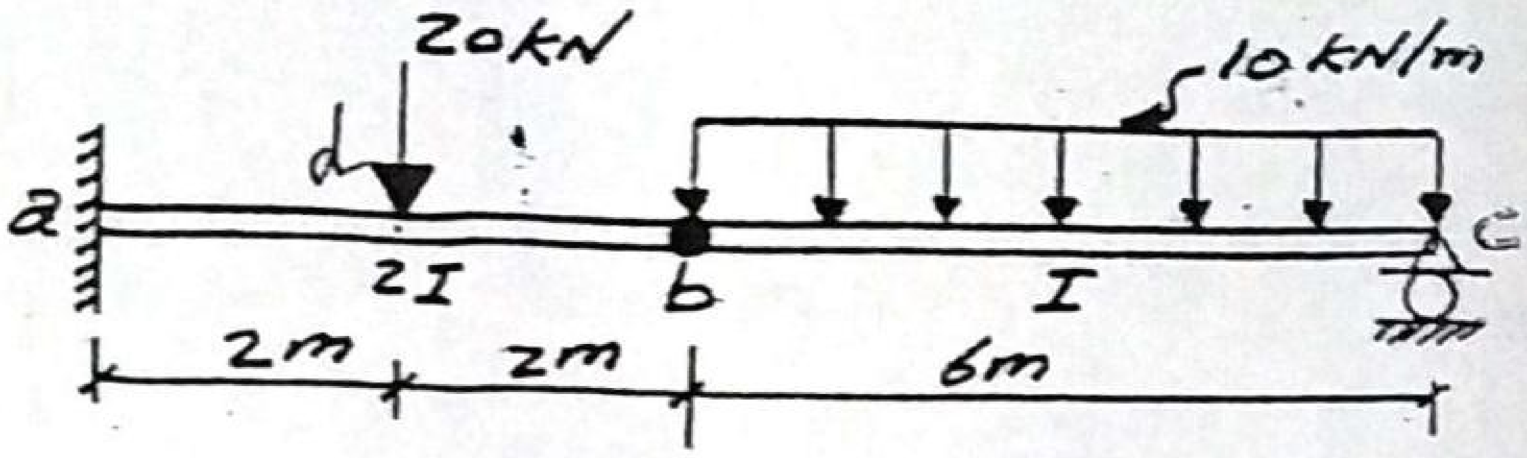
$$= -\frac{67.5}{EI} = \frac{67.5}{EI} \downarrow$$

Ex:- Find θb and Δb for the beam shown?

اول شي نقلو الحمل الموجود ... ونقطع

Sol.

القوة تكون



* ملاحظة :- عدد (θ) في نقطة المفصل الداخلي (Internal hinge) مساوية الى عدد الاضلاع المرتبطة بهذا (I.H).
* هذا يعني في سؤالنا هذا فأن عدد (θ) في نقطة (b) هي أثنان
* هذه الملاحظة خاصة فقط بنقطة (I.H) ولا تطبق في أي نقطة أخرى

[أي أن عدد (θ) في بقية النقاط هي واحدة فقط].

* أما أسماء هذه (θ) فيمكن تسميتها كما يلي :-

في حالة وجود حمل بيه هنج نكون قد القطعه للسنج

1. $(\theta b)_L$ 2. $(\theta b)_R$

* أو قد يتم تسميتها بدلالة الأضلاع المتصلة بها :-

1. θ_{ba} 2. θ_{bc}

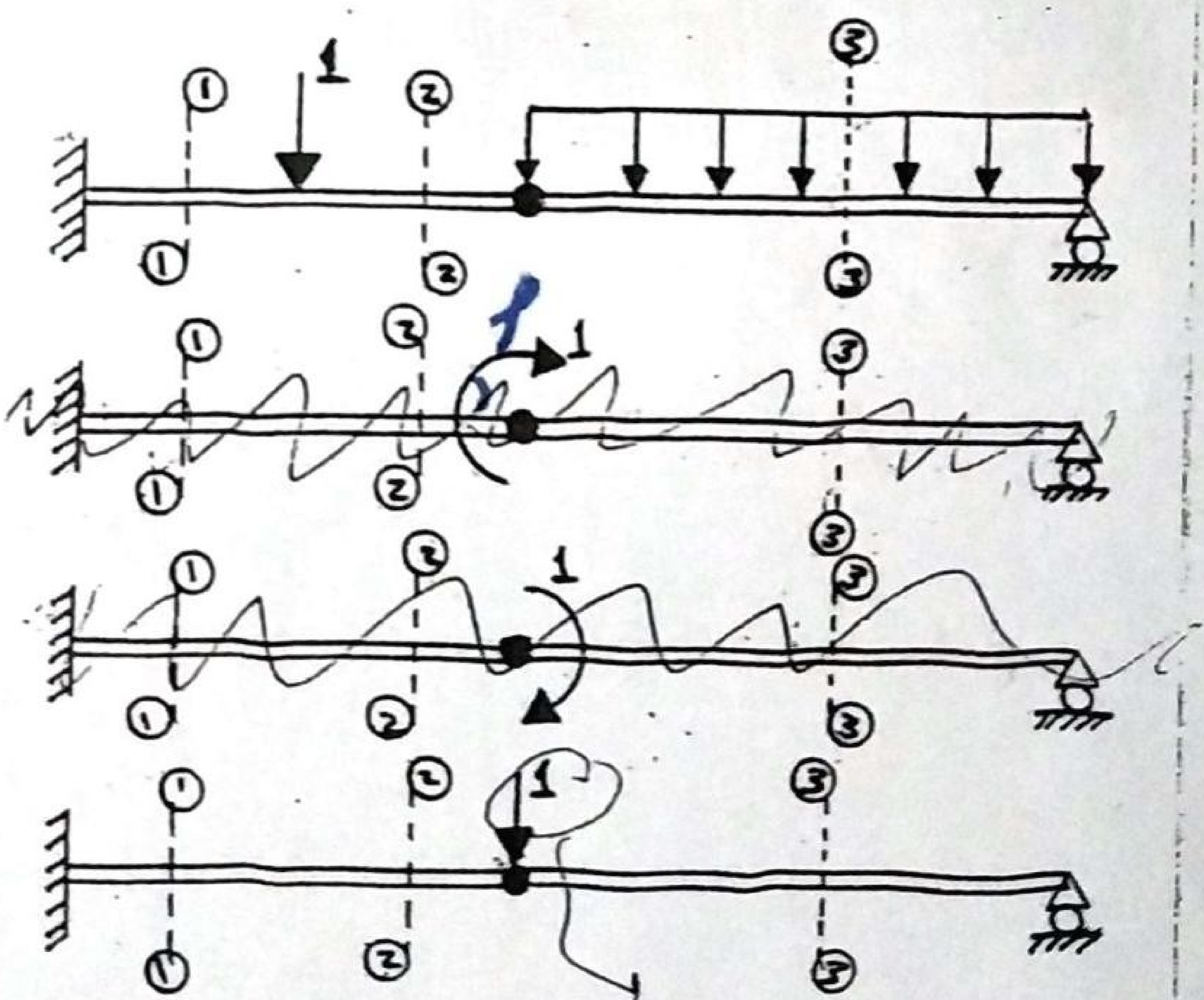
* لغرض التأكد من عدد القطوعات :-

Find M ⇐

Find m_1 (θ_{ba}) ⇐

Find m_2 (θ_{bc}) ⇐

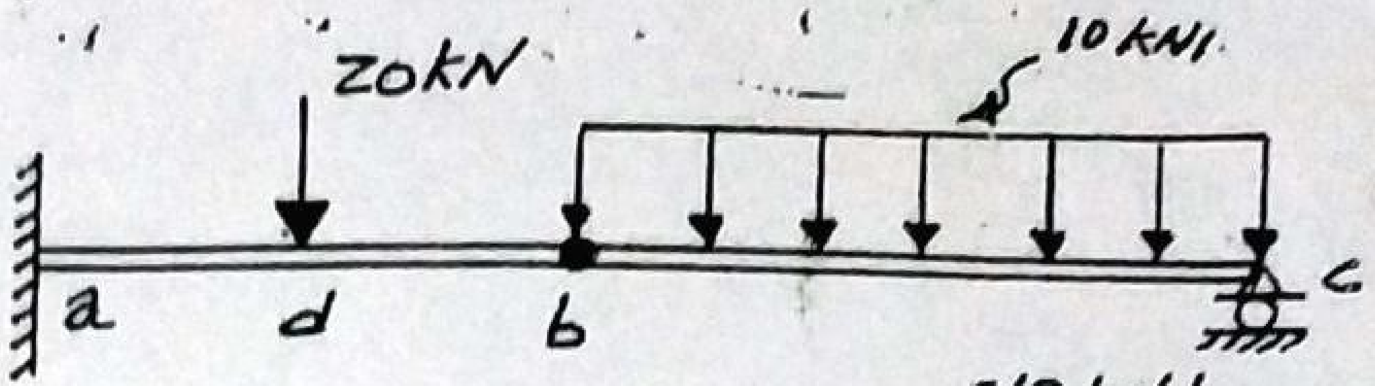
Find m_3 (Δb) ⇐



عند فصل المفصل القوة الموجودة على السنج القوة تكون على النقطة الاكثر مجاهيل

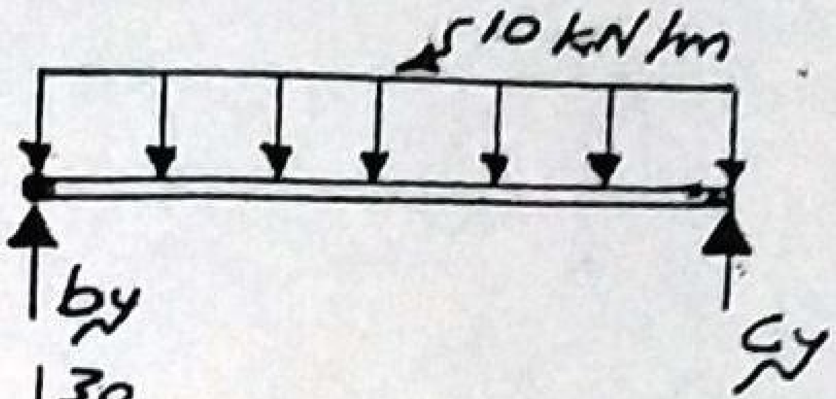
* To find M :-

* To find reaction :-



* for part bc :-

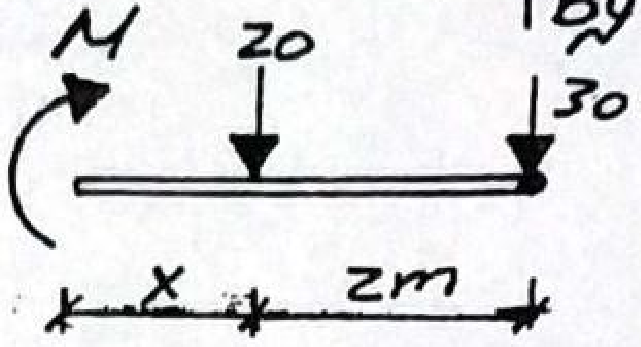
$b_y = c_y = \frac{10*6}{2} = 30 \uparrow$



① for part ad :-

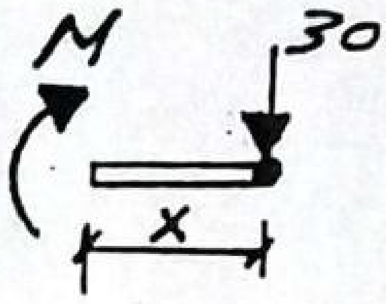
$M = -20(x) - 30(2+x)$

$M = 50x - 60$



② for part db :-

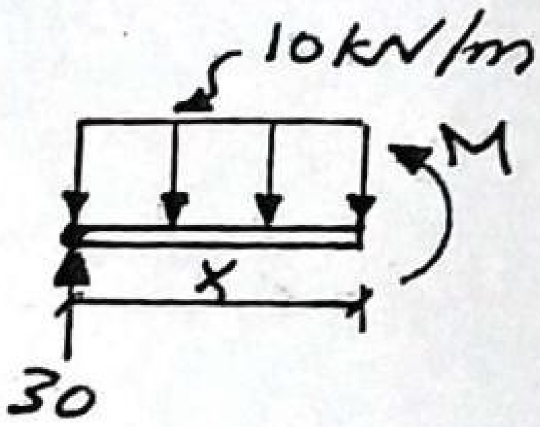
$M = -30x$



③ for part bc :-

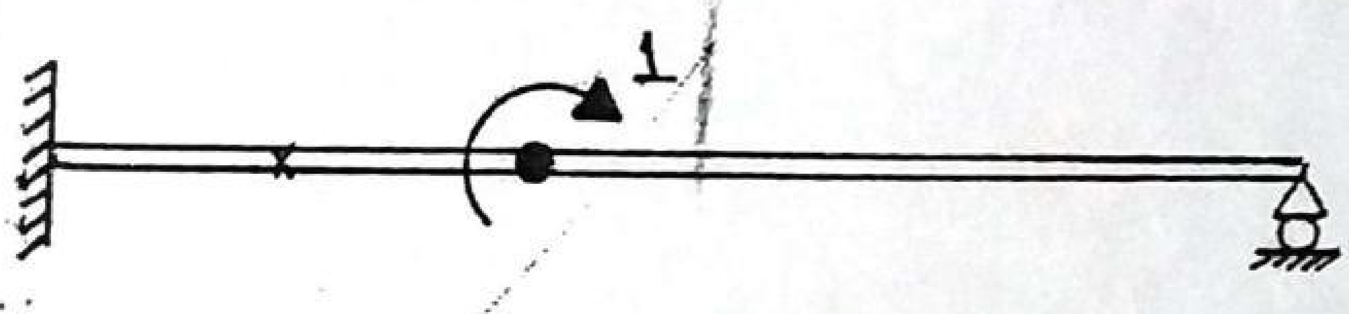
$M = 30x - \frac{10x^2}{2}$

$M = 30x - 5x^2$



* To find M_1 (θba) :-

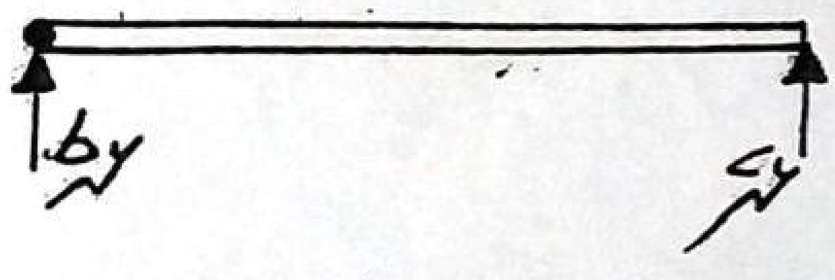
* To find reaction :-



* for part bc :-

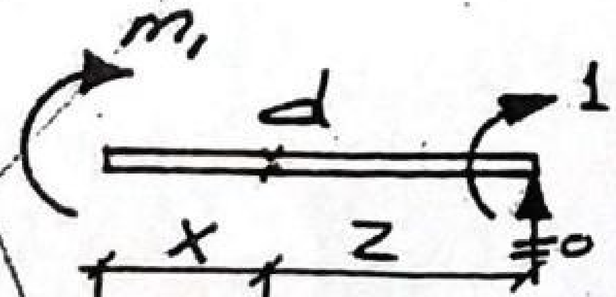
$\sum M_c = 0$

$\therefore b_y = 0 \Rightarrow c_y = 0$



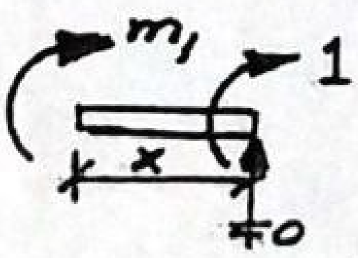
① for part ad :-

$m_1 = -1$



② for part db :-

$m_1 = -1$



③ for part bc :-

$m_1 = 0$