Module Title: Fundamental of Electrical Engineering (DC)

Module Code:	**UOMU024011**

Chapter 5
Lectures (Week 11,12,13,14,15)
DC Network Theorems

Week (11)
DC Network Theorems

Superposition Theorem

5.0 Introduction DC Network Theorems.

There are certain theorems, which when applied to the solution of electric networks, either simplify the network itself or make their analytical solution very easy. These theorems are also equally applicable to ac system, where Ohmic resistance of dc system is replaced by impedance. Some of the theorems are as follows:

5.1 Superposition Theorem

This theorem may be stated as follows:

"In a network of linear resistances containing more than one source of emf, the current through any branch is the algebraic sum of all the currents which would flow at that point if each source were considered separately for a time being all other sources replaced by resistances equal to their internal resistances ".

In other words, current through (or voltage across) any branch of the network is algebraic sum of currents (or voltages) which each emf would have produced while acting singly. It is important to keep in mind that this theorem is applicable to only linear networks.

Illustration:

Two sources of emfs V_1 and V_2 are acting for the network shown below and it is required to find the current (I) through R_2 . This is achieved by superimposing the currents produced by V_1 and V_2 .

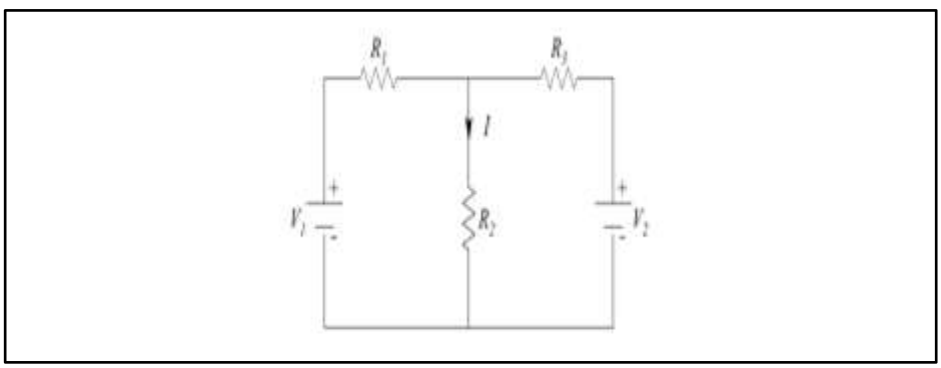
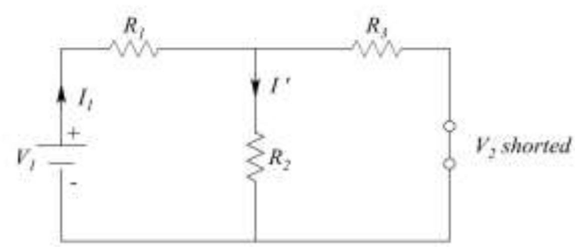


Figure 5.1: Illustration of superposition theorem

While activating V_1 , we replace V_2 by internal resistance (short circuit for ideal voltage source)



The current supplied by V_2 is,

$$I_2 = \frac{V_2}{R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

Using current divider rule,

$$I'' = I_2 \frac{R_2}{R_1 + R_2}$$

Thus, form superposition theorem, the total current when all of the sources acting together is the summation of individual currents. i.e. I = I ' + I ".

<u>Solved problems on superposition theorem:</u>

Example 1: Use superposition theorem to find I_O for the circuit shown in figure 1 below:

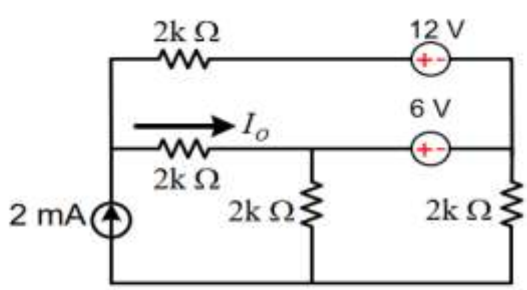


Figure 1

Solution

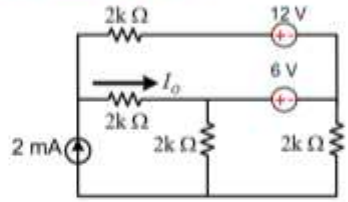


Figure 1

Solution:

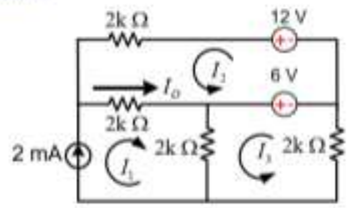


Figure 2

$$\begin{aligned} 2I_2 + 2(I_1 + I_2) + 6 - 12 &= 0 \\ 2I_1 + 4I_2 + 0I_3 &= 6 \end{aligned}$$

$$\begin{aligned} 2(I_1 + I_3) + 2I_3 - 6 &= 0 \\ 2I_1 + 0I_2 + 4I_3 &= 6 \end{aligned}$$

$$\begin{aligned} 2I_1 + 4I_2 + 0I_3 &= 6 \\ 2I_1 + 0I_2 + 4I_3 &= 6 \end{aligned}$$

$$I_1 = 2mA$$

$$\begin{aligned} 4I_2 + 0I_3 &= 2 \\ 0I_2 + 4I_3 &= 2 \end{aligned}$$

On solving

$$I_1 = 2mA,\ I_2 = 0.5\ I_3 = 0.5$$

$$I_O = I_1 + I_2 = 2 + 0.5 = 2.5mA$$

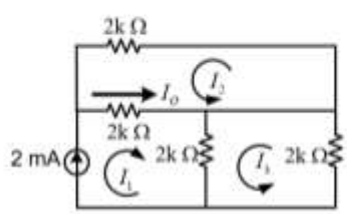


Figure 3

$$\begin{aligned} 2I_2 + 2(I_1 + I_2) &= 0 \\ 2I_1 + 4I_2 + 0I_3 &= 0 \end{aligned}$$

$$\begin{aligned} 2(I_1 + I_3) + 2I_3 &= 0 \\ 2I_1 + 0I_2 + 4I_3 &= 0 \end{aligned}$$

$$\begin{aligned} 2I_1 + 4I_2 + 0I_3 &= 0 \\ 2I_1 + 0I_2 + 4I_3 &= 0 \end{aligned}$$

$$I_1 = 2mA$$

$$\begin{aligned} 4I_2 + 0I_3 &= -4 \\ 0I_2 + 4I_3 &= -4 \end{aligned}$$

On solving

$$I_1 = 2mA,\ I_2 = -1 I_3 = -1$$

$$I_{O1} = I_1 + I_2 = 2 - 1 = 1mA$$

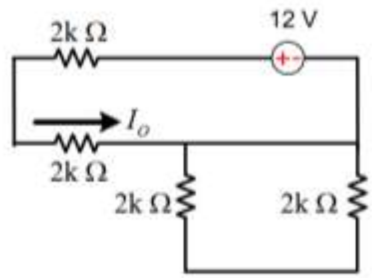


Figure 4

$$I_{02} = \frac{12}{4} = 3mA$$

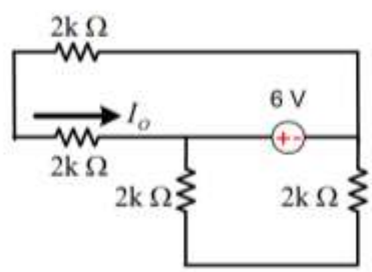
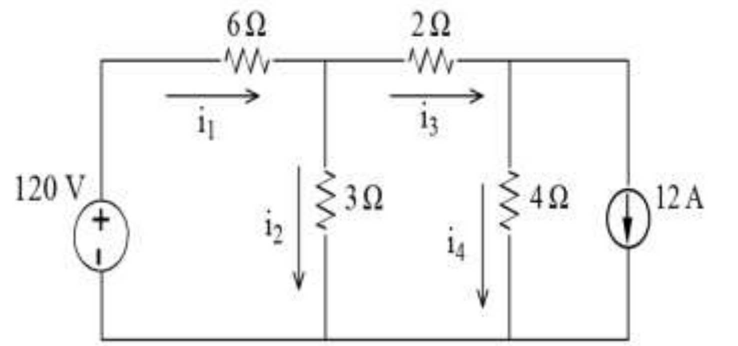


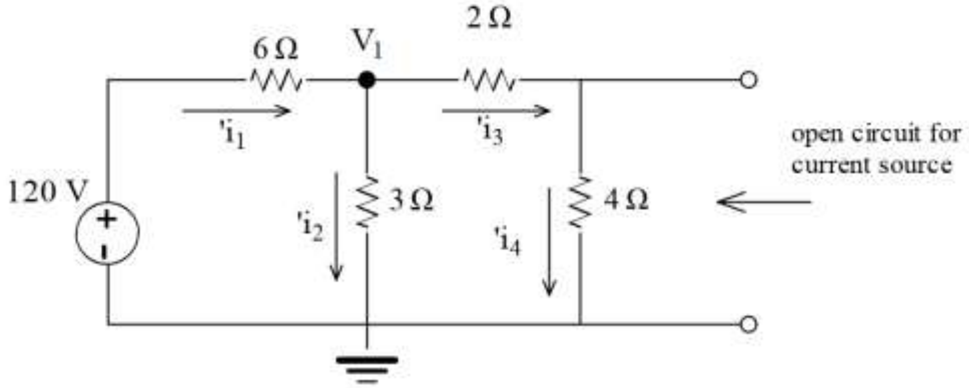
Figure 5

$$I_{03} = \frac{6}{4} = 1.5mA$$

Example 2: Use superposition to find i1, i2, i3, i4?



•Activate independent voltage source 120 V only



•Using KCL at V1 (nodal analysis)

$$'i_1 - 'i_2 - 'i_3 = 0$$

$$\frac{120-V_1}{6}-\frac{V_1}{3}-\frac{V_1}{2+4}=0$$

$$20-V_1\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{6}\right)=0$$

$$\Rightarrow \quad V_1=30\ V$$

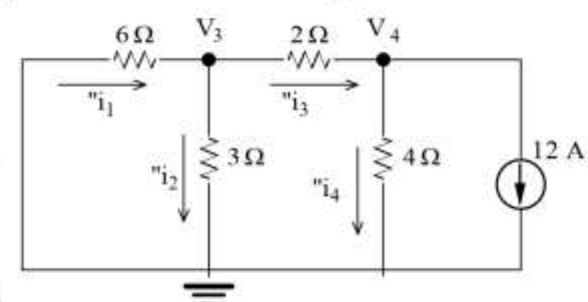
$$'i_1=\frac{120-V_1}{6}=\frac{90}{6}=15\ A$$

$$'i_2=\frac{V_1}{3}=\frac{30}{3}=10\ A$$

$$'i_3=i_4=\frac{V_1}{6}=5\ A$$

* Activate the independent current source only

short circuit for voltage source



KCL at V_3:

$$"i_1-"i_2-"i_3=0$$

$$\frac{-V_3}{6}-\frac{V_3}{3}-\frac{V_3-V_4}{2}=0$$

$$-V_3-2\,V_3-3\,(V_3-V)=0$$

$$-6\,V_3+3V_4=0 \qquad ......(1)$$

KCL at V4: $"i_3 - "i_4 - 12 = 0$

$$\frac{V_3 - V_4}{2} - \frac{V_4}{4} - 12 = 0$$

$$2\,V_3 - 2\,V_4 - V_4 = 48$$

$$2\,V_3 - 3\,V_4 = 48 \qquad \ldots\ldots(2)$$

$$V_3 = -12\ \text{V}$$

$$V_4 = -24\ \text{V}$$

$$"i_1 = \frac{-V_3}{6} = \frac{12}{6} = 2\ \text{A}$$

$$"i_2 = \frac{V_3}{3} = \frac{-12}{3} = -4\ \text{A}$$

$$"i_3 = \frac{V_3 - V_4}{2} = \frac{-12 + 24}{2} = 6\ \text{A}$$

$$"i_4 = \frac{V_4}{4} = \frac{-24}{4} = -6\ \text{A}$$

$$i_1 = 'i_1 + "i_1 = 15 + 2 = 17\ \text{A}$$

$$i_2 = 'i_2 + "i_2 = 10 - 4 = 6\ \text{A}$$

$$i_3 = 'i_3 + "i_3 = 5 + 6 = 11\ \text{A}$$

$$i_4 = 'i_4 + "i_4 = 5 - 6 = -1\ \text{A}$$

Week (12)
DC Network Theorems

Kirchhoff's laws solved problems

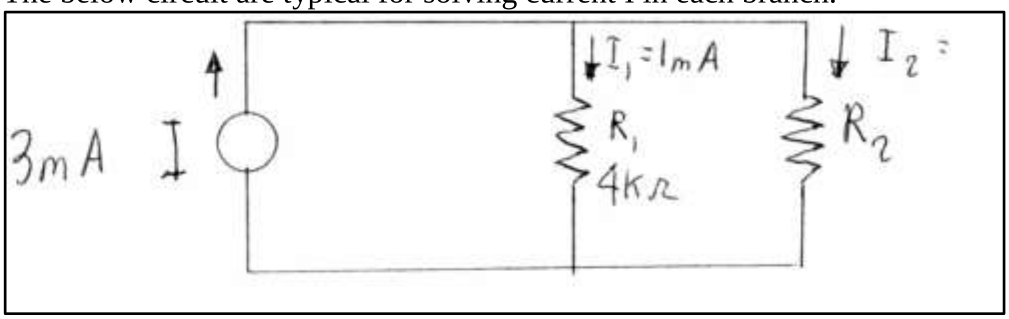
5.2 Kirchhoff's laws solved problems

Introduction:

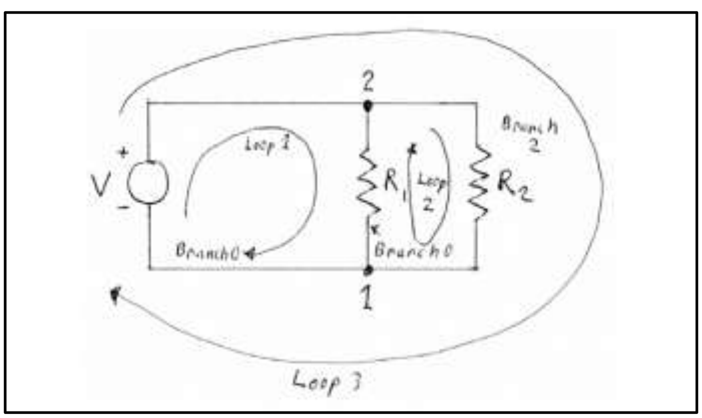
Kirchhoff's Laws and Circuit Analysis are contents: for current I and voltage V at each element .

- Linear circuits: involve resistors, capacitors, inductors
- Initial analysis uses only resistors
- Power sources, constant voltage and current
- Solved using Kirchhoff's Laws (Current and Voltage)

The below circuit are typical for solving current I in each branch.



- **Branches**: lines with devices connecting two nodes
- **Loop**: an independent closed path in a circuit
- There may be several possible closed paths



Using Kirchhoff's Laws (Current and Voltage):

1. Kirchhoff's Current Law (KCL)

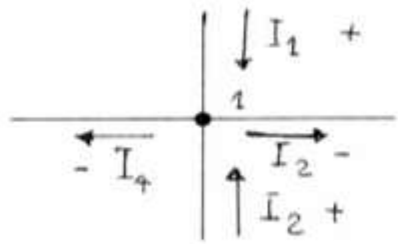
Kirchhoff's Current Law (KCL)

- Kirchhoff's Current Law (KCL)
- The algebraic sum of currents entering any node (junction) is zero

$$\sum_{j=1}^{N} I_j = 0$$

where N = number of lines entering the node

- NOTE: the sign convention:
- Currents are positive when they entering the node
- Currents negative when leaving
- Or the reverse of this.



KCL is called a **Continuity Equation**:
It says current is not created or destroyed at any node

Worked Example: Current Law (KCL)

Example of Kirchhoff's Current Law (KCL)

- Consider the simple parallel resistances below
- At node 1 define current positive into resistors
- Since V on R_1 = 5V the current is

$$I_1 = \frac{V}{R_1} = \frac{5}{1000} = 5\ mA$$

- Same V on R_2 = 5V the current is

$$I_2 = \frac{V}{R_2} = \frac{5}{5000} = 1\ mA$$

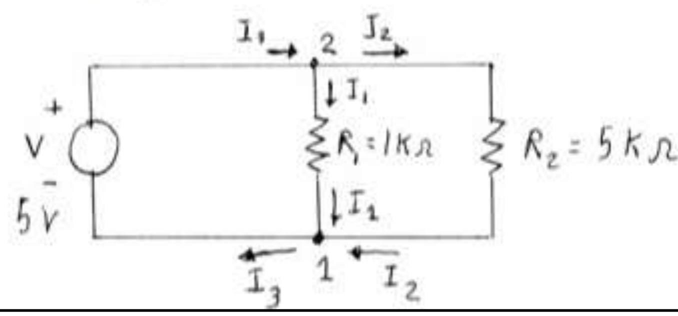
- Thus by KCL at node 1

$$I_1 + I_2 + I_3 = 0.005 + 0.001 + I_3 = 0$$

- Thus the total current is

$$I_3 = -I_1 - I_2 = -6\ mA$$

- Node 2 has the negatives of these values



Worked example: Kirchhoff's Voltage Law (KVL)

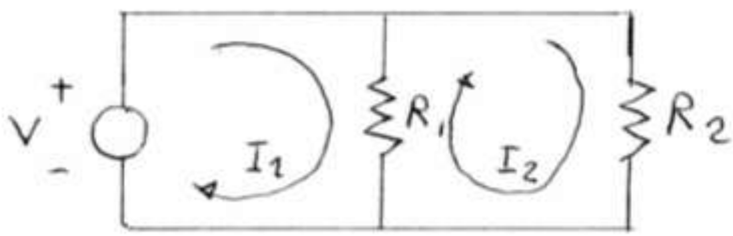
Kirchhoff's Voltage Law (KVL)

- Kirchhoff's Voltage Law (KVL)
- Algebraic sum of the voltage drops around any loop or circuit = 0

$$\sum_{j=1}^{N} V_j = 0$$

where N = number of voltage drops

- NOTE: the sign convention
- Voltage drops are positive in the direction of the set loop current
- Voltage drops negative when opposite loop current
- Voltage sources positive if current flows out of + side
- Voltage sources negative if current flows into + side



- A loop is an independent closed path in the circuit
- Define a "loop current" along that path
- Real currents may be made up of several loop currents

$$I_{R1} = I_1 - I_2$$

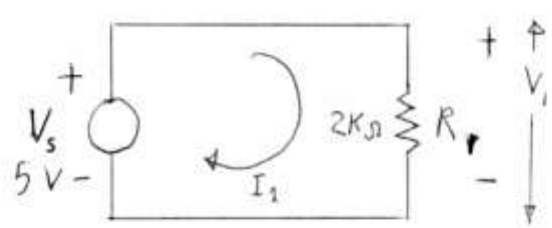
Example on Kirchhoff's Voltage Law

Example Kirchhoff's Voltage Law (KVL)

Consider a simple one loop circuit
Voltages are numbered by the element name
eg. V_1 or V_{R1} : voltage across R_1
Going around loop 1 in the loop direction
Recall by the rules:

- Voltage drops negative when opposite loop current.
- Voltage sources positive if current flows out of + side

$$V_s - V_1 = 0$$



Example Kirchhoff's Voltage Law (KVL) Continued

- Thus voltage directions are easily defined here:

$$V_s - V_1 = 0$$

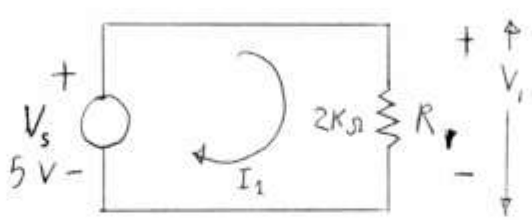
- Why negative V_1? Opposes current flow I_1
- Since

$$V_1 = I_1 R_1$$

$$V_s - I_1 R_1 = 0$$

- Thus this reduces to the Ohms law expression

$$I_1 = \frac{V_s}{R_1}$$



KVL Example Resistor Voltage Divider

- Consider a series of resistors and a voltage source
- Then using KVL

$$V - V_1 - V_2 = 0$$

- Since by Ohm's law

$$V_1 = I_1 R_1 \quad V_2 = I_1 R_2$$

- Then

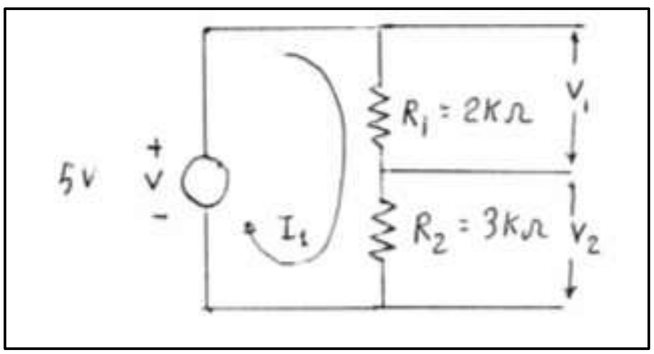
$$V - I_1 R_1 - I_1 R_2 = V - I_1 (R_1 + R_2) = 0$$

- Thus

$$I_1 = \frac{V}{R_1 + R_2} = \frac{5}{2000 + 3000} = 1\ mA$$

- i.e. get the resistors in series formula

$$R_{total} = R_1 + R_2 = 5\ K\Omega$$



KVL Example Resistor Voltage Divider Continued

- What is the voltage across each resistor?
- Now we can relate V_1 and V_2 to the applied V
- With the substitution

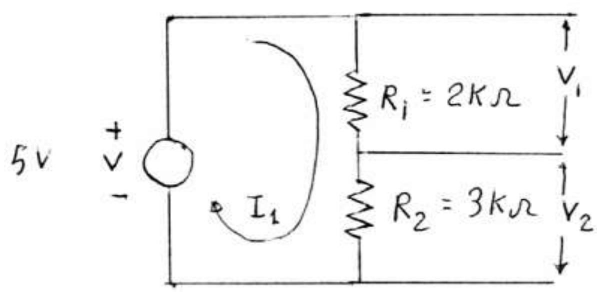
$$I_1 = \frac{V}{R_1 + R_2}$$

- Thus V_1

$$V_1 = I_1 R_1 = \frac{VR_1}{R_1 + R_2} = \frac{5(2000)}{2000 + 3000} = 2\ V$$

- Similarly for the V_2

$$V_1 = I_1 R_2 = \frac{VR_2}{R_1 + R_2} = \frac{5(3000)}{2000 + 3000} = 3\ V$$



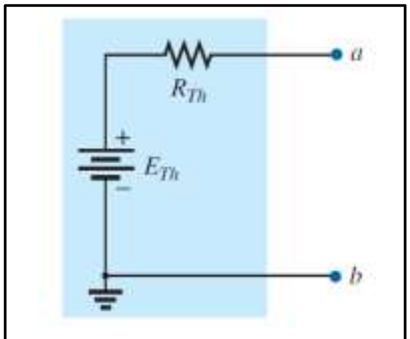
Week (13)
DC Network Theorems

Thevenin's Theorems with solved problems

Introduction to Thevenin's theorem:

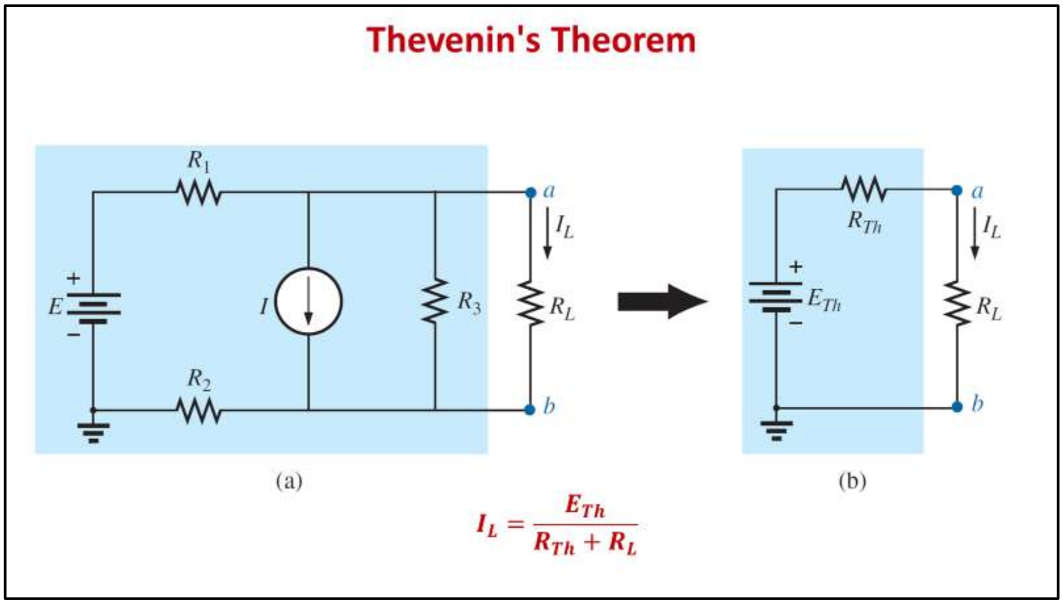
Thevenin's theorem, is probably one of the most interesting in that it permits the reduction of complex networks to a simpler form for analysis and design.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



Thévenin equivalent circuit.

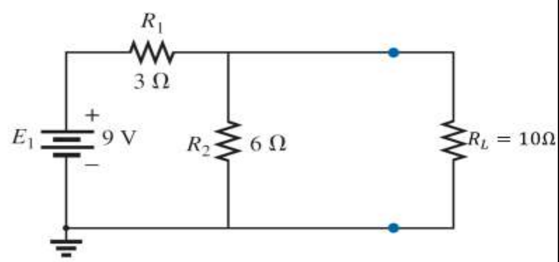

Leon-Charles Thévenin.



Thevenin's Theorem

Thevenin's Theorem Procedure

EXAMPLE 1: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.



Thevenin's Theorem

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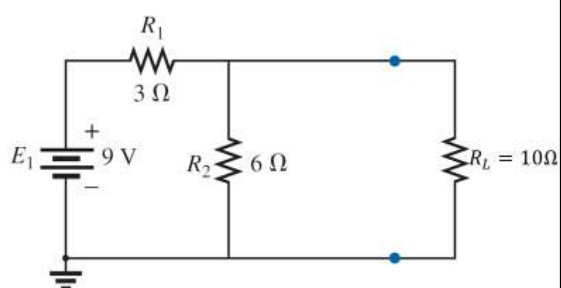
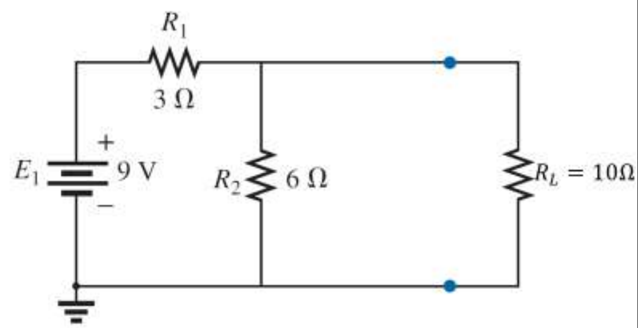


Thevenin's Theorem

Thevenin's Theorem Procedure

EXAMPLE 1: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Step 1:- **Remove that portion of the network where the Thevenin equivalent circuit is found. In Figure below, this requires that the load resistor R_L be temporarily removed from the network.**



Thevenin's Theorem

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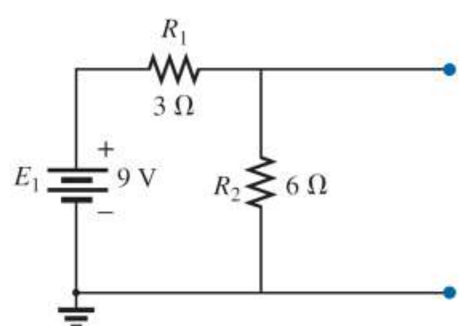
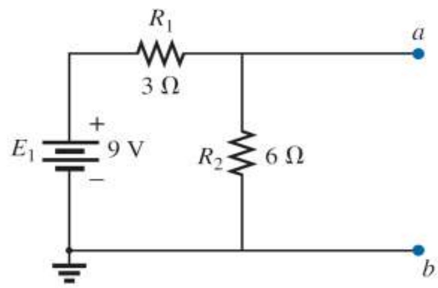


Thevenin's Theorem

Thevenin's Theorem Procedure

EXAMPLE 1: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Step 2:- Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)



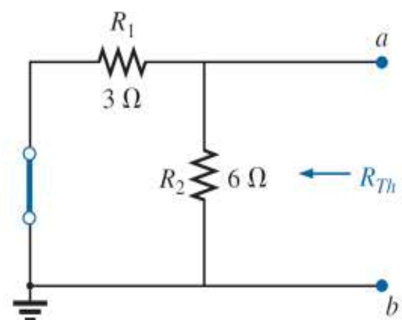
Thevenin's Theorem

Thevenin's Theorem Procedure

EXAMPLE 1: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Step 3:- Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals.

$$R_{Th} = R_1 \| R_2 = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega} = \mathbf{2\,\Omega}$$

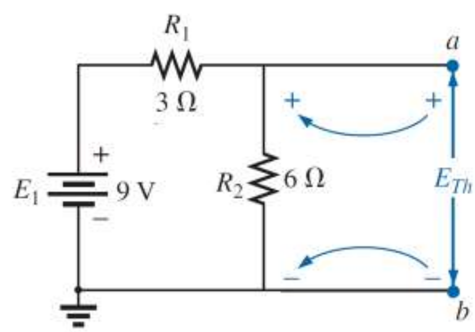


Thevenin's Theorem

Thevenin's Theorem Procedure

EXAMPLE 1: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Step 4:-Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.



Thevenin's Theorem

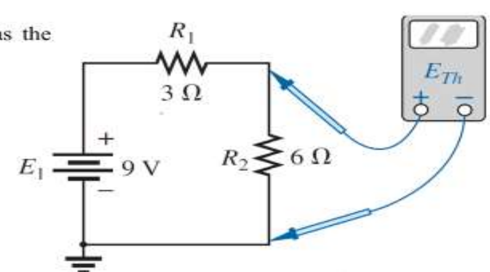
Thevenin's Theorem Procedure

EXAMPLE 1: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Step 4:-Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.

For this case, the open circuit voltage E_{Th} is the same as the voltage drop across the **6 Ω** resistor.
Applying the voltage divider rule,

$$E_{Th} = \frac{R_2E_1}{R_2 + R_1} = \frac{(6\,\Omega)(9\text{ V})}{6\,\Omega + 3\,\Omega} = \frac{54\text{ V}}{9} = \mathbf{6\text{ V}}$$



Thevenin's Theorem

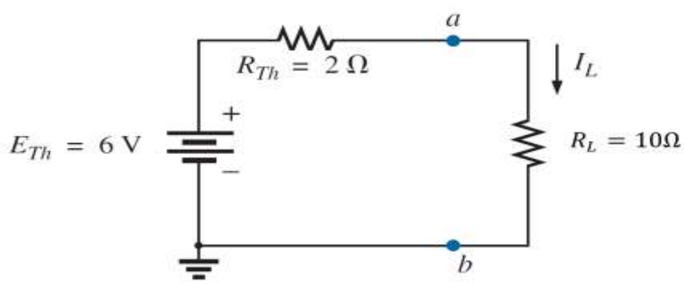
Thevenin's Theorem Procedure

EXAMPLE 1: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Step 5:- Draw the Thevenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

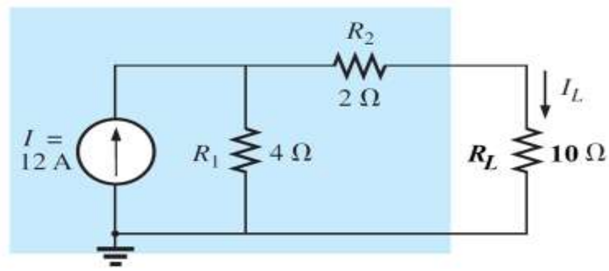
$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$$I_L = \frac{6\,V}{2\Omega + 10\Omega} = \frac{6}{12} = 0.5\,A$$



Thevenin's Theorem

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

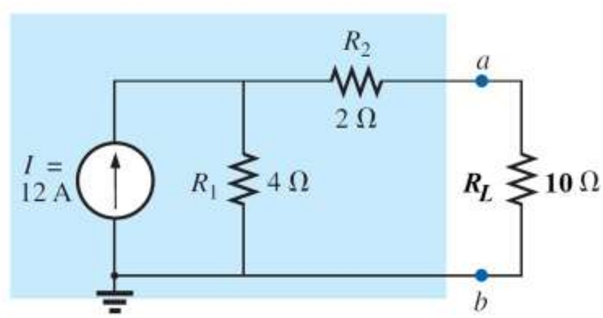


Thevenin's Theorem

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 1 and 2:-

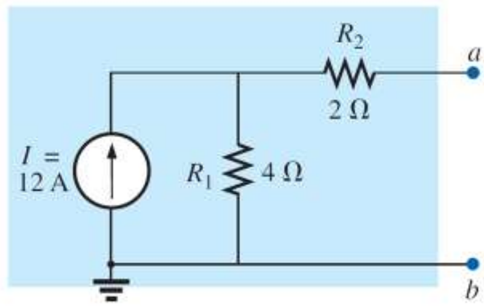


Thevenin's Theorem

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 1 and 2:-



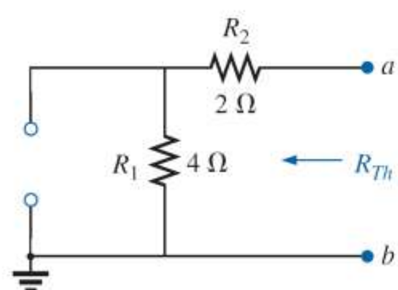
Thevenin's Theorem

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 3:-

$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$

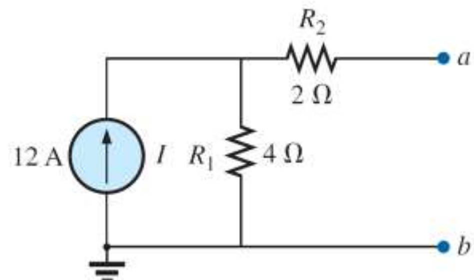


Continue Example 2

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 4:-

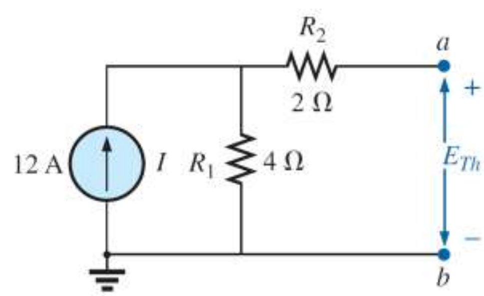


Thevenin's Theorem

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 4:-



Continue example 2

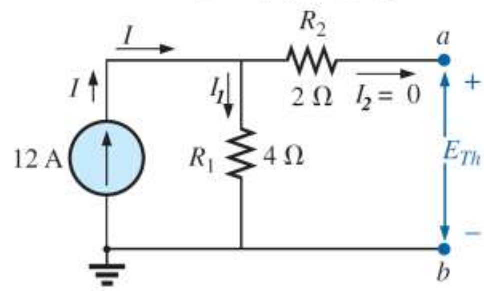
Thevenin's Theorem

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 4:-

$$V_2 = I_2R_2 = (0)R_2 = 0\text{ V}$$



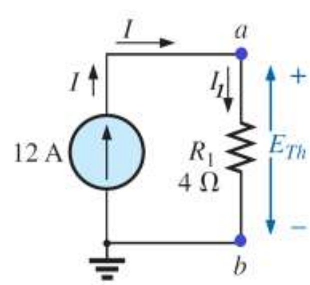
Thevenin's Theorem

EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 4:-

$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\text{ A})(4\ \Omega) = \mathbf{48\ V}$$



Thevenin's Theorem

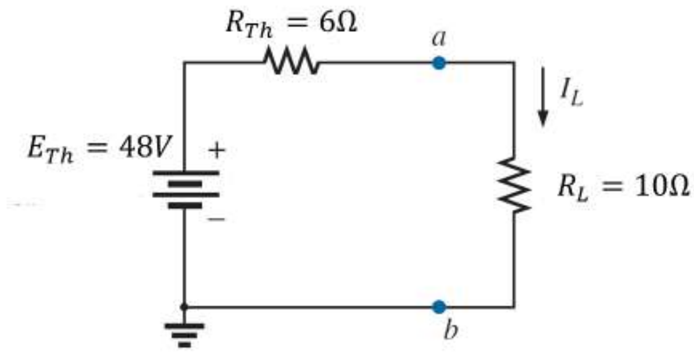
EXAMPLE 2: Find the Thevenin equivalent circuit. Then find the current through R_L for the circuit of figure.

Solution:-

Step 5:-

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

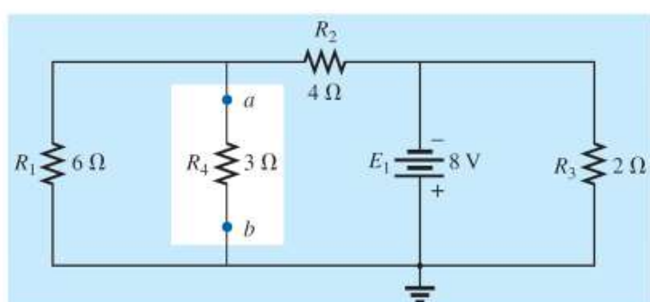
$$I_L = \frac{48\ V}{6\Omega + 10\Omega} = \frac{48}{16} = 3\ A$$



Example 3 :

Thevenin's Theorem

EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through *3 Ω resistor* for the circuit of figure.

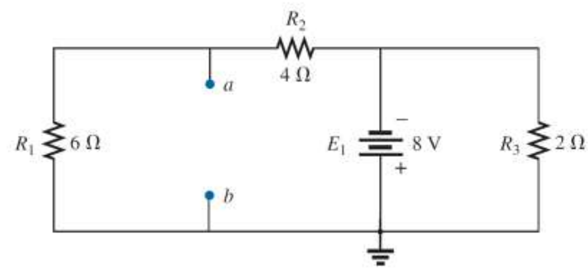


Thevenin's Theorem

EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through *3 Ω resistor* for the circuit of figure.

Solution:-

Step 1 and 2:-

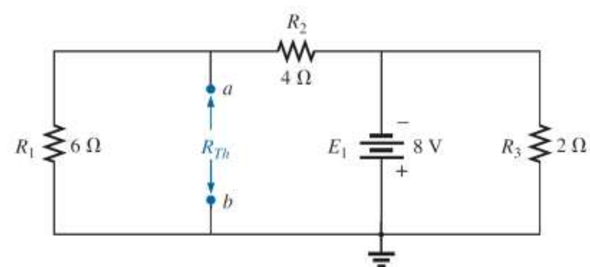


Thevenin's Theorem

EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through *3 Ω resistor* for the circuit of figure.

Solution:-

Step 3:-

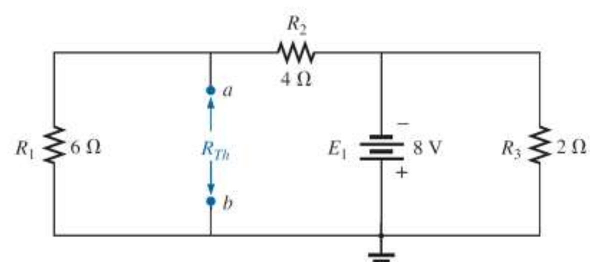


Thevenin's Theorem

EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through *3 Ω resistor* for the circuit of figure.

Solution:-

Step 3:-



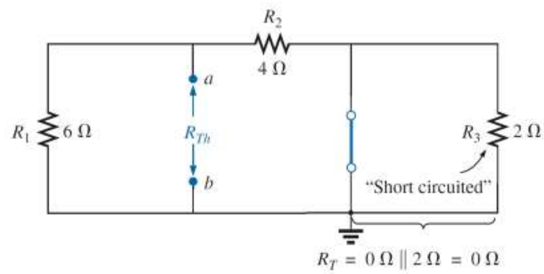
Continue Example 3

Thevenin's Theorem

EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through ***3 Ω resistor*** for the circuit of figure.

Solution:-

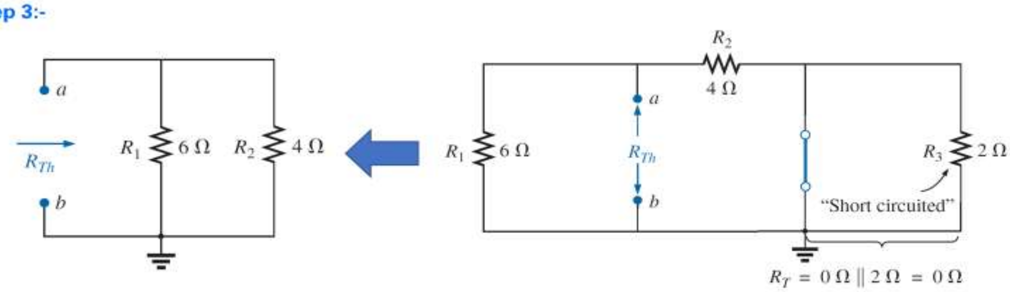
Step 3:-



EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through ***3 Ω resistor*** for the circuit of figure.

Solution:-

Step 3:-



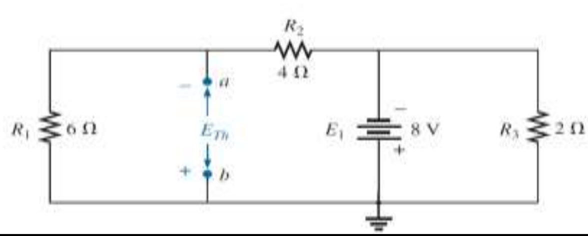
$$R_{Th} = R_1 \parallel R_2 = \frac{(6\,\Omega)(4\,\Omega)}{6\,\Omega + 4\,\Omega} = \frac{24\,\Omega}{10} = \mathbf{2.4\,\Omega}$$

Thevenin's Theorem

EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through ***3 Ω resistor*** for the circuit of figure.

Solution:-

Step 4:-



Continue Example 3 solution

Thevenin's Theorem

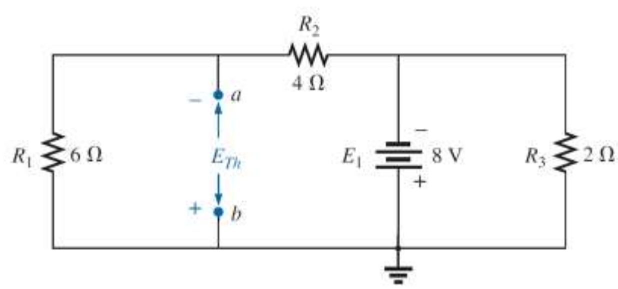
EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through *3 Ω resistor* for the circuit of figure.

Solution:-

Step 4:-

Applying the voltage divider rule,

$$E_{Th} = \frac{R_1E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ V)}{6\ \Omega + 4\ \Omega} = \frac{48\ V}{10} = \mathbf{4.8\ V}$$



Thevenin's Theorem

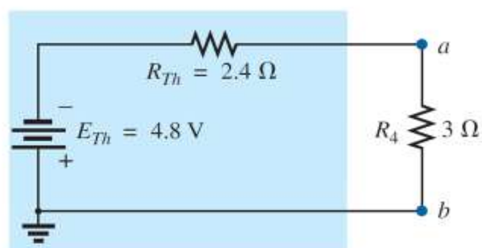
EXAMPLE 3: Find the Thevenin equivalent circuit. Then find the current through *3 Ω resistor* for the circuit of figure.

Solution:-

Step 5:-

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$$I_L = \frac{4.8\ V}{2.4\Omega + 3\Omega} = \frac{4.8}{5.4} = 0.889\ A$$



Week (14)
DC Network Theorems

Norton's Theorems with solved problems

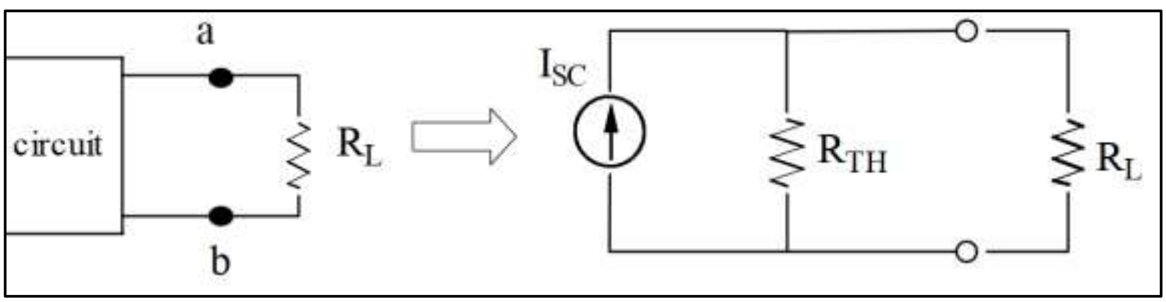
Introduction:

How to solve Norton theorem problems?

1. Step 1: Remove the load resistor and replace it with a short circuit.
2. Step 2: Calculate the Norton current—the current through the short circuit.
3. Step 3: Replace the power sources. All voltage sources are replaced with short circuits, and all current sources are replaced with open circuits.

Norton theorem problems solution process :

A portion of the **circuit at pair of nodes** can be replaced by **a current source Isc** in a **parallel with a resistor RTH**. Isc is the short circuit current at the terminals, and RTH is the Thevenin's equivalent resistance.



Here we will consider (3) cases :

1. Circuit containing only independent sources.

2. Circuit containing only dependent sources.

3. Circuit containing both independent and dependent sources.

Case (1): Circuit containing only independent sources:

Procedure of Thevenin's Theorem:

a. Find the open circuit voltage at the terminals, Voc.

b. Find the Thevenin's equivalent resistance, RTH at the terminals when all independent sources are zero:

- Replacing independent voltage sources by short circuit
- Replacing independent current sources by open circuit

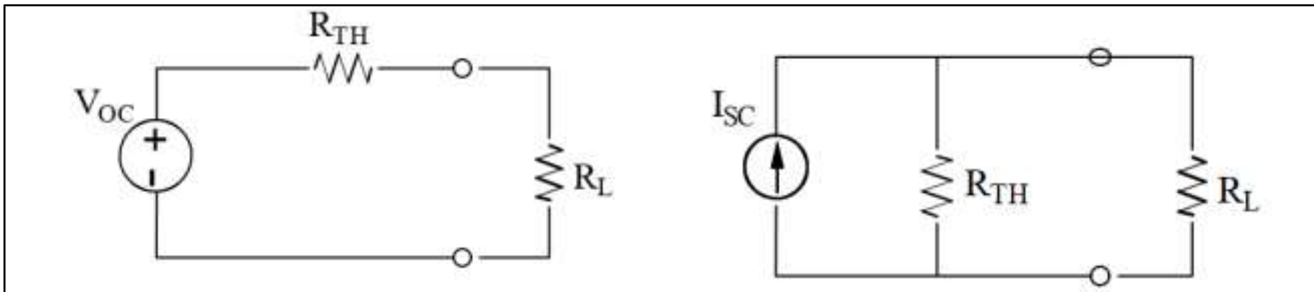
c. Reconnect the load to the Thevenin equivalent circuit

Procedure of Norton's Theorem:

a. Find the short circuit current at the terminals, Isc.

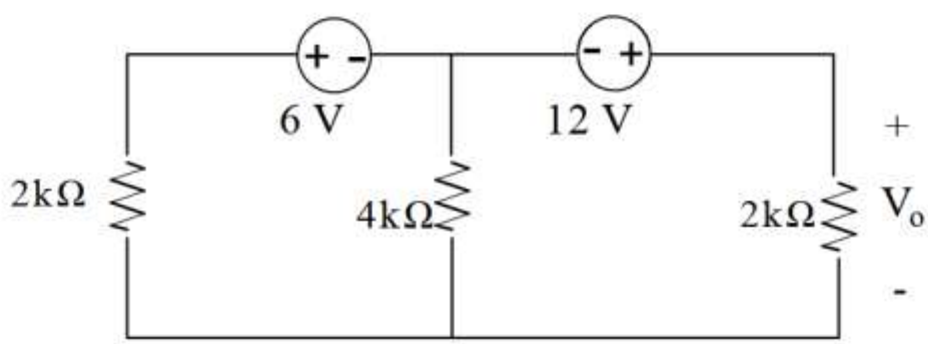
b. Find Thevenin's equivalent resistance, RTH (as before).

c. Reconnect the load to Norton's equivalent circuit.



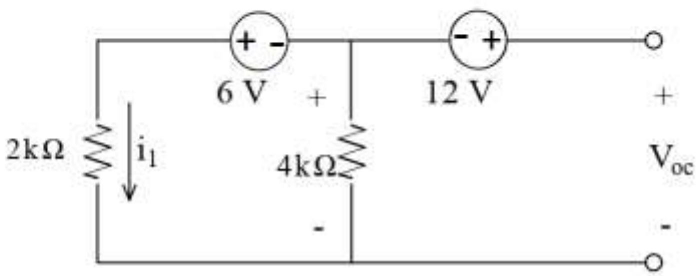
Example :

Use Thevenin's and Norton Theorms to find V_0



Using Thevenin Theorm:

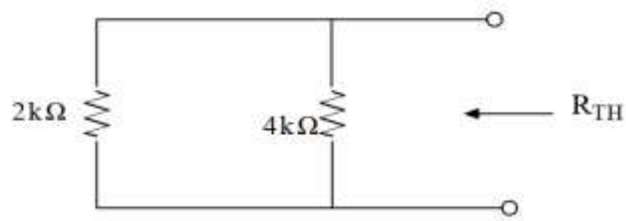
First find V_{OC}:



$$i_1 = \frac{6\ V}{2\ k + 4\ k} = 1\ m\ A \quad \Rightarrow \quad V_{4k\Omega} = i_1\,(4\,k) = -4\ V$$

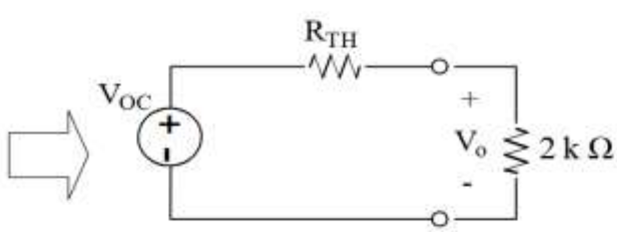
$$V_{oc} = 12\ V - 4\ V = 8\ V$$

Second, find R_{TH}



R_{TH} = 2k//4k = 4/3 k Ω

Thevenin equivalent circuit is



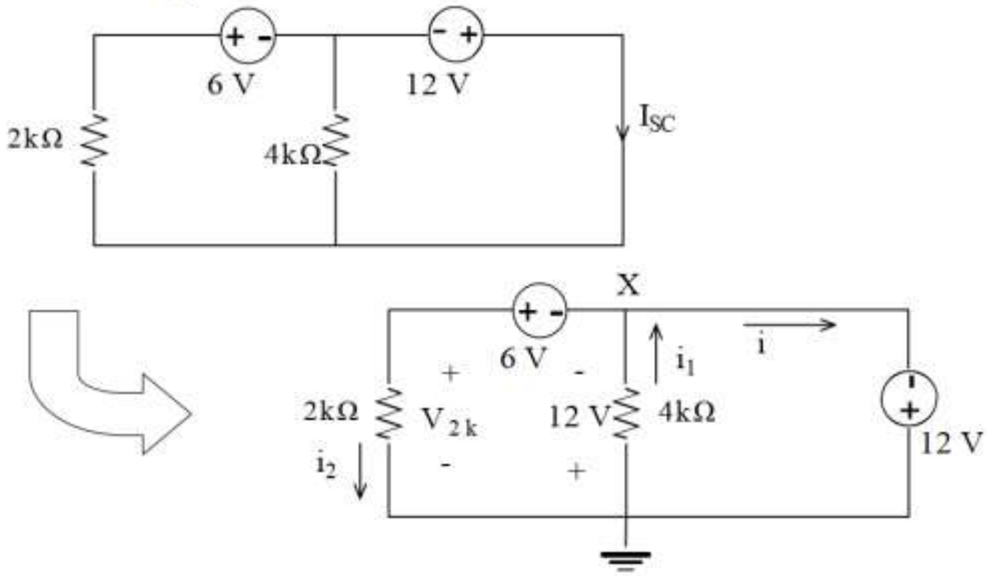
$$V_0 = \frac{2\ k\ \Omega}{2\ k + R_{TH}} V_{oc}$$

$$= \frac{2\ k}{10/3\ k}(8\ V)$$

$$V_0 = 4.8\ V$$

Using Norton Theorm

First find I_{sc}



$$i_1 = \frac{12\ V}{4\ k} = 3\ m\ A$$

KVL around outer loop:

$$12 - 6 + V_{2k} = 0 \quad \Rightarrow \quad V_{2k} = -6\ V$$

$$i_2 = \frac{V_{2k}}{2\ k} = \frac{-6}{2\ k} = -3\ m\ A$$

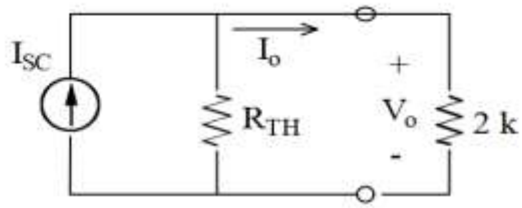
KCL at x :

$$i_1 - i_2 - i = 0$$

$$3\ m + 3\ m - i = 0 \quad \Rightarrow \quad i = 6\ m\ A \quad \Rightarrow I_{sc} = 6\ m\ A$$

Continue

R_{TH} is the same as before:

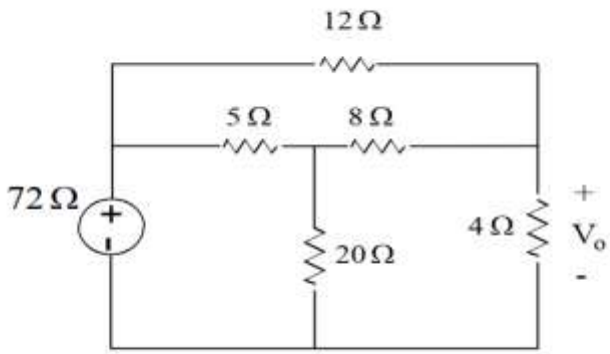


$$I_0 = \frac{R_{TH}}{R_{TH} + 2\,k}(I_{sc}) = \frac{4/3\ k}{4/3\ k + 2\,k}(6\,m) = 2.4\ m\,A$$

$$V_0 = I_0\ (2\,k) = (2.4\,m)\ (2\,k) = 4.8\,V$$

Example 2

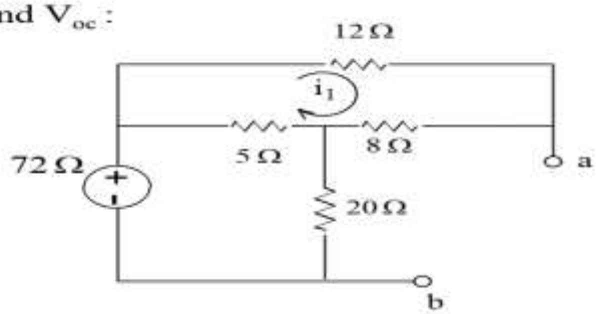
Example :
Use Thevenin and Norton to find V_0



Continue solve example 2

Using Thevenin Theorm:

1. Find V_{oc} :



KVL around the upper loop :

$$12\, i_1 + 8\, i_1 + 5\, (i_1 - i_2) = 0$$

$$25\, i_1 - 5\, i_2 = 0 \quad \ldots\ldots(1)$$

KCL around lower loop :

$$5\, (i_2 - i_1) + 20\, i_2 = 72$$

$$-5 i_1 + 25\, i_2 = 72 \quad \ldots\ldots(2)$$

$$i_1 = 0.6\ A, \quad i_2 = 3\ A$$

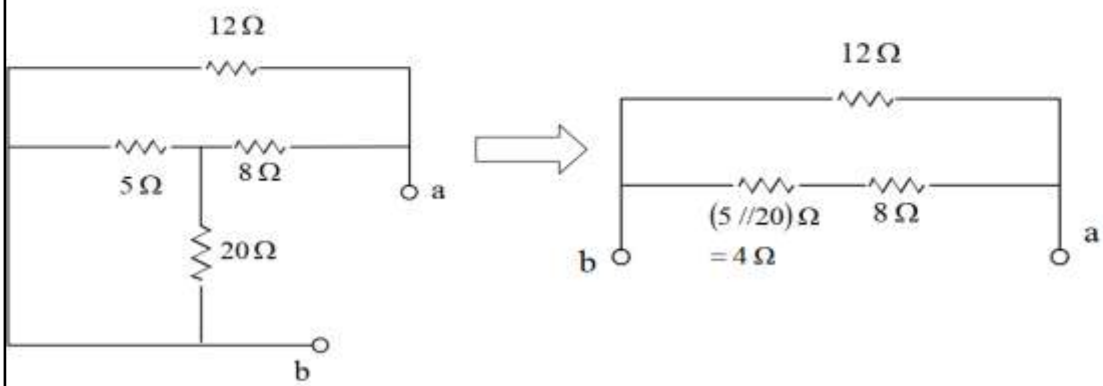
$$V_{oc} = 8\, i_1 + 20\, i_2$$

$$= 8\,(0.6) + 20\,(3)$$

$$V_{oc} = 64.8\ V$$

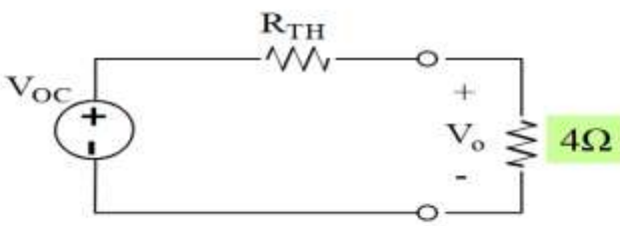
Continue :

2. Find R_{TH}



$R_{TH} = (8+4) // 12 = 12 // 12 = 6\Omega$

3. Reconnect the load :



$$V_o = \frac{4}{4 + R_{TH}} V_{oc}$$

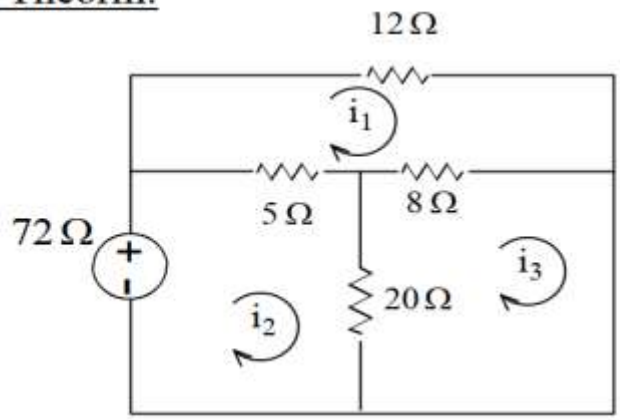
$$= \frac{4}{4+6} (64.8)$$

$$V_o = 25.92\ \text{V}$$

Continue : using Norton Theorm

Using Norton Theorm:

1. Find I_{SC} :



KVL around upper loop :

$$12\,i_1 + 8\,(i_1 - i_3) + 5\,(i_1 - i_2) = 0$$

$$25\,i_1 - 5\,i_2 - 8\,i_3 = 0 \quad \ldots\ldots(1)$$

KVL around lower loop :

$$5\,(i_2 - i_1) + 20\,(i_2 - i_3) = 72$$

$$-5\,i_1 + 25\,i_2 - 20\,i_3 = 72 \quad \ldots\ldots(2)$$

KVL around right loop :

$$8\,(i_3 - i_1) + 20\,(i_3 - i_2) = 0$$

$$-8\,i_1 - 20\,i_2 + 28\,i_3 = 0 \quad \ldots\ldots(3)$$

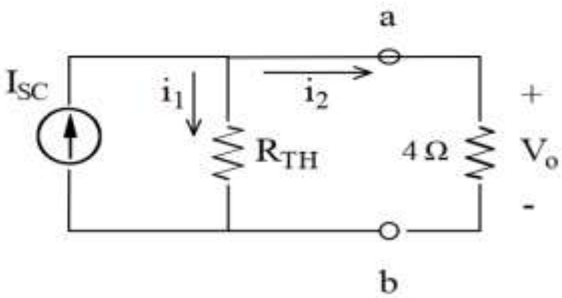
$$i_1 = 6\text{ A}, \quad i_2 = 12.72\text{ A}, \quad i_3 = 10.8\text{ A}$$

$$\Rightarrow \quad I_{SC} = 10.8\text{ A}$$

Find :

From before , $R_{TH} = 6\ \Omega$

3. Reconnect the load



$$V_o = (4\ \Omega)\, i_2$$

$$= (4\ \Omega)\left(\frac{R_{TH}}{R_{TH}+4}\right) I_{SC}$$

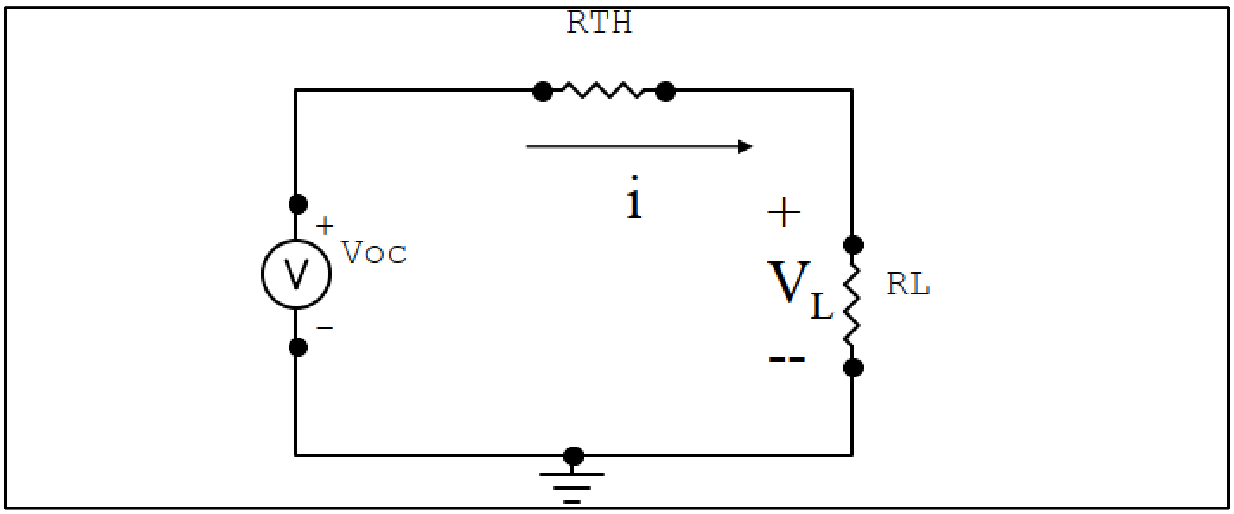
$$V_o = 4\left(\frac{6}{6+4}\right)(10.8) = 25.92\ \text{V}$$

Week (15)
DC Network Theorems

Maximum Power Transfer with solved problems

Introduction

A technique in which the load is selected to maximize the power transfer. This technique is based on the Thevenin equivalent circuit. See figure below .



Load power can be found by:

$$P_L = v_L\, i = i^2 R_L$$
$$= \left(\frac{V_{OC}}{R_{TH} + R_L}\right)^2 R_L$$

We wish to select R_L to maximize P_L:

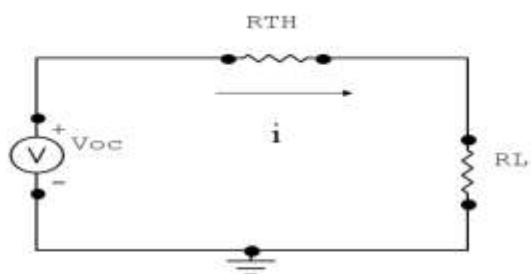
Take $\frac{dP_L}{dR_L} = 0$

$$\frac{dP_L}{dR_L} = \frac{(R_{TH}+R_L)^2 (V_{OC})^2 - R_L (V_{OC})^2\, 2(R_{TH}+R_L)}{(R_{TH}+R_L)^4} = 0$$

$$\frac{V_{oc}^2 (R_{TH}+R_L)\left[(R_{TH}+R_L) - 2R_L\right]}{(R_{TH}+R_L)^4} = 0$$

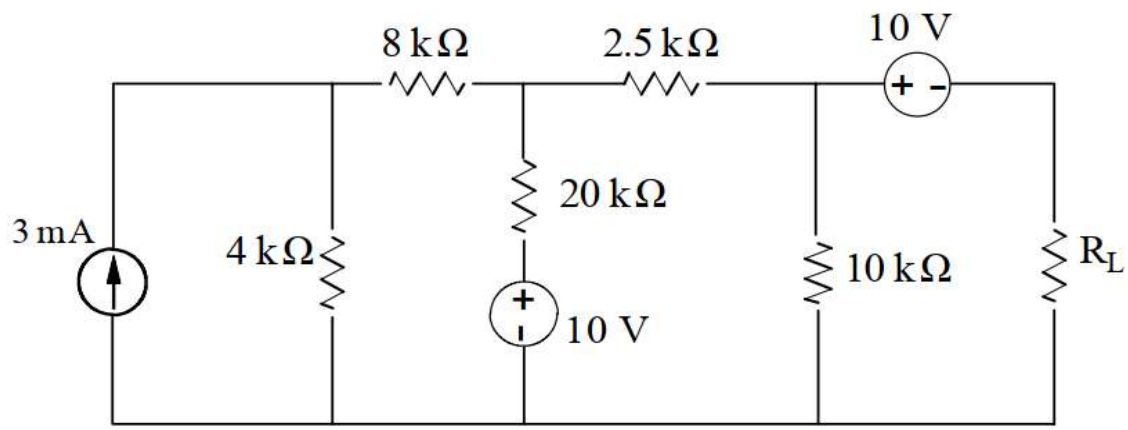
$$\Rightarrow \quad R_{TH} + R_L - 2R_L = 0$$
$$\Rightarrow \quad R_{TH} - R_L = 0$$
$$R_L = R_{TH}$$

If $R_L = R_{TH}$, what is the maximum Power Transfer?



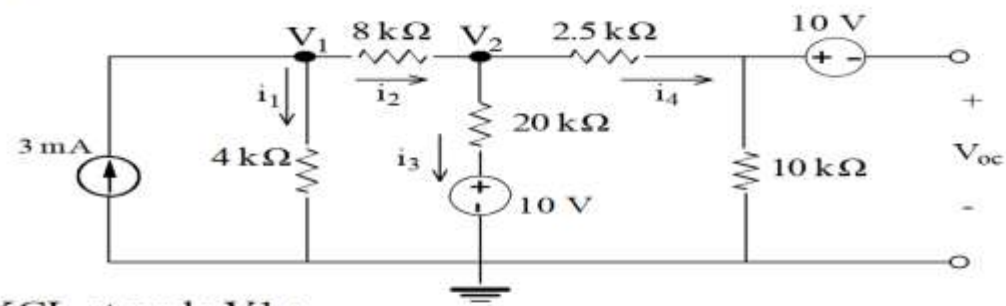
$$P_{L\max} = i^2 R_L$$
$$= \left(\frac{V_{OC}}{2R_{TH}}\right)^2 R_{TH}$$
$$= \frac{(V_{OC})^2 R_{TH}}{4R_{TH}^2} = \frac{(V_{OC})^2}{4R_{TH}}$$
$$P_{L\max} = \frac{V_{OC}^{\;2}}{4R_{TH}}$$

<u>Example 2</u> find the maximum power for the circuit shown below.



- Find R_L for maximum Power Transfer ?
- Find the maximum Power transfer to R_L ?

Let's find Thevenin equivalent circuit .



KCL at node V1 :

$$3\text{ mA} - i_1 - i_2 = 0$$

$$3\text{ mA} - \frac{V_1}{4\text{ k}\Omega} - \frac{V_1 - V_2}{8\text{ k}\Omega} = 0$$

$$V_1\left[\frac{1}{4\,k}+\frac{1}{8\,k}\right]-\left(\frac{1}{8\,k}\right)V_2=3\,m$$

$$0.375\,m\,V_1-0.125\,m\,V_2=3\,m \qquad \ldots\ldots(1)$$

KCL at node V2:

$$i_2-i_3-i_4=0$$

$$\frac{V_1-V_2}{8\,k}-\frac{V_2-10}{20\,k}-\frac{V_2}{12.5\,k}=0$$

$$V_1\left(\frac{1}{8\,k}\right)-\left(\frac{1}{8\,k}+\frac{1}{20\,k}+\frac{1}{12.5\,k}\right)V_2=-0.5\,m$$

$$0.125\,m\,V_1-0.255\,m\,V_2=-0.5\,m \qquad \ldots\ldots(2)$$

$$V_1=10.34\ V$$

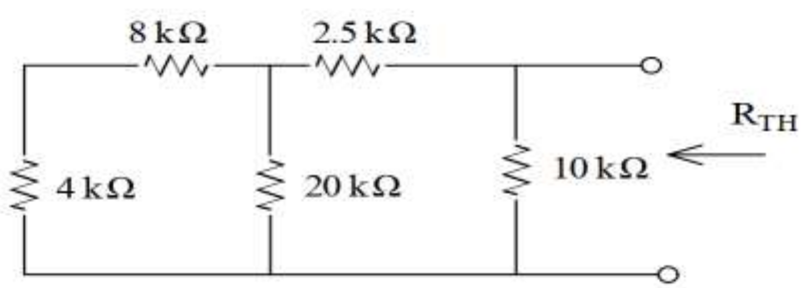
$$V_2=7.03\ V$$

$$V_{OC}=-10+10\,k\,i_4$$

$$=-10+10\,k\left(\frac{V2}{12.5\,k}\right)=-10+\frac{10}{12.5}(7.03)$$

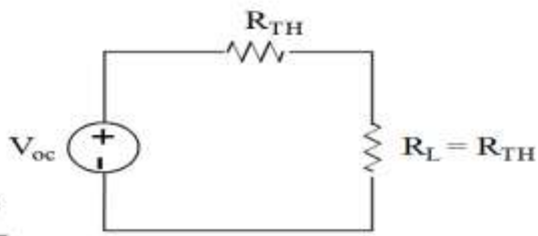
$$V_{OC}=-4.375\ V$$

<u>To find R_{TH}:</u>



Continue solution

$$R_{TH} = \{[(8\,k + 4\,k) // 20\,k] + 2.5\,k\} // 10\,k$$
$$= [(12\,k // 20\,k) + 2.5\,k] // 10\,k$$
$$= (7.5\,k + 2.5\,k) // 10\,k$$
$$= 10\,k // 10\,k$$
$$R_{TH} = 5\,k\,\Omega$$



$$P_{L\,max} = \frac{V_{OC}^{2}}{4\,R_{TH}} = \frac{(-4.375)^2}{4\,(5\,k)}$$
$$P_{L\,max} = 0.957\ m\,W$$