Al-Mustaqbal University / College of Engineering & Technology
Class: first
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Determinants

Matrix:

A set of mn numbers (real or complex), arranged in a rectangular formation (array or table) having m rows and n columns and enclosed by a square bracket [ ] is called m x n matrix (read "m by n matrix").

Order of a Matrix:

The order or dimension of a matrix is the ordered pair having as first component the number of rows and as second component the number of columns in the matrix. If there are 3 rows and 2 columns in a matrix, then its order is written as (3, 2) or (3 x 2) read as three by two.

The Determinant of a Matrix:

The determinant of a matrix is a scalar (number), obtained from the elements of a matrix by specified, operations, which is characteristic of the matrix. The determinants are defined only for square matrices. It is denoted by det (A) or |A| for a square matrix A.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det(A)$

Remarks

(i) For matrix A, |A| is read as determinant of A and not modulus of A.
(ii) Only square matrices have determinants.

Definition of the Determinant Let $A = \lfloor a_{ij} \rfloor_{n\times n}$ be an $n \times n$ matrix.

(1) If $n = 1$, that is $A = [a_{11}]$, then we define $\det(A) = a_{11}$.

(2) If $n > 1$, we define $\det(A) = \sum_{k=1}^{n} (-1)^{1+k} a_{1k} \det(A_{1k})$

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If $A = [5]$, then by part (1) of the definition of the determinant, det (A) = 5.

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, then by parts (2) and (1), $\det(A) = (-1)^{1+1}(2)\det[5] + (-1)^{1+2}(3)\det[4]$

$$= (1)(2)(5) + (-1)(3)(4) = 10 - 12 = -2$$

Examples:

1. Determinant of a matrix of order one

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a.

2. Determinant of a matrix of order two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Then, $\det(A)=|A|=(a_{11}a_{22} - a_{12}a_{21})$

Example: If $A = \begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix}$, find det(A)

Solution: det(A)==2(2)-4(-1)=4+4=8

Example: If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find det(A)

Solution: det(A)==1(4)-2(3)=4-6=-2

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Example : If $A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$ find $|A|$

Solution:

$$|A| = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} = 9 - (-2) = 9 + 2 = 11$$

The determinant of the (3 x 3) matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \text{ denoted by } |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

is given as, det A = |A|

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Determinant of a matrix of order 3 × 3

Determinant of a matrix of order three can be determined by expressing it in terms of second order determinants.

Example 4: If $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{bmatrix}$

find det A by expansion about (a) the first row (b) the first column.

Solution (a)

$$|A| = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix}$$

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$$= 3\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 2\begin{vmatrix} 0 & -2 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix}$$

$$= 3(4+6) - 2(0+2) + 1\,(0-1)$$

$$= 30 - 4 - 1$$

$$|A| \quad = 25$$

Example: If $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 2 & 1 \\ 3 & 6 & 2 \end{bmatrix}$, find det(A)

Solution: det(A)=1(2x2 – 1x6) – 3(4x2 – 1x3) +2(4x6-2x3)

=1(4-6) – 3(8-3) + 2(24-6)

=1(-2) – 3(5) + 2(18)

=-2-15+36

=19

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