Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

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Lecture No.: 3

Lecture Title: (P-series-Telscopy series)

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P-Series Test

$\sum_{n=1}^{\infty} \frac{1}{p^n}$

We know that $\int_1^{\infty} \frac{1}{x^p} dx$ converges if $p > 1$ and diverges if $p \le 1$.

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges for } p > 1, \text{ diverges for } p \le 1.$$

Diverges: For $0 < p \le 1$

Converges: For $p > 1$

Steps to apply:

Step 1: Determine the type of series given.

Step 2: Determine the value of pp based on the type of series.

Step 3: Use the appropriate condition to determine its behavior.

Example 1: Determine if the series converges or diverges. If it converges, determine where the series converges.

1)

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$$

Diverges: for $p = \frac{1}{3} < 1$

2)

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt[9]{n^3}}$$

Converge : for p= $\frac{9}{3} > 1$

3)

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{n}}$$

Diverge : for p= $\frac{1}{5} < 1$

4)

$$\sum_{n=1}^{\infty}\frac{1}{n^{15}}$$

converges : for $p = 15 > 1$.

5)

$$\sum_{n=100}^{\infty}\frac{1}{n^{15}}$$

also diverges since a finite number of terms have no effect whether a series converges or diverge.

A special case of integral test is called (***P – series***):

$$\sum_{n=1}^{\infty}\frac{1}{n^p} \Longrightarrow \int_{1}^{\infty} x^{-p}\,dx$$

Telescoping Series

series is a series in which most of the terms cancel in each of the partial sums, leaving only some of the first terms and some of the last terms.

Consider the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

We discussed this series in the example:

$$\frac{1}{n(n+1)} = \frac{1}{n} + \frac{1}{(n+1)}$$

$$\left(1+\frac{1}{2}\right) - \left(\frac{1}{2}-\frac{1}{3}\right) - \left(\frac{1}{3}-\frac{1}{4}\right) + \cdots$$

$$s_1 = \left(1+\frac{1}{2}\right)$$

$$s_2 = \left(1+\frac{1}{2}\right) - \left(\frac{1}{2}-\frac{1}{3}\right) = \left(1-\frac{1}{3}\right)$$

$$s_3 = \left(1+\frac{1}{2}\right) - \left(\frac{1}{2}-\frac{1}{3}\right) - \left(\frac{1}{3}-\frac{1}{4}\right) = \left(1-\frac{1}{4}\right)$$

For example, any series of the form

$$\sum_{n=1}^{\infty} [b_n - b_{n+1}] = (b1 - b2) + (b2 - b3) + (b3 - b4) + \cdots$$

is a telescoping series. We can see this by writing out some of the partial sums. In particular, we see that

$s_1 = (b1 - b2)$

$s_2 = (b1 - b2) + (b2 - b3) = b1 - b3$

$s_3 = (b1 - b2) + (b2 - b3) + (b3 - b4) = b1 - b4$

Ex:

$$\sum_{n=1}^{\infty} \left[(\cos \frac{1}{n}) - \cos \frac{1}{n+1}) \right]$$

$s_1 = (\cos 1) - \cos \frac{1}{2})$

$s_2 = (\cos 1) - \cos \frac{1}{2}) + (\cos \frac{1}{2}) - \cos \frac{1}{3})$

$= (\cos 1) - \cos \frac{1}{3})$

$s_3 = (\cos 1) - \cos \frac{1}{2}) + (\cos \frac{1}{2}) - \cos \frac{1}{3}) + (\cos \frac{1}{3}) - \cos \frac{1}{4})$

$= (\cos 1) - \cos \frac{1}{4})$

Ex:

$\sum_{n=1}^{\infty} \frac{1}{(n+1)} + \frac{1}{(n+2)}$

$s_n = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right)$

$= \frac{1}{2} - \frac{1}{5}$