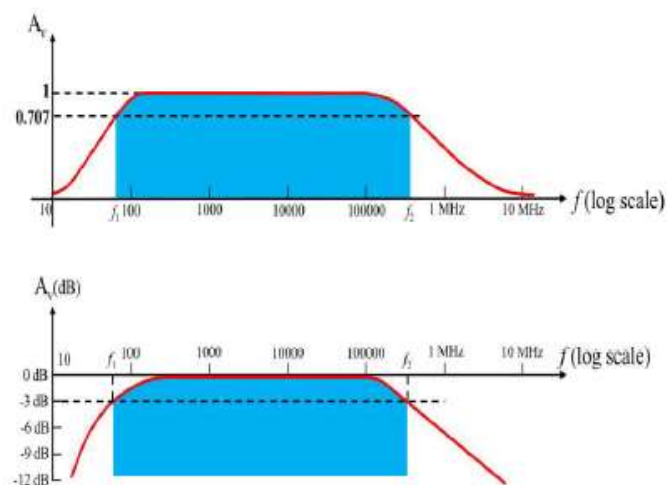




## Electronic Circuit

### Lecture 2 (3<sup>rd</sup> Week)

## General Frequency Considerations





## 1.1. Introduction

We will now investigate the frequency effects introduced by the larger capacitive elements of the network at low frequencies and the smaller capacitive elements of the active device at the high frequencies. Since the analysis will extend through a wide frequency range, the logarithmic scale will be defined and used throughout the analysis.

In addition, since industry typically uses a decibel scale on its frequency plots, the concept of the decibel is introduced in some detail.

في المحاضرات السابقة وعند القيام بعملية تحليل الدوائر الالكترونية , تم اعتبار دوائر المتسعات كدائرة مغلقة واهمال قيم التردد وتأثيرها على الدائرة الالكترونية. وذلك لان قيمة التردد عالي جدا بحيث لا يؤثر عند حساب قيمة الكسب في التيار او الفولتية.

في هذه المحاضرة سيتم دراسة تأثير التردد في قيمة الجهد المراد حسابة في الترددات العالية والواطئة كما سيتم ايضا حة بالتفصيل.

## 1.2 LOGARITHMS

As a first step in clarifying the relationship between the variables of a logarithmic function, consider the following mathematical equations:

بما انه سيتم التعامل مع طيف واسع من الترددات تبدا من الترددات الصغيرة الى الترددات العالية لذلك سيتم استخدام اللوغارثيمات ووحدة الديسيبل لوصف وقياس النتائج عند القيام بتحليل الدوائر الالكترونية.

الخطوة الاولى هي تحديد العلاقة بين مختلف المتغيرات التي نحتاجها في معادلات اللوغارثيمات وكما موضح في المعادلات ادناه.



$$a = b^x, \quad x = \log_b a \quad (11.1)$$

but

$$a = b^x = (10)^2 = 100$$
$$x = \log_b a = \log_{10} 100 = 2$$

fact for the vast majority of scientific research, the base in the logarithmic equation is chosen as either 10 or the number  $e = 2.71828$

$$\text{Common logarithm: } x = \log_{10} a \quad (11.2)$$

$$\text{Natural logarithm: } y = \log_e a \quad (11.3)$$

The two are related by

$$\log_e a = 2.3 \log_{10} a \quad (11.4)$$

**EXAMPLE 9.1** Using the calculator, determine the logarithm of the following numbers to the base indicated:

a.  $\log_{10} 10^6$ .

b.  $\log_e e^3$ .

c.  $\log_{10} 10^{-2}$ .

d.  $\log_e e^{-1}$ .

**Solution:**

a. 6

b. 3

c. 2

d. 1



the logarithm of a number taken to a power is simply the power of the number if the number matches the base of the logarithm

Tab 9.1 shows how the logarithm of a number increases only as the exponent of the number

$\log_{10} 10^0$	= 0
$\log_{10} 10$	= 1
$\log_{10} 100$	= 2
$\log_{10} 1,000$	= 3
$\log_{10} 10,000$	= 4
$\log_{10} 100,000$	= 5
$\log_{10} 1,000,000$	= 6
$\log_{10} 10,000,000$	= 7
$\log_{10} 100,000,000$	= 8
etc.	

Table 9.1

Because the remaining analysis of this chapter employs the common logarithm, we review a few properties of logarithms using solely the common logarithm. In general, however, the same relationships hold true for logarithms to any base. First, note that

$$\log_{10} 1 = 0 \quad (9.5)$$

as clearly revealed by Table 9.1, because  $10^0 = 1$ . Next,

$$\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b \quad (9.6)$$

which for the special case of  $a = 1$  becomes

$$\log_{10} \frac{1}{b} = -\log_{10} b \quad (9.7)$$

which shows that for any  $b$  greater than 1, the logarithm of a number less than 1 is always negative. Finally,

$$\log_{10} ab = \log_{10} a + \log_{10} b \quad (9.8)$$

In each case, the equations employing natural logarithms have the same format.



## Semilog graph paper

The log of 2 to the base 10 is approximately 0.3. The distance from 1 ( $\log_{10} 1 = 0$ ) to 2 is therefore 30% of the span. The log of 3 to the base 10 is 0.4771 or almost 48% of the span (very close to one-half the distance between power-of-10 increments on the (log scale)).

يلاحظ ان التخطيط يحتوي على محور تدرجه طبيعي يبدأ من الواحد الى الستة ومحور تدرجه لوغاريتمي ومن هنا اصل التسمية. يلاحظ ايضا عدم وجود صفر او قيمة سالبة في المحور اللوغاريتمي. ملاحظة اخرى هي ان لوغاريتم الواحد يساوي صفر لذلك يوضع بمكان الصفر وهكذا لبقية القيم. الملاحظة الاخيرة هي ان المسافة بين القيم ليست مسافة حقيقة وانما مسافة لوغاريتمية.

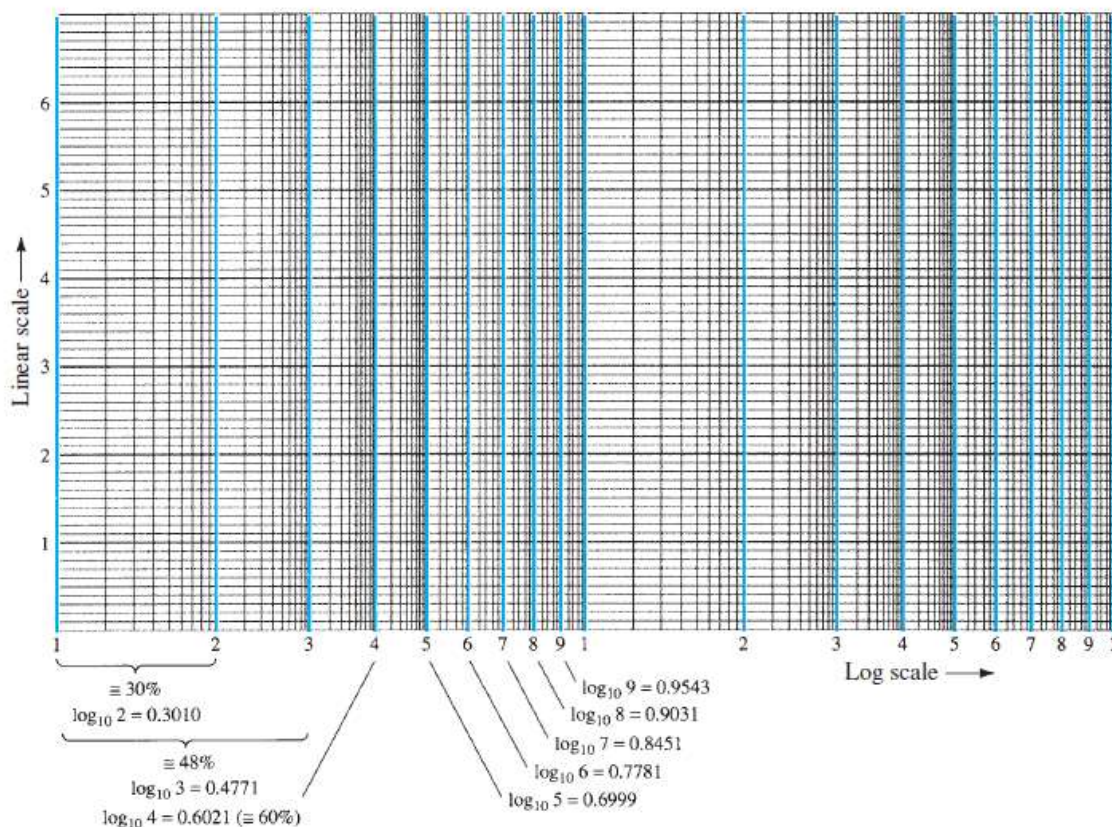


FIG. 9.1  
Semilog graph paper.





The following equation can be used to determine the logarithmic level at a particular point between known levels using a ruler or simply estimating the distances.

يمكن استخدام المعادلة التالية لتحديد المستوى اللوغاريتمي عند نقطة معينة بين المستويات المعروفة باستخدام مسطرة أو ببساطة تقدير المسافات.

$$\text{Value} = 10^x \times 10^{d_1/d_2}$$

**EXAMPLE 9.5** Determine the value of the point appearing on the logarithmic plot in Fig. below using the measurements made by a ruler (linear).

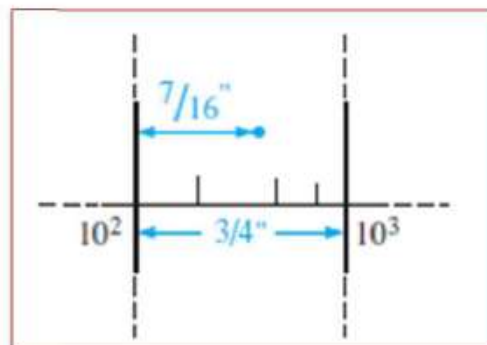
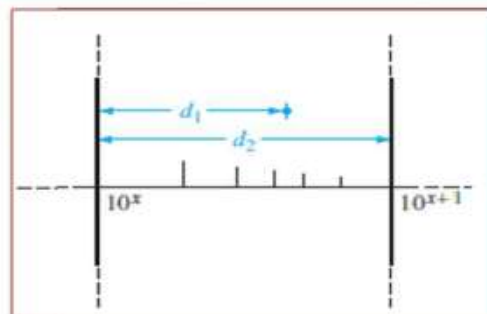
$$\frac{d_1}{d_2} = \frac{7/16''}{3/4''} = \frac{0.438''}{0.750''} = 0.584$$

Using a calculator:

$$10^{d_1/d_2} = 10^{0.584} = 3.837$$

Applying Eq. (9.9):

$$\begin{aligned} \text{Value} &= 10^x \times 10^{d_1/d_2} = 10^2 \times 3.837 \\ &= 383.7 \end{aligned}$$





## DECIBELS

The term bel is derived from the surname of Alexander Graham Bell. For standardization, the bel (B) is defined by the following equation relating two power levels, P<sub>1</sub> and P<sub>2</sub>

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} \quad \text{dB} \quad (11.10)$$

$$G_{dBm} = 10 \log_{10} \frac{P_2}{1 \text{ mW} |_{600 \Omega}} \quad \text{dBm} \quad (11.11)$$

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_2}{V_1^2/R_1} = 10 \log_{10} \left( \frac{V_2}{V_1} \right)^2$$

and

$$G_{dB} = 20 \log_{10} \frac{V_2}{V_1} \quad \text{dB} \quad (11.12)$$

هي وحدة لوغاريتمية تعطي النسبة بين كميتين فيزيائيتين ، مثل القدرة أو الشدة وذلك بالنسبة إلى قيمة معيارية . يستخدم في الصوت وفي الإلكترونيات . تعني هذه الوحدة اللوغاريتمية أنه إذا زادت القدرة أو الشدة إلى الضعف ، يزداد الديسيبل بمقدار 3 dB .

- The power and audio levels are related on a logarithmic basis.
- For an increase in power level of 4w to 64w , the audio level will increase by a factor of 3 because  $4^3 = 64$ . In logarithmic form the relationship can be written as
- $\log_4 64 = 3$



**EXAMPLE 9.7** The input power to a device is 10,000 W at a voltage of 1000 V. The output power is 500 W and the output impedance is 20  $\Omega$ .

- Find the power gain in decibels.
- Find the voltage gain in decibels.
- Explain why parts (a) and (b) agree or disagree.

**Solution:**

$$\begin{aligned} \text{a. } G_{\text{dB}} &= 10 \log_{10} \frac{P_o}{P_i} = 10 \log_{10} \frac{500 \text{ W}}{10 \text{ kW}} = 10 \log_{10} \frac{1}{20} = -10 \log_{10} 20 \\ &= -10(1.301) = -13.01 \text{ dB} \end{aligned}$$

$$\begin{aligned} \text{b. } G_v &= 20 \log_{10} \frac{V_o}{V_i} = 20 \log_{10} \frac{\sqrt{PR}}{1000} = 20 \log_{10} \frac{\sqrt{(500 \text{ W})(20 \Omega)}}{1000 \text{ V}} \\ &= 20 \log_{10} \frac{100}{1000} = 20 \log_{10} \frac{1}{10} = -20 \log_{10} 10 = -20 \text{ dB} \end{aligned}$$

$$\text{c. } R_i = \frac{V_i^2}{P_i} = \frac{(1 \text{ kV})^2}{10 \text{ kW}} = \frac{10^6}{10^4} = 100 \Omega \neq R_o = 20 \Omega$$

## GENERAL FREQUENCY CONSIDERATIONS

At low frequencies, we shall find that the coupling and bypass capacitors can no longer be replaced by the short-circuit approximation because of the increase in reactance of these elements. The frequency-dependent parameters of the small-signal equivalent circuits and the stray capacitive elements associated with the active device and the network will limit the high-frequency response of the system. An increase in the number of stages of a cascaded system will also limit both the high- and low-frequency responses.





### Low-Frequency Range

the larger capacitors of a system will have an important impact on the response of a system in the low-frequency range and can be ignored for the high-frequency region

Variation in  $X_C = \frac{1}{2\pi fC}$  with frequency for a 1- $\mu F$  capacitor

$f$	$X_C$	
10 Hz	15.91 k $\Omega$	} Range of possible effect
100 Hz	1.59 k $\Omega$	
1 kHz	159 $\Omega$	
10 kHz	15.9 $\Omega$	} Range of lesser concern (= short-circuit equivalence)
100 kHz	1.59 $\Omega$	
1 MHz	0.159 $\Omega$	
10 MHz	15.9 m $\Omega$	
100 MHz	1.59 m $\Omega$	

Variation in  $X_C = \frac{1}{2\pi fC}$  with frequency for a 5 pF capacitor

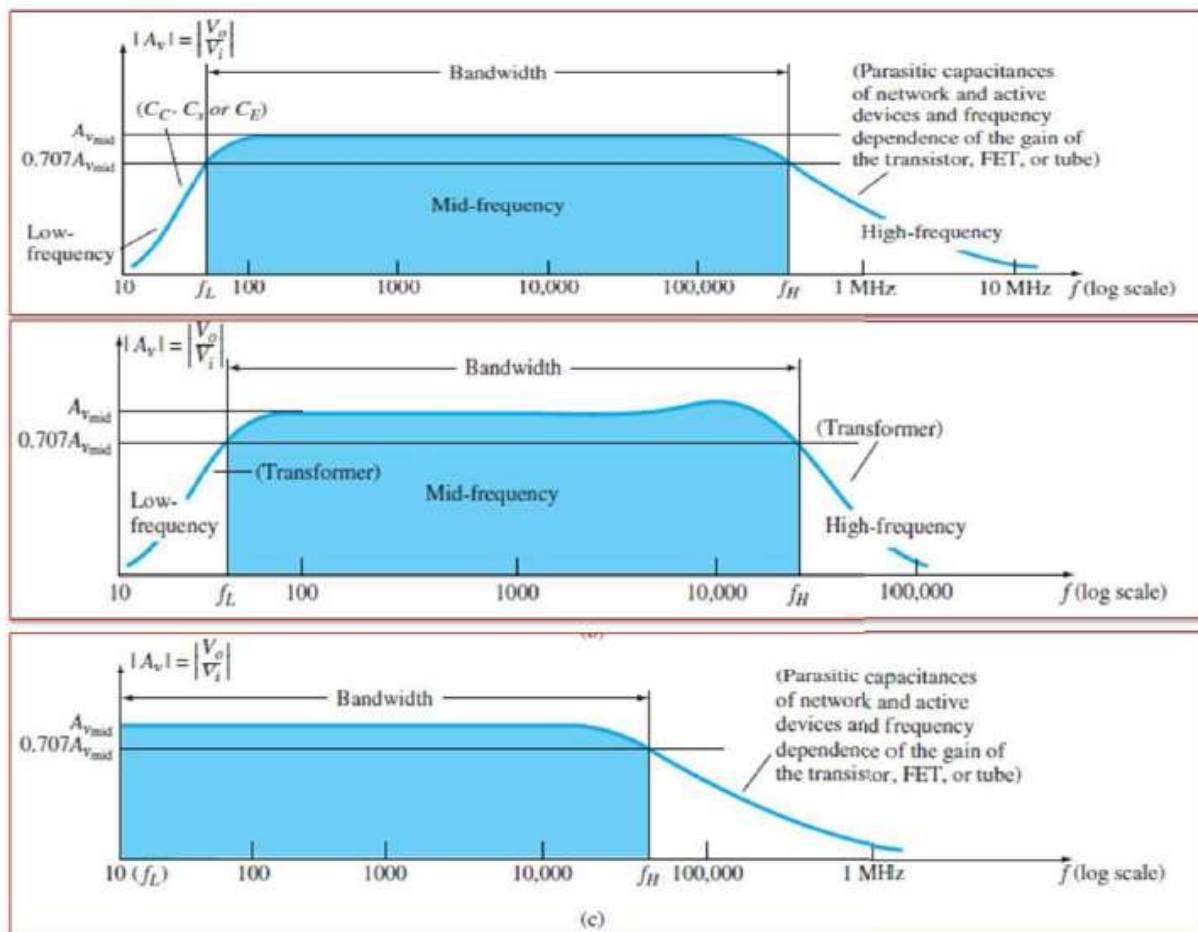
$f$	$X_C$	
10 Hz	3,183 M $\Omega$	} Range of lesser concern (= open-circuit equivalent)
100 Hz	318.3 M $\Omega$	
1 kHz	31.83 M $\Omega$	
10 kHz	3.183 M $\Omega$	} Range of possible effect
100 kHz	318.3 k $\Omega$	
1 MHz	31.83 k $\Omega$	
10 MHz	3.183 k $\Omega$	
100 MHz	318.3 $\Omega$	

### Mid-frequency range

the effect of the capacitive elements in an amplifier are ignored for the mid-frequency range when important quantities such as the gain and impedance levels are determine



## Typical Frequency Response





## Band of frequencies

For each system of Fig. 9.8 , there is a band of frequencies in which the magnitude of the gain is either equal or relatively close to the midband value. To fix the frequency boundaries of relatively high gain,  $0.707A_{v_{mid}}$  was chosen to be the gain at the cutoff levels. The corresponding frequencies  $f_1$  and  $f_2$  are generally called the corner, cutoff, band, break, or half-power frequencies. The multiplier 0.707 was chosen because at this level the output power is half the midband power output, that is, at midfrequencies

$$P_{o_{mid}} = \frac{|V_o^2|}{R_o} = \frac{|A_{v_{mid}} V_i|^2}{R_o}$$

and at the half-power frequencies,

$$P_{o_{HPF}} = \frac{|0.707 A_{v_{mid}} V_i|^2}{R_o} = 0.5 \frac{|A_{v_{mid}} V_i|^2}{R_o}$$

and

$$P_{o_{HPF}} = 0.5 P_{o_{mid}} \quad (9.18)$$

The bandwidth (or passband) of each system is determined by  $f_H$  and  $f_L$ , that is,

$$\text{bandwidth (BW)} = f_H - f_L \quad (9.19)$$

with  $f_H$  and  $f_L$  defined in each curve of Fig. 9.8.

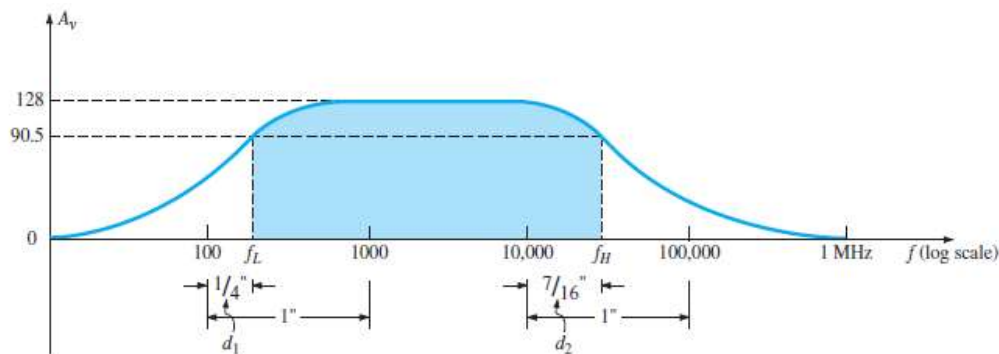
the effect of the capacitive elements in an amplifier are ignored for the mid-frequency range when important quantities such as the gain and impedance levels are determined.

The band frequencies define a level where the gain or quantity of interest will be 70.7% or its maximum value.



**EXAMPLE 9.9** Given the frequency response of Fig. 9.10 :

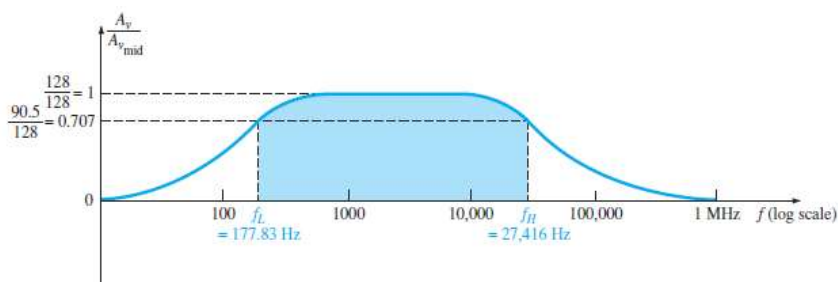
- Find the cutoff frequencies  $f_L$  and  $f_H$  using the measurements provided.
- Find the bandwidth of the response.
- Sketch the normalized response.



**FIG. 9.10**  
Gain plot for Example 9.8.

**Solution:**

- For  $f_L$ :  $\frac{d_1}{d_2} = \frac{1/4''}{1''} = 0.25$   
 $10^{d_1/d_2} = 10^{0.25} = 1.7783$   
 $\text{Value} = 10^x \times 10^{d_1/d_2} = 10^2 \times 1.7783 = 177.83 \text{ Hz}$
- For  $f_H$ :  $\frac{d_1}{d_2} = \frac{7/16''}{1''} = 0.438$   
 $10^{d_1/d_2} = 10^{0.438} = 2.7416$   
 $\text{Value} = 10^x \times 10^{d_1/d_2} = 10^4 \times 2.7416 = 27,416 \text{ Hz}$
- The bandwidth:  
 $\text{BW} = f_H - f_L = 27,416 \text{ Hz} - 177.83 \text{ Hz} \approx 27.24 \text{ KHz}$
- The normalized response is determined by simply dividing each level of Fig. 9.10 by the midband level of 128, as shown in Fig. 9.11. The result is a maximum value of 1 and cutoff levels of 0.707.

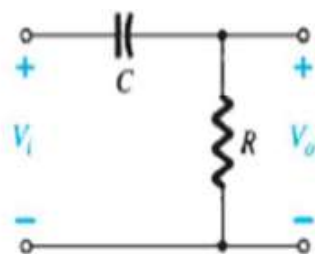


**FIG. 9.11**  
Normalized plot of Fig. 9.10.

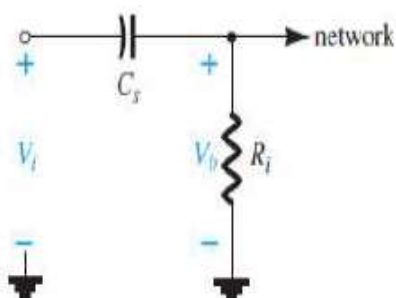


## LOW-FREQUENCY ANALYSIS- BODE PLOT

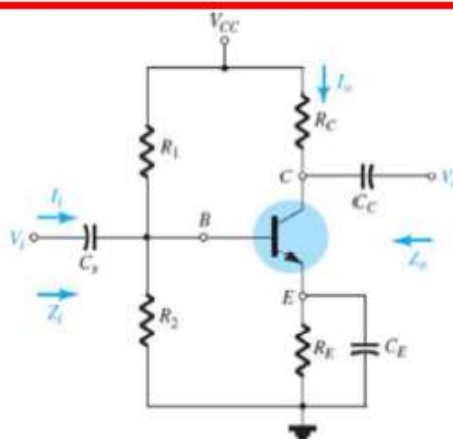
In the low-frequency region of the single-stage BJT or FET amplifier, it is the RC combinations formed by the network capacitors  $C_C$ ,  $C_E$ , and  $C_S$  and the network resistive parameters that determine the cutoff frequencies. In fact, an RC network similar to Fig. 9.14 can be established for each capacitive element, and the frequency at which the output voltage drops to 0.707 of its maximum value can be determined. Once the cutoff frequencies due to each capacitor are determined, they can be compared to establish which will determine the low cutoff frequency for the system.



**FIG. 9.14**  
*RC combination that will define a low-cutoff frequency.*



**FIG. 9.16**  
*Equivalent input circuit for the network of Fig. 9.15.*



**FIG. 9.15**  
*Voltage-divider bias configuration.*





## LOW-FREQUENCY ANALYSIS - BODE PLOT

At very high frequencies,

$$X_C = \frac{1}{2\pi fC} \cong 0 \Omega$$

$$V_o \cong V_i$$

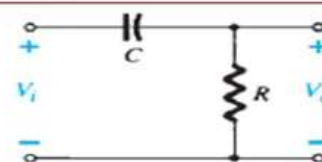


FIG. 9.14

RC combination that will define a low-cutoff frequency.

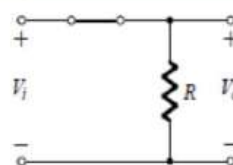


Figure 11.9 R-C circuit of Figure 11.8 at very high frequencies.

At  $f = 0$  Hz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(0)C} = \infty \Omega$$

$$V_o = 0 \text{ V.}$$

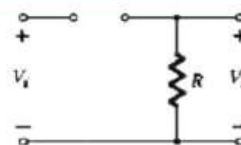
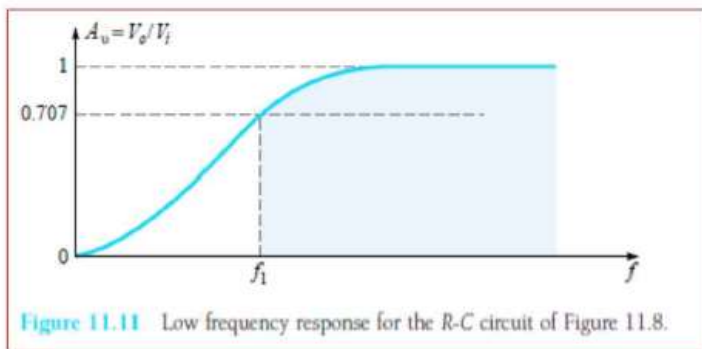


Figure 11.10 R-C circuit of Figure 11.8 at  $f = 0$  Hz.





The output and input voltages are related by the voltage-divider rule in the following manner:

$$V_o = \frac{RV_i}{R + X_C}$$

with the magnitude of  $V_o$  determined by

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}}$$

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}} = \frac{RV_i}{\sqrt{R^2 + R^2}} = \frac{RV_i}{\sqrt{2R^2}} = \frac{RV_i}{\sqrt{2}R} = \frac{1}{\sqrt{2}}V_i$$

and

$$|A_v| = \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707|_{X_C=R} \quad (11.19)$$

$$X_C = \frac{1}{2\pi f_1 C} = R$$

and

$$f_1 = \frac{1}{2\pi RC} \quad (11.20)$$

In terms of logs,

$$G_v = 20 \log_{10} A_v = 20 \log_{10} \frac{1}{\sqrt{2}} = -3 \text{ dB}$$

while at  $A_v = V_o/V_i = 1$  or  $V_o = V_i$  (the maximum value),

$$G_v = 20 \log_{10} 1 = 20(0) = 0 \text{ dB}$$

If the gain equation is written as

$$A_v = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{1}{1 - j(X_C/R)} = \frac{1}{1 - j(1/\omega CR)} = \frac{1}{1 - j(1/2\pi fCR)}$$

and using the frequency defined above,

$$A_v = \frac{1}{1 - j(f_1/f)} \quad (11.21)$$

In the magnitude and phase form,

$$A_v = \frac{V_o}{V_i} = \underbrace{\frac{1}{\sqrt{1 + (f_1/f)^2}}}_{\text{magnitude of } A_v} \underbrace{\angle \tan^{-1}(f_1/f)}_{\text{phase } \angle \text{ by which } V_o \text{ leads } V_i} \quad (11.22)$$



For the magnitude when  $f = f_1$ ,

$$|A_v| = \frac{1}{\sqrt{1 + (1)^2}} = \frac{1}{\sqrt{2}} = 0.707 \rightarrow -3 \text{ dB}$$

In the logarithmic form, the gain in dB is

$$\begin{aligned} A_{v(\text{dB})} &= 20 \log_{10} \frac{1}{\sqrt{1 + (f_1/f)^2}} = -20 \log_{10} \left[ 1 + \left( \frac{f_1}{f} \right)^2 \right]^{1/2} \\ &= -\left(\frac{1}{2}\right)(20) \log_{10} \left[ 1 + \left( \frac{f_1}{f} \right)^2 \right] \\ &= -10 \log_{10} \left[ 1 + \left( \frac{f_1}{f} \right)^2 \right] \end{aligned}$$

For frequencies where  $f \ll f_L$  or  $(f_L/f)^2 \gg 1$ , the equation above can be approximated by

$$A_{v(\text{dB})} = -10 \log_{10} \left( \frac{f_L}{f} \right)^2$$

and finally,

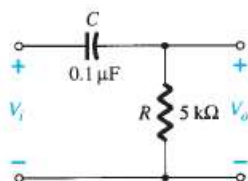
$$A_{v(\text{dB})} = -20 \log_{10} \frac{f_1}{f} \quad f \ll f_1 \quad (11.23)$$

At  $f = f_1$ :  $\frac{f_1}{f} = 1$  and  $-20 \log_{10} 1 = 0 \text{ dB}$

At  $f = \frac{1}{2} f_1$ :  $\frac{f_1}{f} = 2$  and  $-20 \log_{10} 2 \cong -6 \text{ dB}$

At  $f = \frac{1}{4} f_1$ :  $\frac{f_1}{f} = 4$  and  $-20 \log_{10} 4 \cong -12 \text{ dB}$

At  $f = \frac{1}{10} f_1$ :  $\frac{f_1}{f} = 10$  and  $-20 \log_{10} 10 = -20 \text{ dB}$



**FIG. 9.23**  
Example 9.10.

**EXAMPLE 9.10** For the network of Fig. 9.23:

- Determine the break frequency.
- Sketch the asymptotes and locate the  $-3$ -dB point.
- Sketch the frequency response curve.
- Find the gain at  $A_{v(\text{dB})} = -6$  dB.

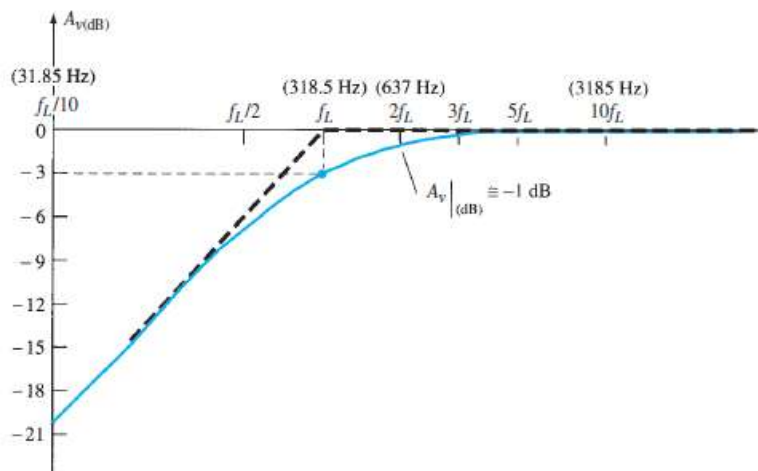
**Solution:**

$$\text{a. } f_L = \frac{1}{2\pi RC} = \frac{1}{(6.28)(5 \times 10^3 \Omega)(0.1 \times 10^{-6} \text{ F})}$$
$$\cong 318.5 \text{ Hz}$$

b. and c. See Fig. 9.24.

$$\text{d. Eq. (9.27): } A_v = \frac{V_o}{V_i} = 10^{A_{v(\text{dB})}/20}$$
$$= 10^{(-6/20)} = 10^{-0.3} = 0.501$$

and  $V_o = 0.501 V_i$  or approximately 50% of  $V_i$ .



**FIG. 9.24**  
Frequency response for the RC circuit of Fig. 9.23.