PHYSICS

Engineering Mechanics

Lecture 2

Force Systems and Principle of Transmissibility

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THE FORCE

- **Force: An action of one body on another.** the force is a ***vector quantity***, because its effect depends on the direction as well as on the magnitude of the action.

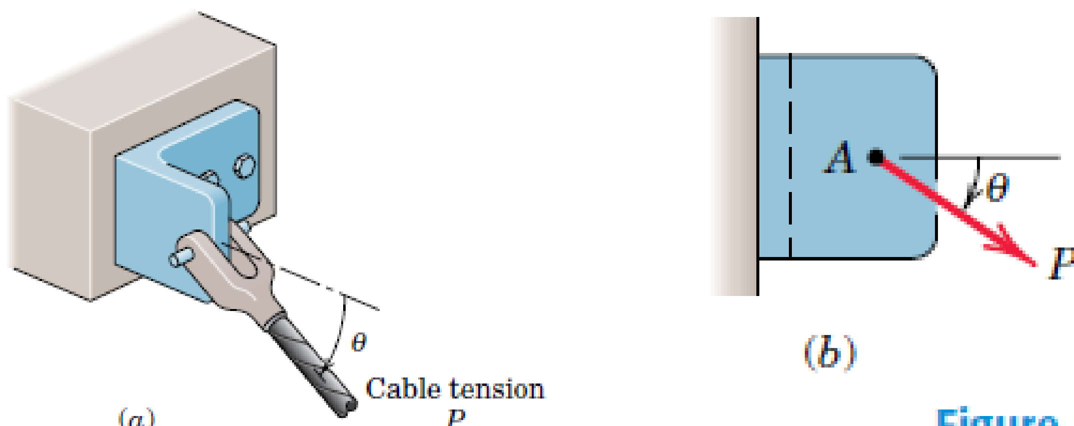


Figure 2/1

- The effect of cable tension on the bracket in fig 2/1 depends on the force vector P, the angle θ, and the location of the point of application A.

- Changing any one of these three specifications will alter the effect on the bracket,

PRINCIPLE OF TRANSMISSIBILITY

- **The principle of transmissibility**, states that a force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts.

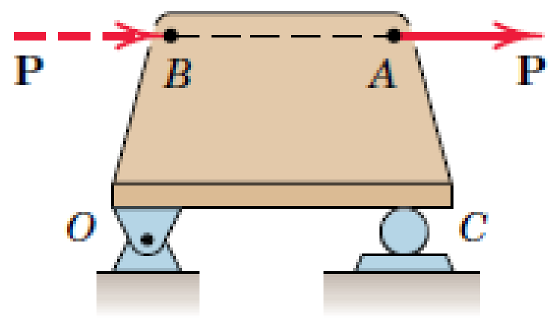


Figure 2/2

FORCE CLASSIFICATION

- **Contact force**. A contact force is produced by direct physical contact; **an example is the force exerted on a body by a supporting surface.**

- **Body force** is generated by virtue of the position of a body within a force field such as a gravitational, electric, or magnetic field. An example of a body force is your weight.

- **Concentrated or distributed force**. The force to be concentrated at a point with negligible loss of accuracy. **Force can be distributed over an area**.

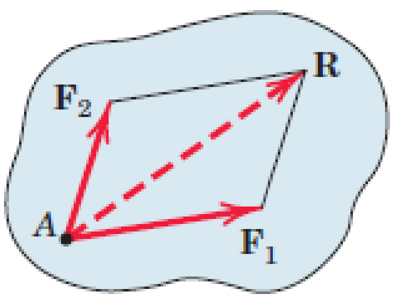
- **The weight** of a body is the force of gravitational attraction distributed over its volume and may be taken as a concentrated force acting through the center of gravity.

- **Action and Reaction:** According to Newton's third law, the **action** of a force is always accompanied by an **equal** and **opposite reaction**.

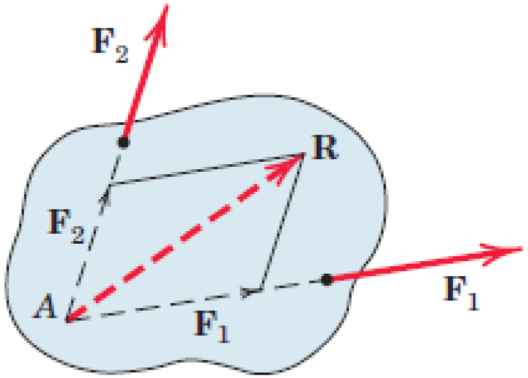
CONCURRENT FORCES

- Two or more forces are said to be **concurrent** *at a point* if their lines of action intersect at that point. The forces **F1** and **F2** shown in Fig. 2/3*a*.

- Suppose the two concurrent forces lie in the same plane but are applied **at two different points as in Fig. 2/3*b***.



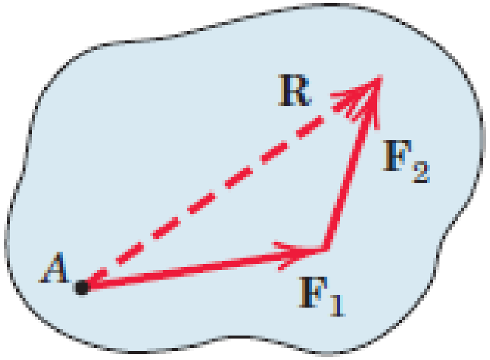
(*a*)



(*b*)

- By the principle of **transmissibility**. We can replace F1 and F2 with the resultant R without altering the external effects on the body upon which they act.

- We can also use the triangle law to obtain R, but we need to move the line of action of one of the forces, as shown in Fig. 2/3*c*.



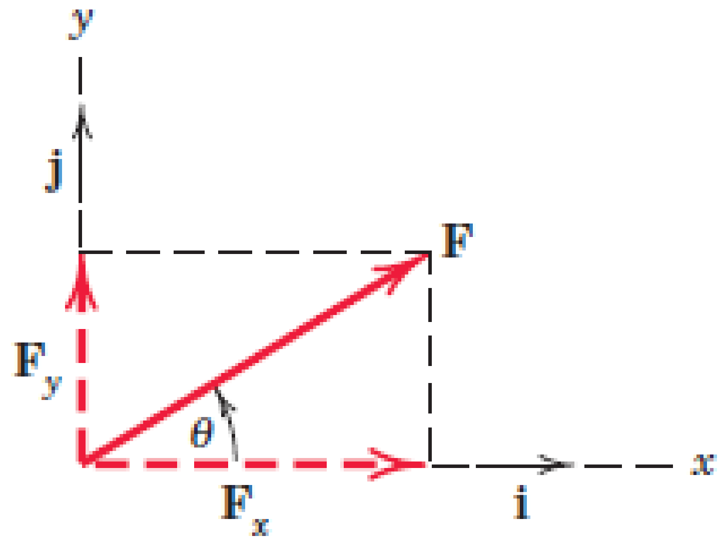
(*c*)

RECTANGULAR COMPONENTS

- The most common two-dimensional resolution of a force vector is into rectangular components. the vector **F** may be written as:

$$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y}$$

Where $\mathbf{F}x$ and $\mathbf{F}y$ are *vector* components of **F** in the *x*- and *y*-directions.



- In terms of the unit vectors **i** and **j** we may write:

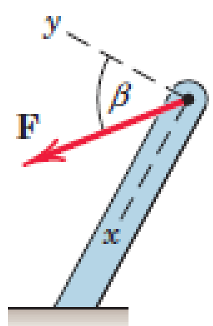
$$\mathbf{F} = \mathbf{F_x i} + \mathbf{F_y j}$$

Where:

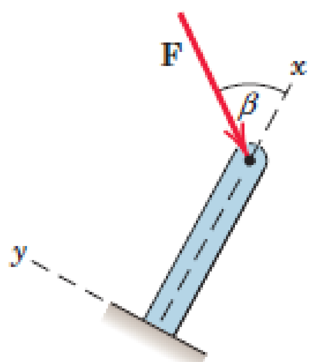
$$F_x = F\cos\theta \qquad F = \sqrt{{F_x}^2 + {F_y}^2}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

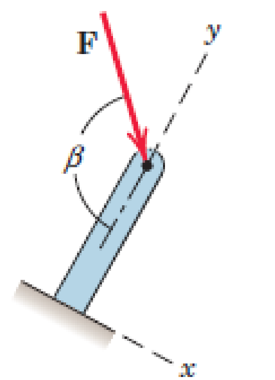
CONVENTIONS FOR DESCRIBING VECTOR COMPONENTS



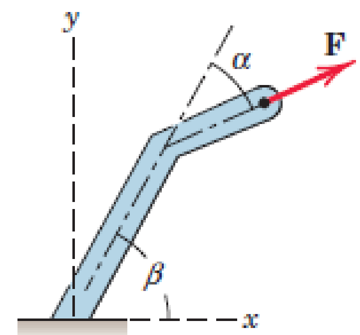
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$



$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$



$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

DETERMINING THE COMPONENTS OF A FORCE

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

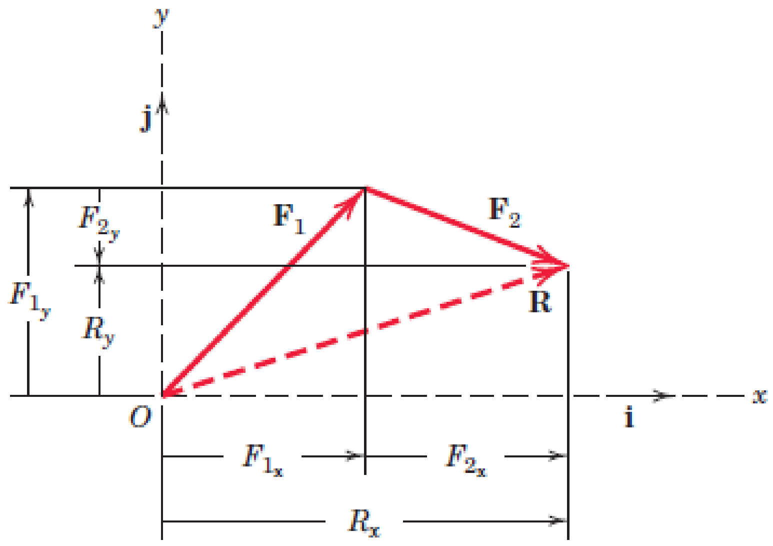
or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

from which we conclude that

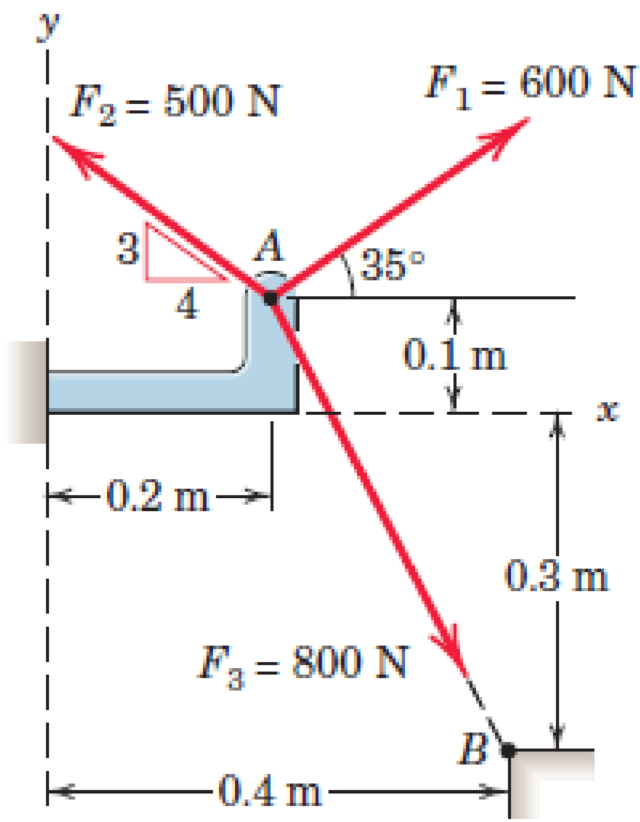
$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$

$$R_y = F_{1_y} + F_{2_y} = \Sigma F_y$$



Sample Problem 2/1

The forces **F**1, **F**2, and **F**3, all of which act on point *A* of the bracket, are specified in three different ways. Determine the *x* and *y* scalar components of each of the three forces.



Solution. The scalar components of $\mathbf{F}_1$, from Fig. *a*, are

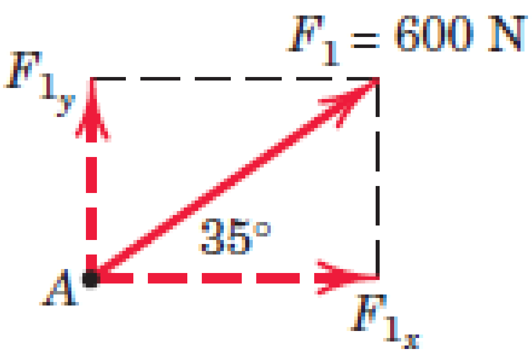
$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad \textit{Ans.}$$

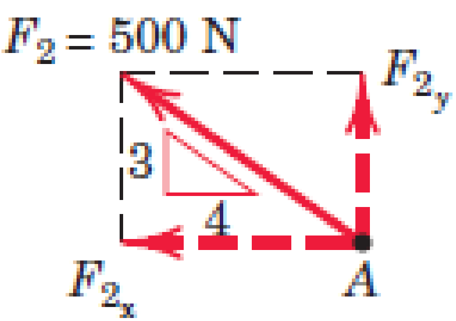
The scalar components of $\mathbf{F}_2$, from Fig. *b*, are

$$F_{2_x} = -500\left(\frac{4}{5}\right) = -400 \text{ N} \qquad \textit{Ans.}$$

$$F_{2_y} = 500\left(\frac{3}{5}\right) = 300 \text{ N} \qquad \textit{Ans.}$$



(*a*)



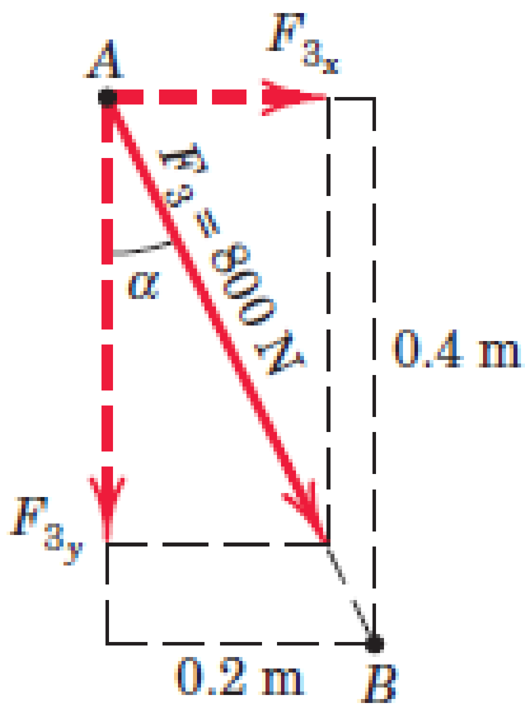
(*b*)

The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. *c*.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

① Then $F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N}$ *Ans.*

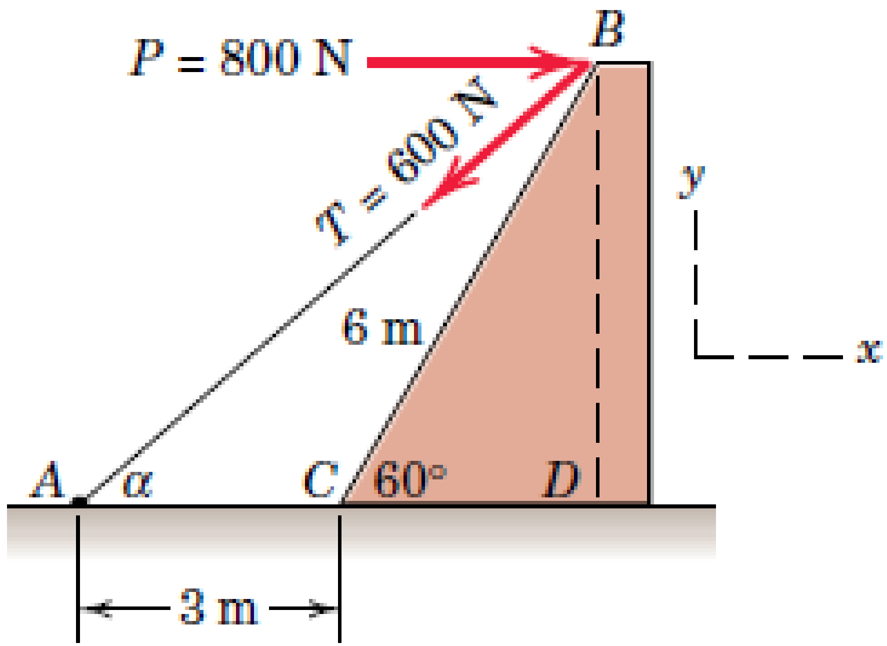
$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N}$$ *Ans.*



(*c*)

Sample Problem 2/2

Combine the two forces **P** and **T**, which act on the fixed structure at *B*, into a single equivalent force **R**.



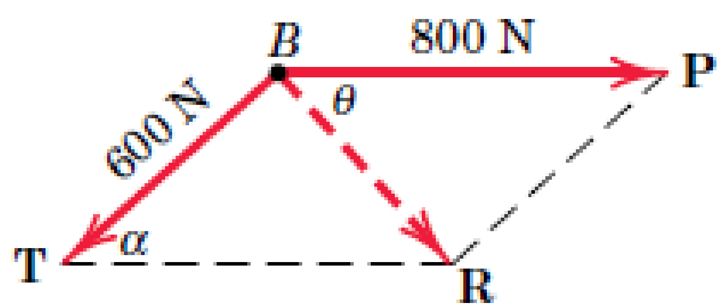
①

Graphical solution. The parallelogram for the vector addition of forces **T** and **P** is constructed as shown in Fig. *a*. The scale used here is 1 cm = 400 N; a scale of 1 cm = 100 N would be more suitable for regular-size paper and would give greater accuracy. Note that the angle α must be determined prior to construction of the parallelogram. From the given figure

$$\tan\alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6\sin 60^\circ}{3 + 6\cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$

Measurement of the length R and direction θ of the resultant force **R** yields the approximate results

$$R = 525\text{ N} \qquad \theta = 49^\circ \qquad \textit{Ans.}$$



(*a*)

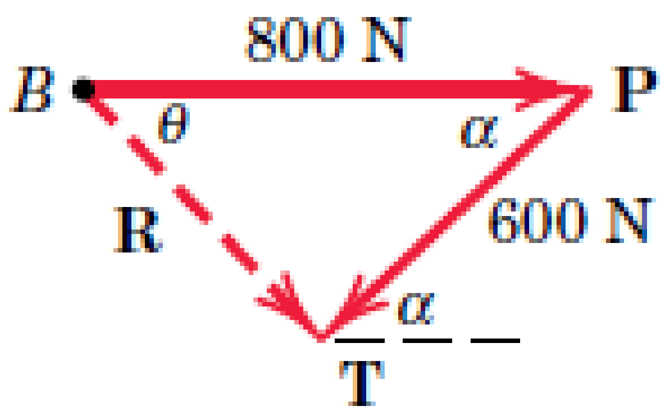
Geometric solution. The triangle for the vector addition of **T** and **P** is shown in Fig. *b*. The angle α is calculated as above. The law of cosines gives

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9° = 274{,}300$$

$$R = 524 \text{ N} \qquad\qquad \textit{Ans.}$$

From the law of sines, we may determine the angle θ which orients **R**. Thus,

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9°} \qquad \sin\theta = 0.750 \qquad \theta = 48.6° \qquad\qquad \textit{Ans.}$$



(*b*)

Algebraic solution. By using the x-y coordinate system on the given figure, we may write

$$R_x = \Sigma F_x = 800 - 600 \cos 40.9° = 346 \text{ N}$$

$$R_y = \Sigma F_y = -600 \sin 40.9° = -393 \text{ N}$$

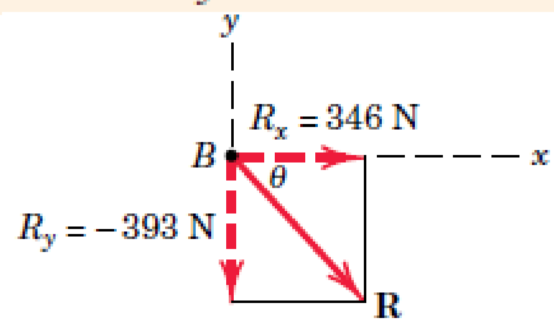
The magnitude and dipection of the resultant force **R** as shown in Fig. *c* are then

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ N} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1} \frac{|R_y|}{|R_x|} = \tan^{-1} \frac{393}{346} = 48.6° \qquad \textit{Ans.}$$

The resultant **R** may also be written in vector notation as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = 346\mathbf{i} - 393\mathbf{j} \text{ N} \qquad \textit{Ans.}$$



(*c*)

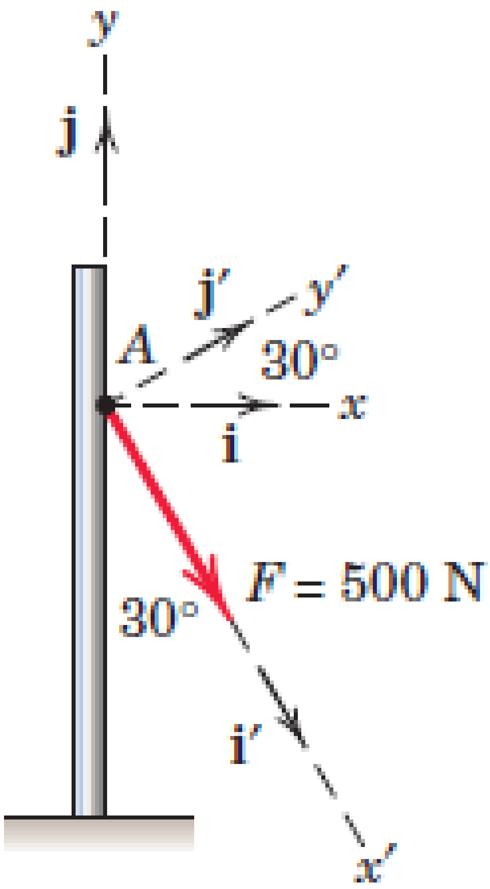
Sample Problem 2/3

The 500-N force **F** is applied to the vertical pole as shown.

(1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components.

(2) Determine the scalar components of the force vector **F** along the *x̆*- and *ý*-axes.

(3) Determine the scalar components of **F** along the *x*- and *ý*-axes.



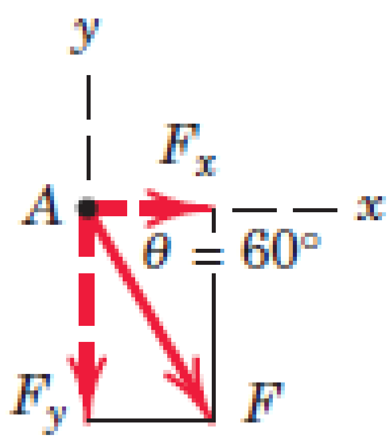
Solution. *Part (1).* From Fig. *a* we may write **F** as

$$\mathbf{F} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$

$$= (500\cos 60°)\mathbf{i} - (500\sin 60°)\mathbf{j}$$

$$= (250\mathbf{i} - 433\mathbf{j})\text{ N} \qquad \textit{Ans.}$$

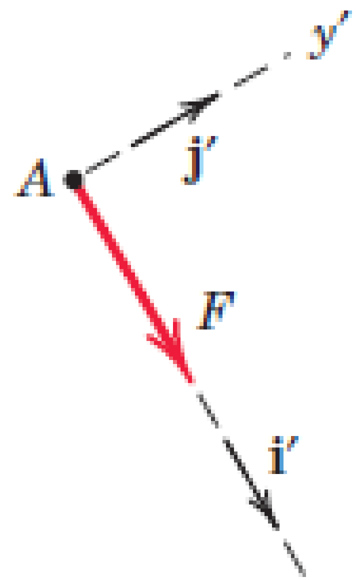
The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F}_x = 250\mathbf{i}$ N and $\mathbf{F}_y = -433\mathbf{j}$ N.



(*a*)

Part (2). From Fig. *b* we may write **F** as $\mathbf{F} = 500\mathbf{i}'$ N, so that the required scalar components are

$$F_{x'} = 500 \text{ N} \qquad F_{y'} = 0 \qquad \textit{Ans.}$$



(*b*)

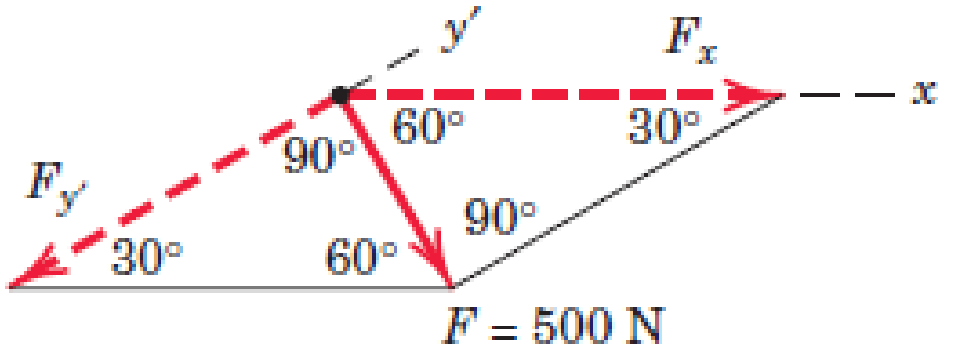
Part (3). The components of **F** in the x- and y'-directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. *c*. The magnitudes of the components may be calculated by the law of sines. Thus,

① $$\frac{|F_x|}{\sin 90°} = \frac{500}{\sin 30°} \qquad |F_x| = 1000 \text{ N}$$

$$\frac{|F_{y'}|}{\sin 60°} = \frac{500}{\sin 30°} \qquad |F_{y'}| = 866 \text{ N}$$

The required scalar components are then

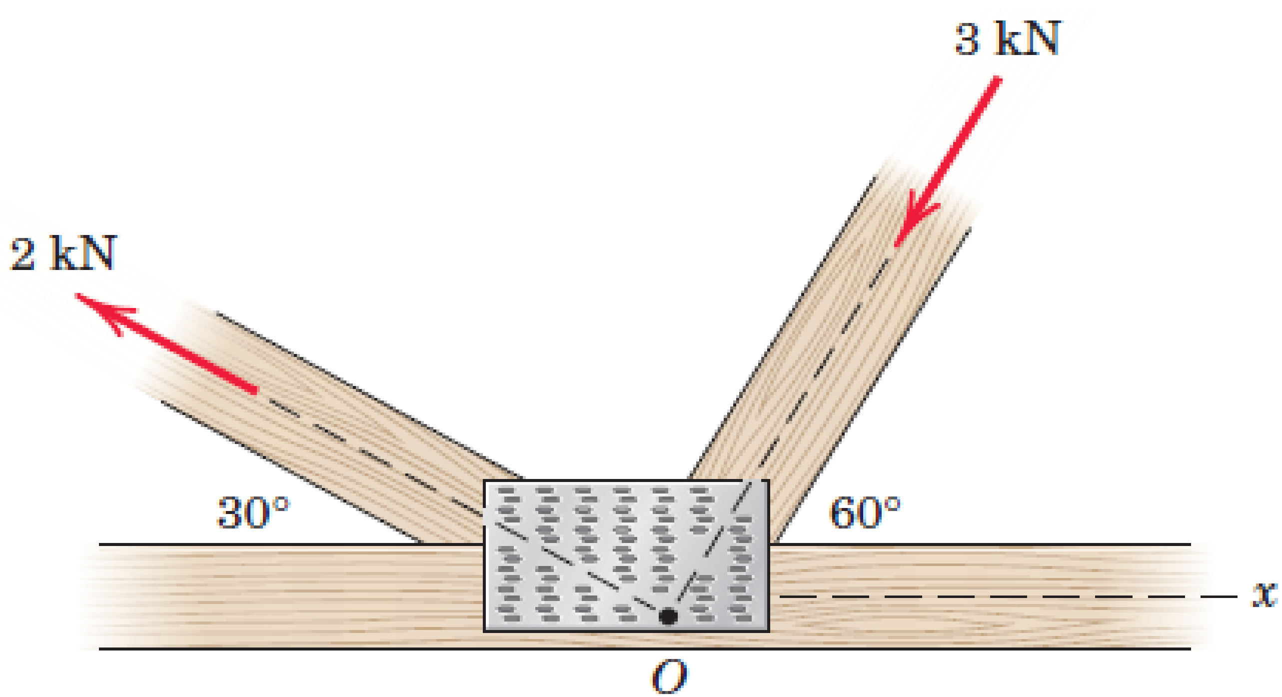
$$F_x = 1000 \text{ N} \qquad F_{y'} = -866 \text{ N}$$ *Ans.*



(*c*)

2/7 The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant $\mathbf{R}$ of the two forces and the angle θ which $\mathbf{R}$ makes with the positive x-axis.

Ans. $R = 3.61$ kN, $\theta = 206°$



Problem 2/7

2/7

$$R_x = \Sigma F_x = -2\cos 30^\circ - 3\cos 60^\circ$$

$$R_x = -3.23 \text{ KN}$$

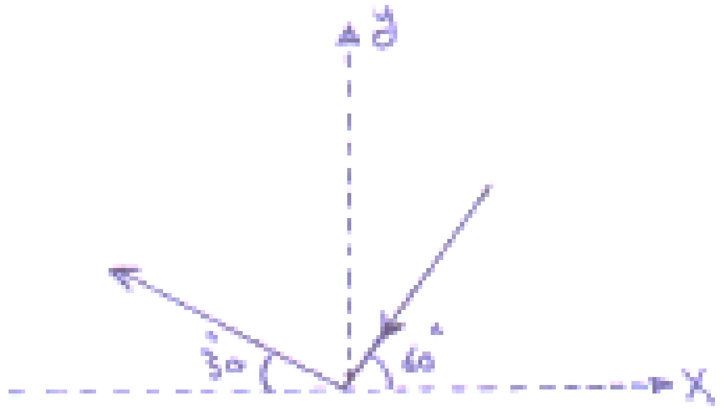
$$R_y = \Sigma F_y = 2\sin 30^\circ - 3\sin 60^\circ$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-3.23)^2 + (-1.598)^2}$$

$$R = 3.61 \text{ KN}$$

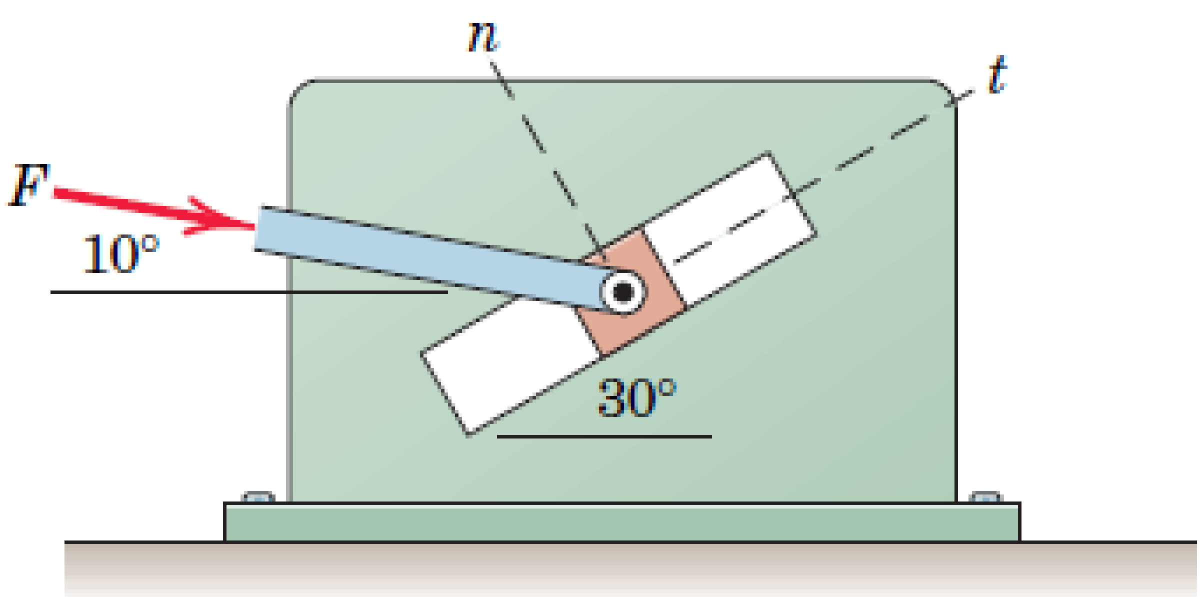
$$\theta = \tan^{-1} \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{-1.598}{-3.23}\right) = 206^\circ$$



2/11 The t-component of the force $\mathbf{F}$ is known to be 75 N. Determine the n-component and the magnitude of $\mathbf{F}$.

Ans. $F_n = -62.9$ N, $F = 97.9$ N



Problem 2/11

2/11

$$\tan 40 = \frac{|F_n|}{F_t}$$

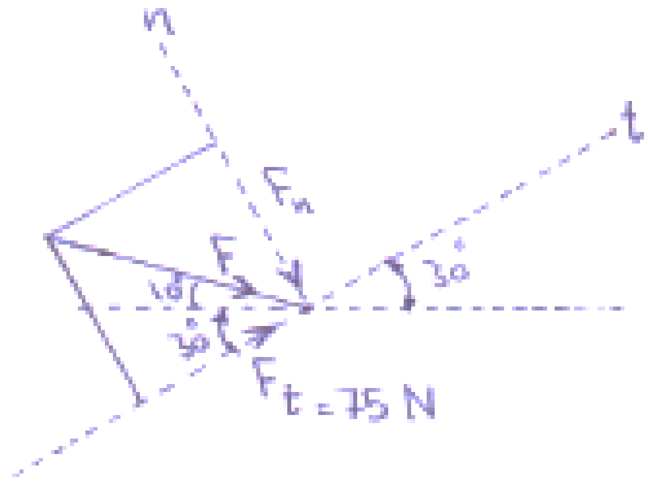
$$|F_n| = F_t \tan 40$$

$$|F_n| = 75 \tan 40$$

$$F_n = -62.9 \text{ N}$$

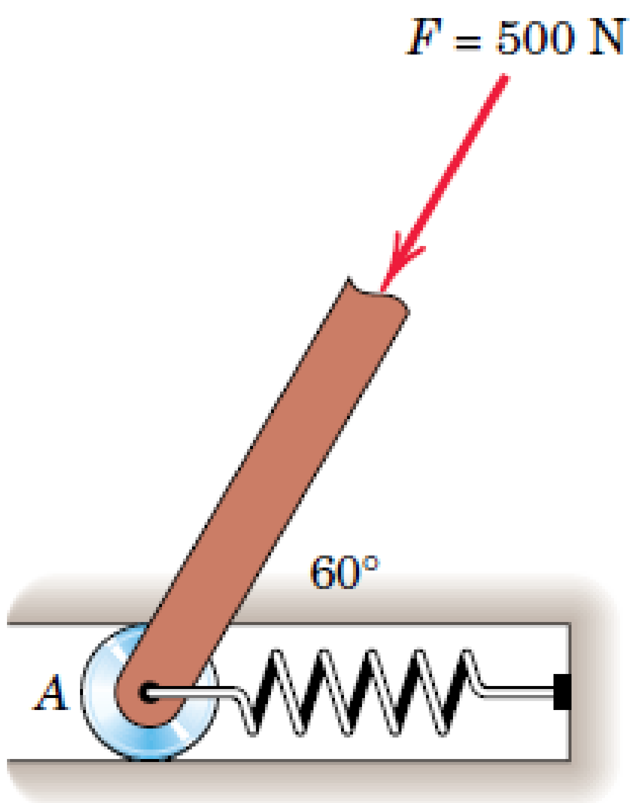
$$\cos 40 = \frac{F_t}{F}$$

$$F = \frac{F_t}{\cos 40} = \frac{75}{\cos 40} = 97.9 \text{ N}$$



2/15 Determine the magnitude F_s of the tensile spring force in order that the resultant of $\mathbf{F}_s$ and $\mathbf{F}$ is a vertical force. Determine the magnitude R of this vertical resultant force.

Ans. $F_s = 250$ N, $R = 433$ N



Problem 2/15

2/15

$$\cos 60 = \frac{F_s}{F}$$

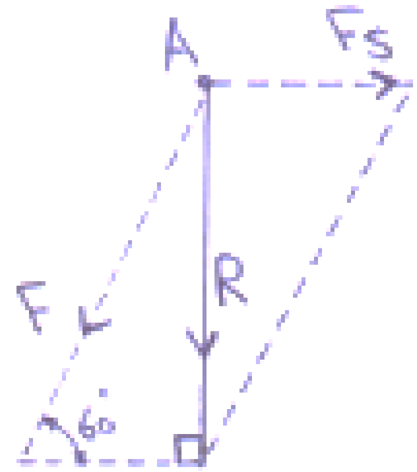
$$F_s = F \cos 60$$

$$F_s = 500 \cos 60 =$$

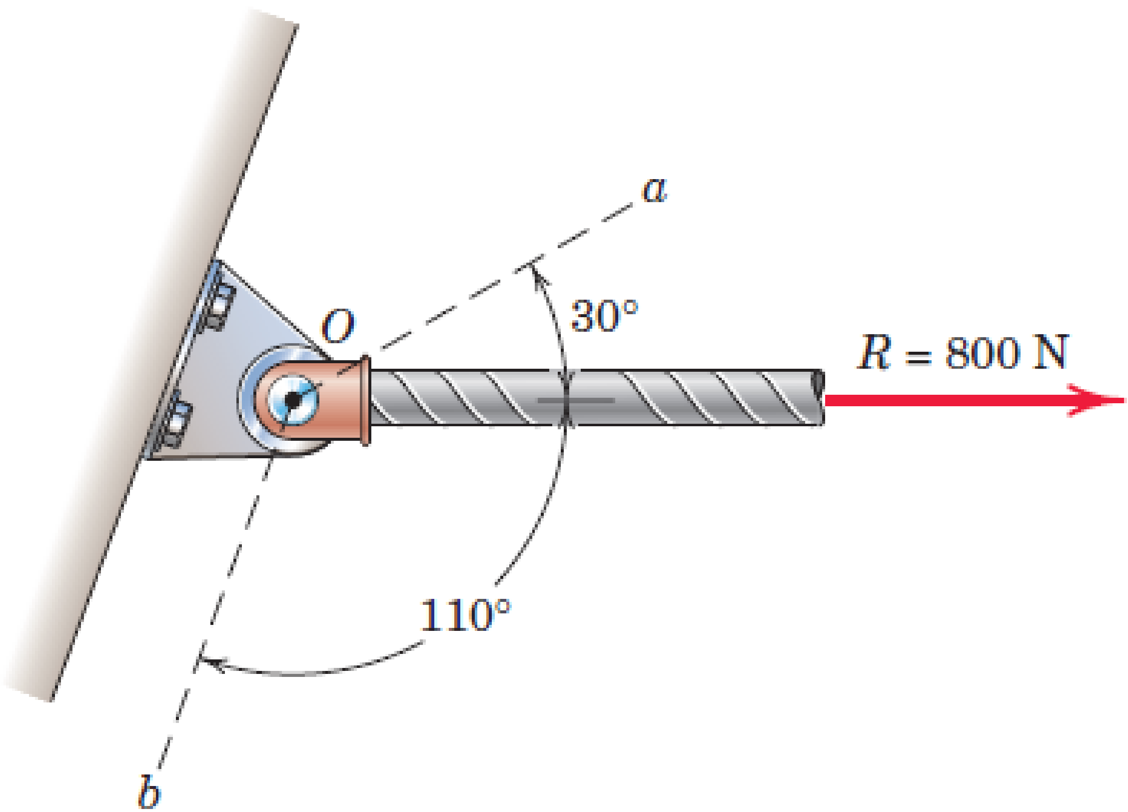
$$\sin 60 = \frac{R}{F}$$

$$R = F \sin 60 = 500 \sin 60$$

$$R =$$



2/18 Determine the scalar components R_a and R_b of the force **R** along the nonrectangular axes a and b. Also determine the orthogonal projection P_a of **R** onto axis a.



Problem 2/18

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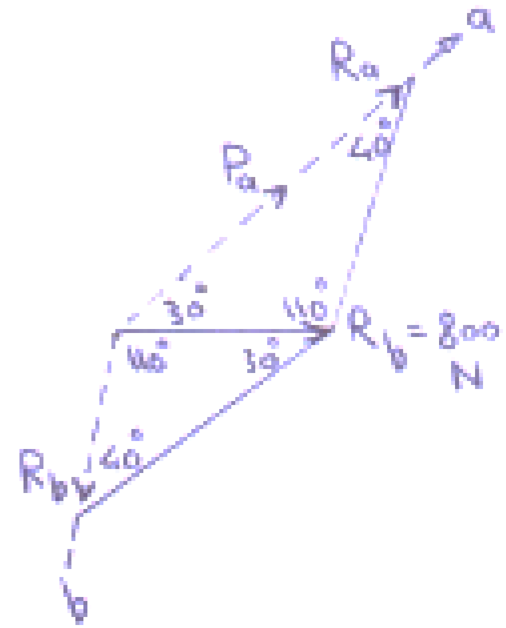
Law of Sines

$$\frac{\sin 40}{800} = \frac{\sin 110}{R_a} = \frac{\sin 30}{R_b}$$



$R_a = 1170$ N

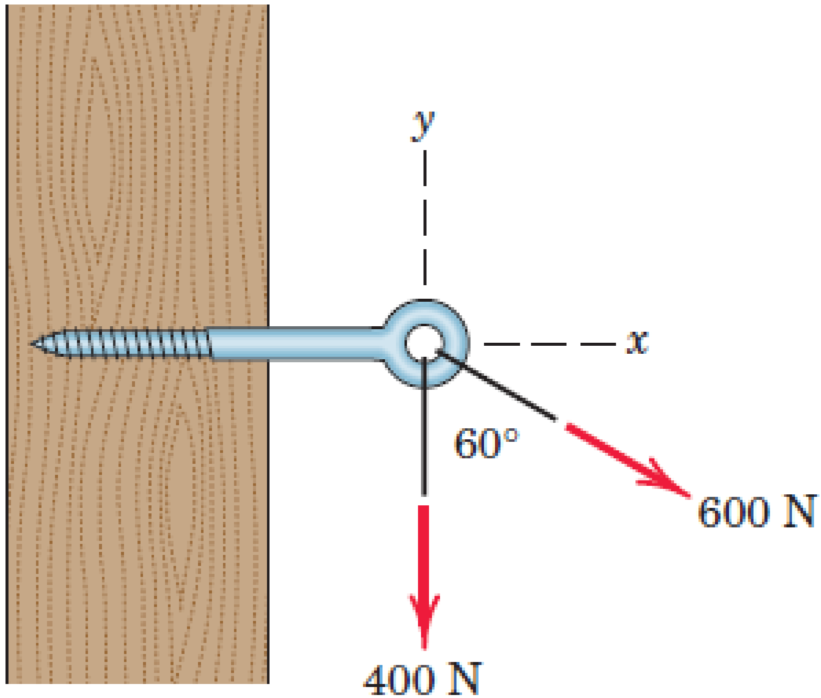
$R_b = 622$ N

$P_a = R \cos 30 = 800 \cos 30$

$P_a = 693$ N

2/19 Determine the resultant $\mathbf{R}$ of the two forces shown by (a) applying the parallelogram rule for vector addition and (b) summing scalar components.

Ans. $\mathbf{R} = 520\mathbf{i} - 700\mathbf{j}$ N



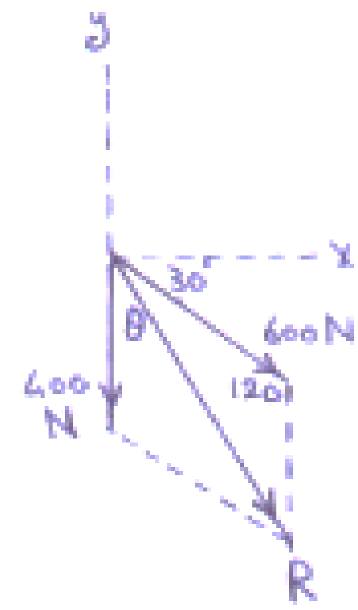
Problem 2/19

2/19

Law of cosines:

$$R^2 = 400^2 + 600^2 - 2(400)(600)\cos 120$$

$$R = 872 \text{ N}$$



Law of Sines:

$$\frac{\sin\theta}{600} = \frac{\sin 120}{872}$$

$$\theta = 36.6^\circ$$

$$R_x = \sum F_x = 600 \cos 30$$

$$R_x = 520 \text{ N}$$

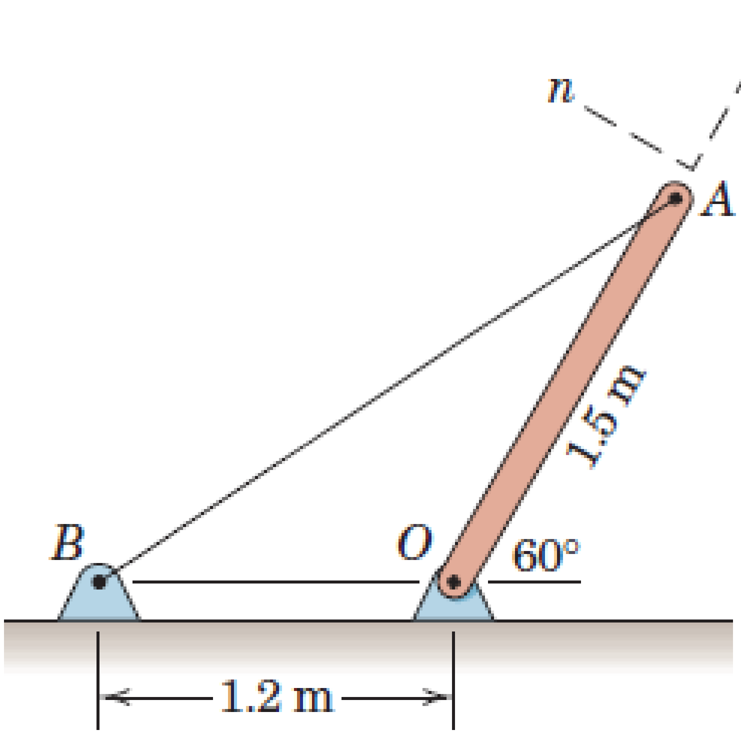
$$R_y = \sum F_y$$

$$R_y = -600 \sin 30 - 400$$

$$R_y = -700 \text{ N}$$

$$\therefore R = 520 i - 700 j \text{ N}$$

2/24 The cable AB prevents bar OA from rotating clockwise about the pivot O. If the cable tension is 750 N, determine the n- and t-components of this force acting on point A of the bar.



Problem 2/24

2/24

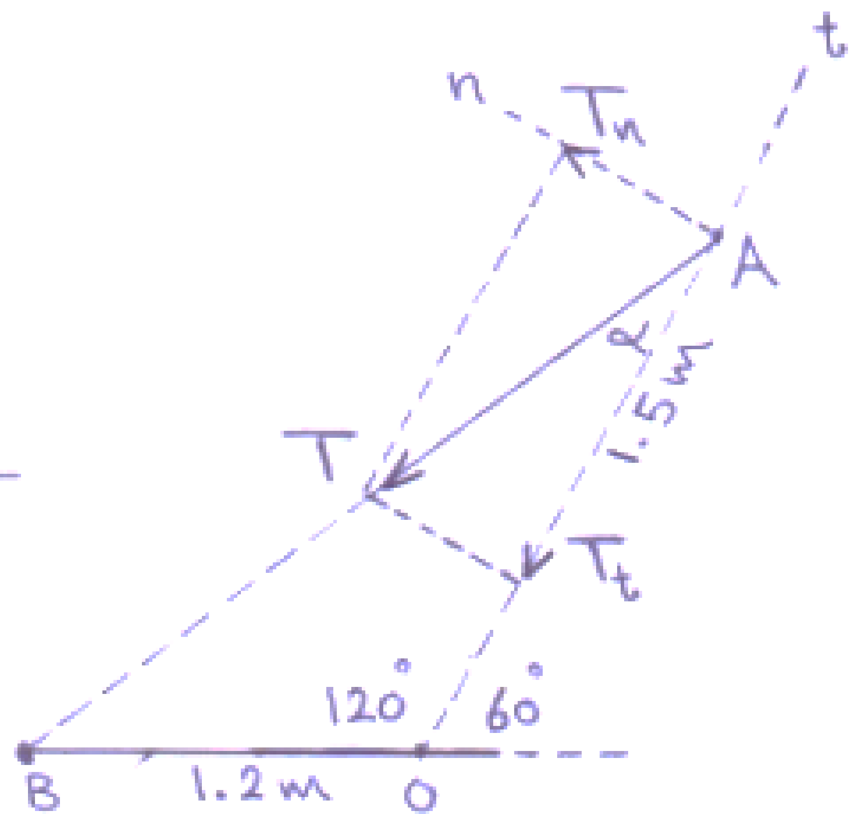
$$\overline{AB}^2 = 1.2^2 + 1.5^2 - 2(1.5)(1.2)\cos 120$$

$$\overline{AB} = 2.34\ m$$

Law of Sines:

$$\frac{\sin\alpha}{1.2} = \frac{\sin 120}{2.34}$$

$$\alpha = 26.3^\circ$$



$$T_n = T \sin \alpha$$

$$T_n = 750 \sin 26.3 = 333 \text{ N}$$

$$T_t = -T \cos \alpha$$

$$T_t = -750 \cos 26.3$$

$$T_t = -672 \text{ N}$$