

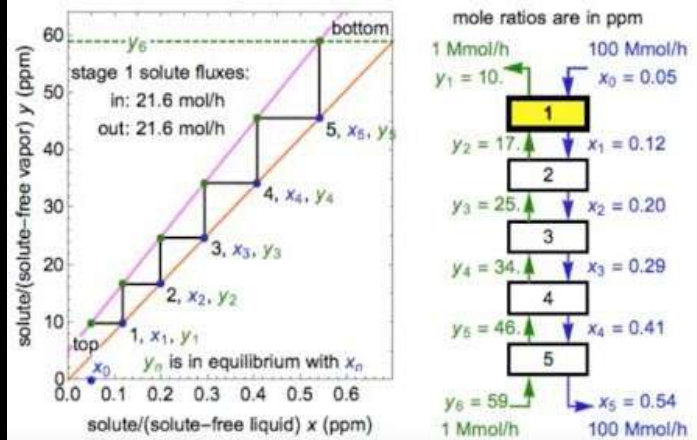
Unit Operation I

Spring 2025

Lecture #2

Dr. Haydar Aljaafari

GRAPHICAL METHOD

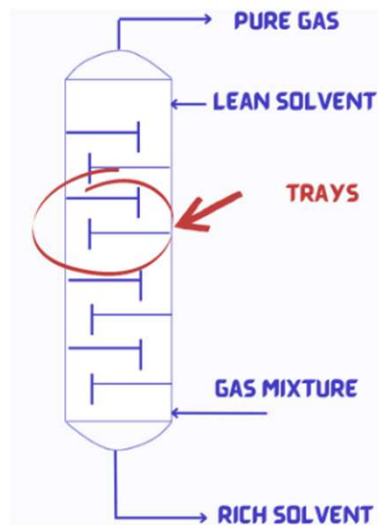


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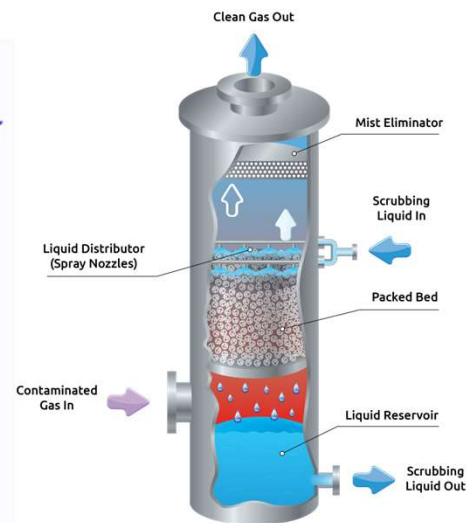
Absorption Process

Equipment

- Tray Tower
- Packed Tower
- Spray Tower
- Bubble Column



Tray Tower



Pack Tower

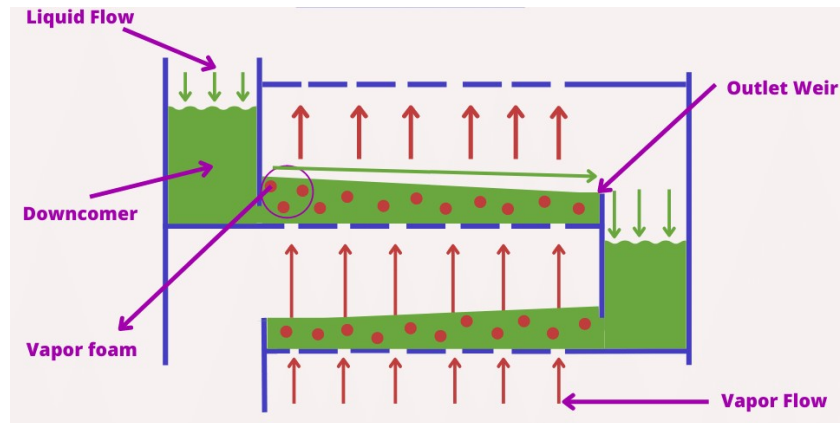


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Tray Tower

A tray tower is a vertical cylindrical vessel in which the liquid flows down by gravity across trays over the outlet weir and into a downcomer to the tray below, and the gas flows up through the opening in each tray, bubbling through the liquid.



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Tray Tower

The absorption process can be carried out in a **single-stage** or **multistage** unit.

A- Single Stage :

Consider the following:

- The unit runs under isobaric, isothermal, and steady-state flow conditions.
- The liquid phase enters from the top while gas enters from the bottom, and both are with known amounts and compositions.
- The two exit streams (Liquid & Gas) are **leaving at the equilibrium**.

Where

L : molar flow rate of the liquid phase (moles/time)

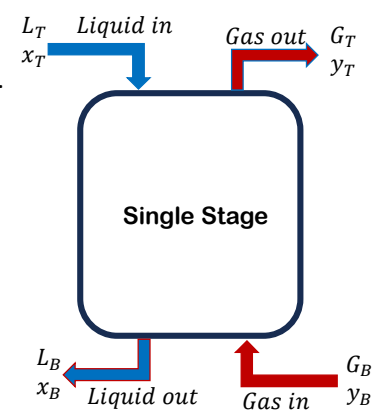
G : molar flow rate of gas phase (moles/time)

x : is the mole fraction of solute A in the **liquid** phase

y : is the mole fraction of solute A in the **gas** phase

T : for the streams in the **top**

B : for the streams in the **bottom**



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Tray Tower

A- Single Stage Unit:

Overall Material Balance

$$L_T + G_B = L_B + G_T \quad \text{.....(1)}$$

Component (A) is transferring from gas to liquid

Component (A) Material Balance

$$L_T \cdot x_T + G_B \cdot y_B = L_B \cdot x_B + G_T \cdot y_T \quad \text{.....(2)}$$

Since only component (A) is transferring between the two phases then

$$\bar{G} = G_T (1 - y_T); \quad G_T = \frac{\bar{G}}{(1 - y_T)}$$

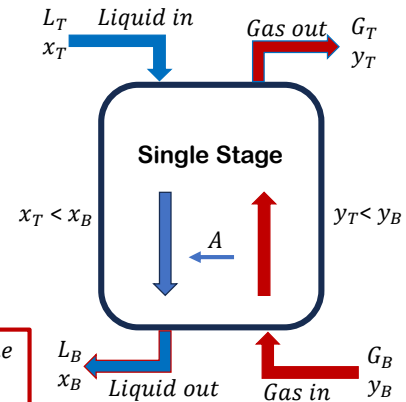
$$\bar{G} = G_B (1 - y_B); \quad G_B = \frac{\bar{G}}{(1 - y_B)}$$

$$\bar{L} = L_T (1 - x_T); \quad L_T = \frac{\bar{L}}{(1 - x_T)}$$

$$\bar{L} = L_B (1 - x_B); \quad L_B = \frac{\bar{L}}{(1 - x_B)}$$

\bar{G} is the inert rate (everything in the gas other than A), (moles/time)

\bar{L} is the solvent rate (everything in the liquid other than A), (moles/time)



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A- Single Stage Unit:

$$L_T \cdot x_T + G_B \cdot y_B = L_B \cdot x_B + G_T \cdot y_T \quad \text{.....(2)}$$

$$\frac{\bar{L}}{(1 - x_T)} \cdot x_T + \frac{\bar{G}}{(1 - y_B)} \cdot y_B = \frac{\bar{L}}{(1 - x_B)} \cdot x_B + \frac{\bar{G}}{(1 - y_T)} \cdot y_T \quad \text{.....(3) (Operating Eq.)}$$

$$\text{Assume } \frac{y_i}{(1 - y_i)} = Y_i; \quad \frac{x_i}{(1 - x_i)} = X_i$$

$$\bar{L} \cdot X_T + \bar{G} \cdot Y_B = \bar{L} \cdot X_B + \bar{G} \cdot Y_T \quad \text{.....(4)}$$

Since only component (A) is transferring between the two phases then

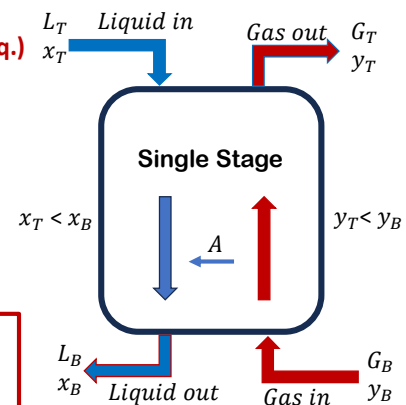
The inert rate \bar{G} and the solvent rate \bar{L} are constants.

Notes:

If $(y_B \text{ or } x_T) \geq 0.1$ then the solution is considered concentrated

If $(y_B \text{ or } x_T) < 0.1$ then the solution is considered diluted

Equilibrium eq. for dilute solutions $P_A = H \cdot x$ or $\frac{P_A}{P_{total}} = \frac{H}{P_{total}} \cdot x \rightarrow y = \bar{H} \cdot x$



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Tray Tower

Example 1: In a single-stage absorption process, a gas mixture at 1 atm pressure containing air and H_2S is contacted with pure water at 30°C . The exit gas and liquid streams reach equilibrium. The inlet gas flow rate is 200 kmol/hr, with a 0.2 mole fraction of H_2S , while the liquid flow rate entering is 600 kmol/hr. Calculate the amount and composition of the two outlet phases. Assume water does not vaporize into the gas phase.

Given: Henry's constant $H = 0.0609 \times 10^4$ atm/mole fraction.

Solution

Since the inlet liquid is pure water, thus, $x_T = 0$ And, $\bar{L} = L_T = 600$ kmole/hr (pure water)

$$\bar{G} = G_B (1 - y_B) = 200 * (1 - 0.2) = 160 \text{ kmole/hr}$$

Applying the operating equation of the single-stage absorption

$$\frac{\bar{L}}{(1 - x_T)} \cdot x_T + \frac{\bar{G}}{(1 - y_B)} \cdot y_B = \frac{\bar{L}}{(1 - x_B)} \cdot x_B + \frac{\bar{G}}{(1 - y_T)} \cdot y_T$$

$$\frac{600}{(1 - 0)} \cdot 0 + \frac{160}{(1 - 0.2)} \cdot 0.2 = \frac{600}{(1 - x_B)} \cdot x_B + \frac{160}{(1 - y_T)} \cdot y_T$$

$$40 = \frac{600}{(1 - x_B)} \cdot x_B + \frac{160}{(1 - y_T)} \cdot y_T \quad \dots\dots\dots (*)$$

Since x_B is in equilibrium with y_T and Henry's constant is given then



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Tray Tower

Example 1: $H = 0.0609 \times 10^4$ atm/mole fraction.

Solution

$$40 = \frac{600}{(1 - x_B)} \cdot x_B + \frac{160}{(1 - y_T)} \cdot y_T \quad \dots\dots\dots (*)$$

Since x_B is in equilibrium with y_T and Henry's constant is given then

$$y = \frac{H}{P_{total}} \cdot x \rightarrow y = \frac{0.0609 \times 10^4}{1} \cdot x \rightarrow y_T = 609 \cdot x_B \text{ sub in } (*)$$

$$40 = \frac{600}{(1 - x_B)} \cdot x_B + \frac{160}{(1 - 609x_B)} \cdot 609x_B$$

Solve for x_B

$$x_B = 0.000327 \text{ then } y_T \text{ from the equilibrium equation } y_T = 609 \cdot 0.000327 = 0.199$$

Then the flow rates are:

$$G_T = \frac{\bar{G}}{(1 - y_T)} = \frac{160}{(1 - 0.199)} = 199.8 \frac{\text{Kmole}}{\text{hr}} ; L_B = \frac{\bar{L}}{(1 - x_B)} = \frac{600}{(1 - 0.000327)} = 600.196 \frac{\text{Kmole}}{\text{hr}}$$



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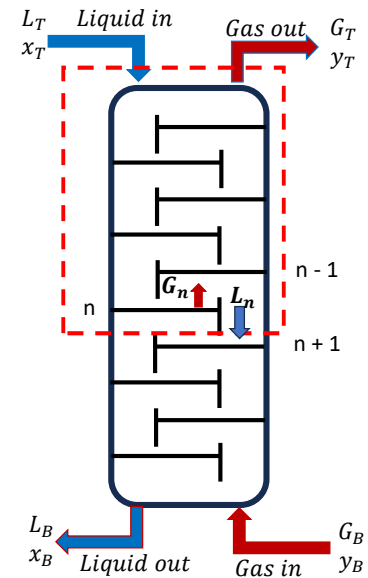
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Tray Tower

B- Multiple Stages:

Consider the following:

- The tower is running under isobaric, isothermal and steady-state flow.
- Phase equilibrium is assumed to be achieved at each tray between the vapor and liquid streams leaving the tray. (each tray is treated as equilibrium stage)
- Assume that the only component transferred from one phase to the other is solute A



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Tray Tower

B- Multiple Stages:

Overall Material Balance

$$L_T + G_{n+1} = L_n + G_T$$

Component (A) Material Balance

$$L_T \cdot x_T + G_{n+1} \cdot y_{n+1} = L_n \cdot x_n + G_T \cdot y_T$$

Since only component (A) is transferring between the two phases then

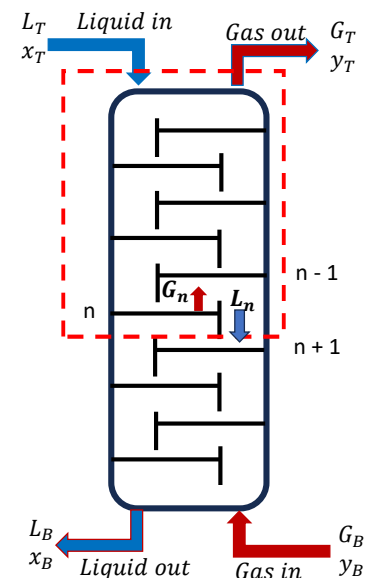
$$\frac{\bar{L}}{(1-x_T)} \cdot x_T + \frac{\bar{G}}{(1-y_{n+1})} \cdot y_{n+1} = \frac{\bar{L}}{(1-x_n)} \cdot x_n + \frac{\bar{G}}{(1-y_T)} \cdot y_T$$

$$\text{Assume } \frac{y_i}{(1-y_i)} = Y_i ; \quad \frac{x_i}{(1-x_i)} = X_i$$

$$\bar{L} \cdot X_T + \bar{G} \cdot Y_{n+1} = \bar{L} \cdot X_n + \bar{G} \cdot Y_T \quad \text{Divide by } \bar{G}$$

$$\frac{\bar{L}}{\bar{G}} \cdot X_T + Y_{n+1} = \frac{\bar{L}}{\bar{G}} \cdot X_n + Y_T \rightarrow Y_{n+1} = \frac{\bar{L}}{\bar{G}} \cdot X_n + (Y_T - \frac{\bar{L}}{\bar{G}} \cdot X_T) \quad \text{Operating Equation}$$

This is the equation of straight line, with a slope of $\frac{\bar{L}}{\bar{G}}$ and intercept of $(Y_T - \frac{\bar{L}}{\bar{G}} \cdot X_T)$



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Tray Tower

B- Multiple Stages:

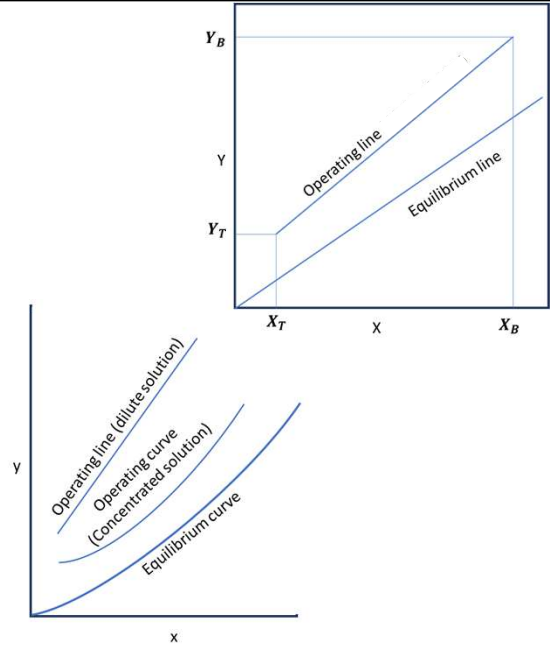
$$Y_{n+1} = \frac{\bar{L}}{\bar{G}} \cdot X_n + (Y_T - \frac{\bar{L}}{\bar{G}} \cdot X_T) \quad \text{Operating Equation}$$

This is the equation of straight line, with a slope of $\frac{\bar{L}}{\bar{G}}$ and intercept of $(Y_T - \frac{\bar{L}}{\bar{G}} \cdot X_T)$

When using the normal axis (y, x) the operating line equation will appear as a curve and not as a straight line unless we are **dealing with dilute solutions**, otherwise we have to convert the equilibrium data to (Y, X) and use the axis as (Y, X) to draw the operating line and the equilibrium data.

When dealing with **dilute** solutions, we can use the ordinary axis (y, x) and the operating line equation will appear as a straight line also the equilibrium data as

$$\frac{y_i}{(1-y_i)} = Y_i ; \quad \frac{x_i}{(1-x_i)} = X_i$$



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Tray Tower

ملاحظات مهمة:

- إذا ذكر بالسؤال (gas rate in) بدون تحديد المكان معناه هذا المقدار يمثل G_B
- إذا ذكر بالسؤال (Inert gas rate in) معناه هذا المقدار يمثل \bar{G}
- إذا ذكر بالسؤال ان (liquid rate in) بدون تحديد المكان معناه هذا المقدار يمثل L_T
- إذا ذكر بالسؤال ان السائل المستخدم (fresh, pure, or free) معناها ان ال ($x_T = 0$) و قيمة ($\bar{L} = L_T$)
- عادة بالسؤال قيمة y_B تكون معلومة
- قيمة ال y_T في بعض الاحيان تعطى بالسؤال وفي احيان اخرى يذكر بالسؤال مقدار ال Recovery (R) ليتم من خلالها ايجاد قيمة ال y_T
- من المعادلة [$y_T = (1-R) y_B$]
- المقصود ب (amount) هي قيم ال G و L
- المقصود ب (Composition) x و y



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Tray Tower

Number of plates in a Tray tower

I) Graphical method

- 1- Complete the material balance to calculate all compositions and flow rates of the inlet and the outlet streams
- 2- Draw the equilibrium curve from the **given** information:
data of x and y , equilibrium equation $y=x^m + c$, or Henry's constant
- 3- Draw the operating line equation
- 4- Draw a vertical line from the point 1 (x_B, y_B) to touch the equilibrium curve point 2. Then draw a horizontal line from point 2 to point 3, connecting the operating line. The triangular formed will represent the plate number **one**.
- 5- Continue drawing the vertical lines and horizontal lines as in step 4 to cross point (x_T, y_T).
- 6- The number of the formed triangles with a base on the operation line represents (**theoretical plate number**).
- 7- To calculate **the actual number of plates**, divide (**theoretical plate number/efficiency of plate**)

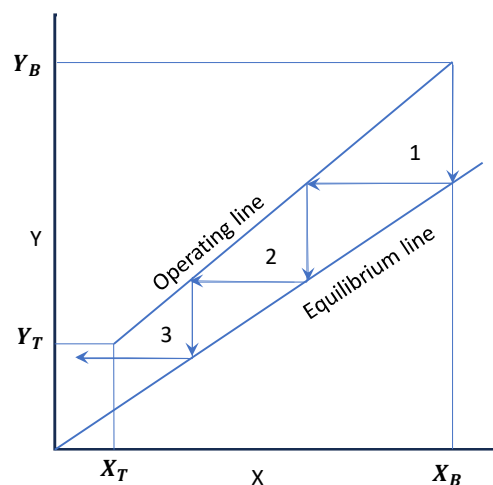


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B- Multiple Stages:



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Tray Tower

Number of plates in a Tray tower

i) Graphical method

عند عدم وجود معلومة عن x_B و \bar{L} في هذه الحالة نجد قيمة $(\frac{\bar{L}}{G})_{min}$ وذلك من خلال الرسم وكالاتي:

1- Draw the equilibrium data.

2- Set the point P (X_T, Y_T)

3- Use the equilibrium curve to find X_B^* which is in equilibrium with Y_B

4- Connect the line between P (X_T, Y_T) and M (X_B^*, Y_B)

5- Calculate the slope of P-M line which is equal to $(\frac{\bar{L}}{G})_{min}$

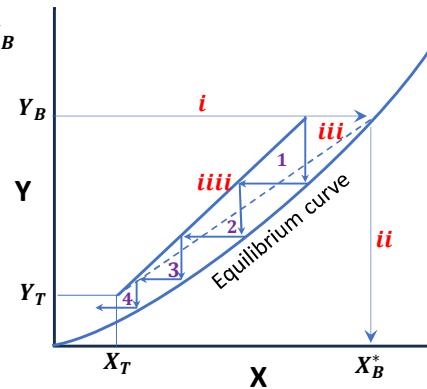
6- Calculate the $(\frac{\bar{L}}{G})_{Operation} = 1.5 (\frac{\bar{L}}{G})_{min}$

7- Calculate the new value of x_B from the equation

$$(\frac{\bar{L}}{G})_{Operation} = \frac{Y_B - Y_T}{X_B - X_T}$$

8- Draw the operation line between (X_T, Y_T) and (X_B, Y_B)

9- Calculate the number of plate



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Tray Tower

Example 2: In production of a liquor containing ethyl alcohol, a CO₂- rich vapor containing a small amount of ethyl alcohol is feed to absorption tray tower in which water was used to recover the alcohol. Determine the number of equilibrium stages required between liquid and gas and the amount of output streams, assuming isothermal, isobaric conditions in the tower and neglecting mass transfer of all components except ethyl alcohol. Given that:

Entering gas: 180 kmole/hr; 98 mol% CO₂, 2 mol% ethyl alcohol at 30 °C, 110 kPa

Entering liquid: 100 % water; 30 °C, 110 kPa

Required recovery of ethyl alcohol (R) = 97%

Equilibrium relationship: $y = 0.57x$

Solution

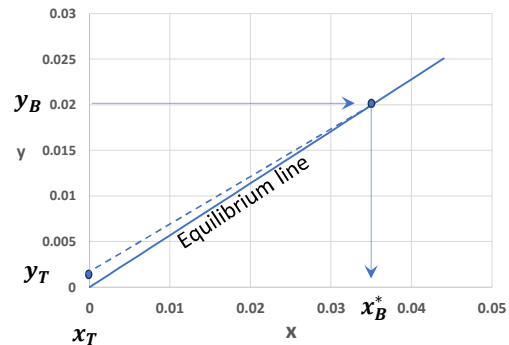
Since $y_B < 0.1$ then normal axis (y, x) can be used

$$y_T = (1-R) y_B \rightarrow y_T = (1-0.97) 0.02 = 0.0006$$

$x_T = 0$ (100 % water)

$x_B = ?$

x_B^* can be found from the graph as shown or from the given equilibrium relation $y_B = 0.57x_B^* \rightarrow x_B^* = 0.02 / 0.57 = 0.035$



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Tray Tower

Example 2:

Solution

$$\left(\frac{L}{G}\right)_{min} = \frac{y_B - y_T}{x_B^* - x_T} = \frac{0.2 - 0.0006}{0.035 - 0} = 0.553$$

Assume the operating factor = 1.5

$$\left(\frac{L}{G}\right)_{operation} = 1.5 \left(\frac{L}{G}\right)_{min} = 1.5 * 0.553 = 0.8293$$

$$\left(\frac{L}{G}\right)_{operation} = \frac{y_B - y_T}{x_B - x_T} \rightarrow 0.8293 = \frac{0.2 - 0.0006}{x_B - 0}$$

$x_B = 0.0234$ (mole fraction of the liquid exits from the bottom)

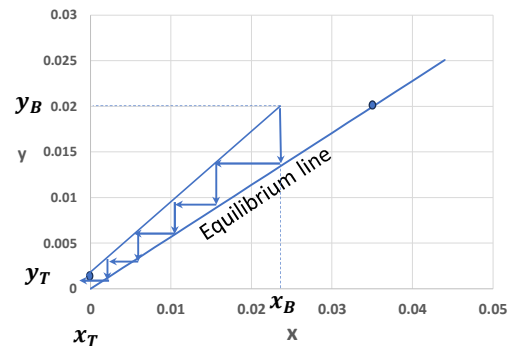
Draw the operating line between (x_B, y_B) and (x_T, y_T)

From the figure calculate the number of plates = 5

$$\bar{G} = G_B (1 - y_B) = 180 (1 - 0.02) = 176.4 \text{ kmole/hr}$$

$$G_T = \frac{\bar{G}}{1 - y_T} = \frac{176.4}{1 - 0.0006} = 176.5 \text{ kmole/hr}$$

$$\left(\frac{L}{G}\right)_{op} = 0.8293 \rightarrow L_B = 0.8293 * 180 = 149.3 \text{ kmole/hr}$$



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H.W.1

Determine the number of equilibrium stages required between liquid and gas and the amount of output streams for **Example 2**, if the entering gas: 150 kmole/hr; 85 mol% CO₂, 15 mol% ethyl alcohol at 30 °C, 110 kPa
 Entering liquid: 100 % water; 30 °C, 110 kPa.
 Required recovery of ethyl alcohol (R) = 95%
 Equilibrium relationship: $y = 0.57 * x$



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