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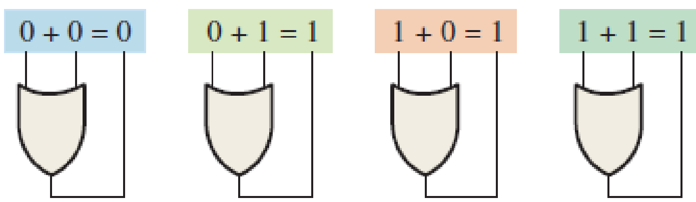
Lecture Three

Boolean Algebra and Logic Simplification

Boolean algebra is the mathematics of digital logic.

Boolean Addition

Boolean addition is equivalent to the OR operation. The basic rules are illustrated with their relation to the OR gate in the figure below.



In Boolean algebra, a **sum term** is a sum of literals. In logic circuits, a sum term is produced by an OR operation. Some examples of sum terms are:

$$A + B, A + \overline{B}, A + B + \overline{C}, \text{ and } \overline{A} + B + C + \overline{D}.$$

Example:

Determine the values of A, B, C, and D that make the sum term $A + \overline{B} + C + \overline{D}$ equal to 0.

Solution

For the sum term to be 0, each of the literals in the term must be 0. Therefore, $A = \mathbf{0}$, $B = \mathbf{1}$ so that $\overline{B} = 0$, $C = \mathbf{0}$, and $D = \mathbf{1}$ so that $\overline{D} = 0$.

$$A + \overline{B} + C + \overline{D} = 0 + \overline{1} + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$$

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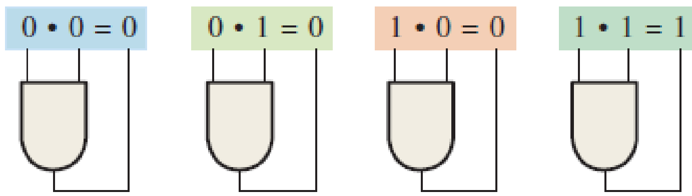
Boolean Multiplication

Boolean multiplication is equivalent to the AND operation.

The basic rules are illustrated with their relation to the AND gate in the figure below.



Example:

Determine the values of A, B, C, and D that make the product term $A\overline{B}C\overline{D}$ equal to 1.

Solution

For the product term to be 1, each of the literals in the term must be 1. Therefore, $A = \mathbf{1}$, $B = \mathbf{0}$ so that $\overline{B} = 1$, $C = \mathbf{1}$, and $D = \mathbf{0}$ so that $\overline{D} = 1$.

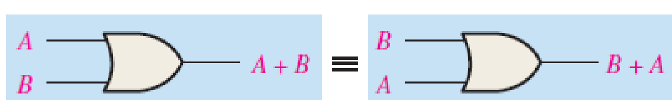
$$A\overline{B}C\overline{D} = 1 \cdot \overline{0} \cdot 1 \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

Laws and Rules of Boolean Algebra

1- Commutative Laws

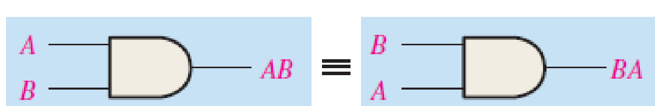
The *commutative law of addition* for two variables is written as

$$\mathbf{A + B = B + A}$$



The *commutative law of multiplication* for two variables is

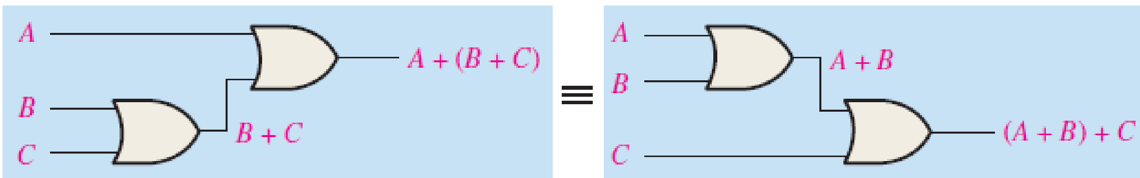
$$AB = BA$$



2- **Associative Laws**

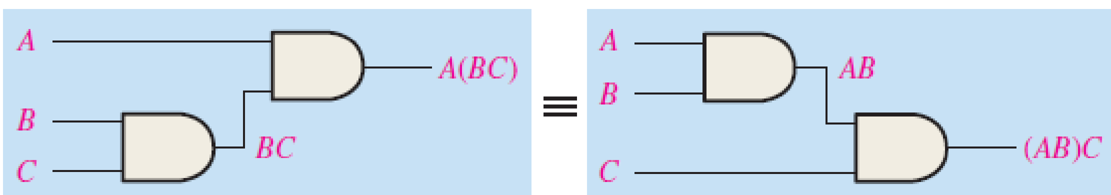
The *associative law of addition* is written as follows for three variables

$$\boldsymbol{A + (B + C) = (A + B) + C}$$



The *associative law of multiplication* is written as follows for three variables:

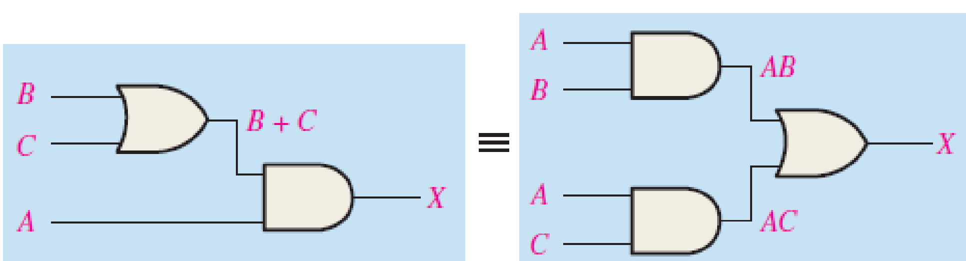
$$\boldsymbol{A(BC) = (AB)C}$$



3- **Distributive Law**

The distributive law is written for three variables as follows:

$$\boldsymbol{A(B + C) = AB + AC}$$



$X = A(B + C)$ $\qquad$ $X = AB + AC$

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Rules of Boolean Algebra

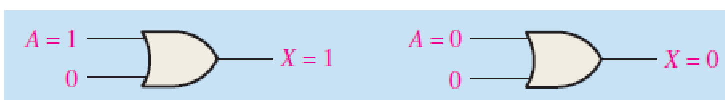
TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

A, B, or C can represent a single variable or a combination of variables.

***Rule 1*:** A + 0 = A

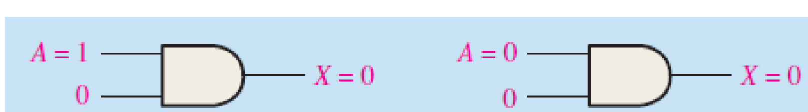


$X = A + 0 = A$

Rule 2: A + 1 = 1

$X = A + 1 = 1$

Rule 3: A . 0 = 0



$X = A \cdot 0 = 0$

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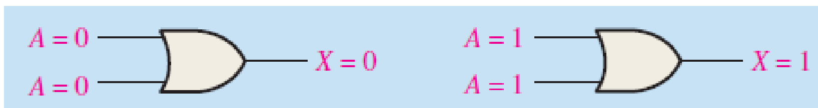
Rule 4: A . 1 = A

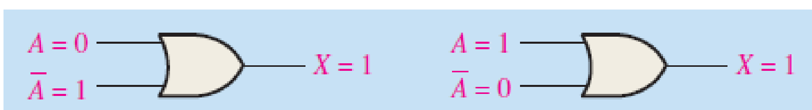
$X = A \cdot 1 = A$

Rule 5: A + A = A



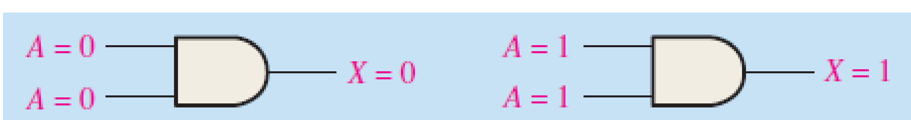
$X = A + A = A$

Rule 6: A + Ā = 1



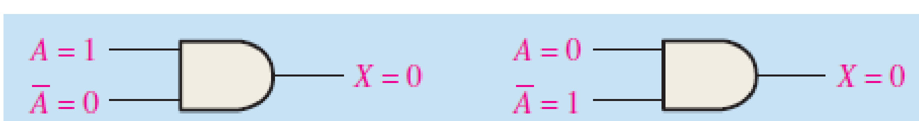
$X = A + \overline{A} = 1$

Rule 7: A . A = A



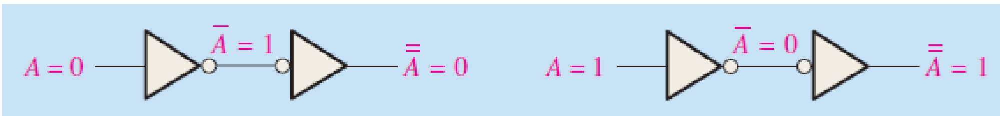
$X = A \cdot A = A$

Rule 8: A . Ā = 0



$X = A \cdot \overline{A} = 0$

Rule 9: $\overline{\overline{A}} = A$



$\overline{\overline{A}} = A$

Rule 10: A + AB = A

$$\begin{aligned} A + AB &= A \cdot 1 + AB = A(1 + B) \\ &= A \cdot 1 \\ &= A \end{aligned}$$

Rule 11: $A + \overline{A}B = A + B$

$$\begin{aligned} A + \overline{A}B &= (A + AB) + \overline{A}B && \text{Rule 10: } A = A + AB \\ &= (AA + AB) + \overline{A}B && \text{Rule 7: } A = AA \\ &= AA + AB + A\overline{A} + \overline{A}B && \text{Rule 8: adding } A\overline{A} = 0 \\ &= (A + \overline{A})(A + B) && \text{Factoring} \\ &= 1 \cdot (A + B) && \text{Rule 6: } A + \overline{A} = 1 \\ &= A + B && \text{Rule 4: drop the 1} \end{aligned}$$

Rule 12: $(A + B)(A + C) = A + BC$

$$\begin{aligned} (A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\ &= A + AC + AB + BC && \text{Rule 7: } AA = A \\ &= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\ &= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\ &= A(1 + B) + BC && \text{Factoring (distributive law)} \\ &= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\ &= A + BC && \text{Rule 4: } A \cdot 1 = A \end{aligned}$$

Logic Simplification Using Boolean Algebra

Example 1

Using Boolean algebra techniques, simplify this expression:

AB + A(B + C) + B(B + C)

Solution

The following is not necessarily the only approach.

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 ($AB + AB = AB$) to the first two terms.

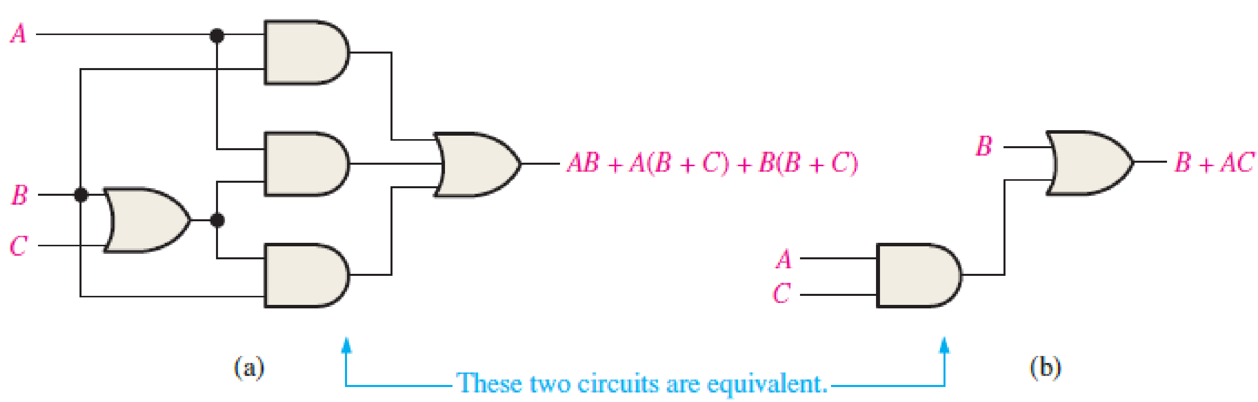
$$AB + AC + B + BC$$

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$$B + AC$$



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Example 2

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

Solution

Step 1: Apply the distributive law to the terms within the brackets.

$$(A\overline{B}C + A\overline{B}BD + \overline{A}\,\overline{B})C$$

Step 2: Apply rule 8 ($\overline{B}B = 0$) to the second term within the parentheses.

$$(A\overline{B}C + A \cdot 0 \cdot D + \overline{A}\,\overline{B})C$$

Step 3: Apply rule 3 ($A \cdot 0 \cdot D = 0$) to the second term within the parentheses.

$$(A\overline{B}C + 0 + \overline{A}\,\overline{B})C$$

Step 4: Apply rule 1 (drop the 0) within the parentheses.

$$(A\overline{B}C + \overline{A}\,\overline{B})C$$

Step 5: Apply the distributive law.

$$A\overline{B}CC + \overline{A}\,\overline{B}C$$

Step 6: Apply rule 7 ($CC = C$) to the first term.

$$A\overline{B}C + \overline{A}\,\overline{B}C$$

Step 7: Factor out $\overline{B}C$.

$$\overline{B}C(A + \overline{A})$$

Step 8: Apply rule 6 ($A + \overline{A} = 1$).

$$\overline{B}C \cdot 1$$

Step 9: Apply rule 4 (drop the 1).

$$\overline{B}C$$

H.W

Simplify the following Boolean expression:

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$