



Laplace transform:

is a mathematical technique that changes a function of time into a function in the frequency domain.

$$L\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} \cdot f(t) dt$$

Some properties of Laplace transform:

$$(1) L\{C_1 f_1(t) + C_2 f_2(t)\} = C_1 L\{f_1(t)\} + C_2 L\{f_2(t)\} \\ = C_1 F_1(s) + C_2 F_2(s)$$

$$(2) L\{e^{at} \cdot f(t)\} = F(s-a)$$

$$(3) L\left\{\int_0^t f(u) du\right\} = \frac{F(s)}{s}$$

$$(4) L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} f(u) du$$

$$(5) L\{f'(t)\} = s F(s) - f(0)$$

$$L\{f''(t)\} = s^2 F(s) - s f(0) - f'(0)$$

$$L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots$$

$$(6) \text{ If } L\{f(t)\} = F(s) \text{ then}$$

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$



$F(s)$	$f(t)$
$\frac{1}{s}$	1
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{n!}{s^{n+1}}$	t^n
$\frac{1}{s+a}$	e^{-at}
$\frac{a}{s^2+a^2}$	$\sin at$
$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{a}{s^2-a^2}$	$\sinh at$
$\frac{s}{s^2-a^2}$	$\cosh at$



$$\text{ex.1/ } L\{4t^5 - 2t + 6t^3\}$$

sol/

$$F(s) = 4 \frac{5!}{s^{5+1}} - 2 \frac{1}{s^{1+1}} + 6 \frac{3!}{s^{3+1}}$$

$$F(s) = \frac{480}{s^6} - \frac{2}{s^2} + \frac{36}{s^4}$$

$$\text{Ex.2/ } L\{e^{2t} \cdot t^2\}$$

$$F(s) = L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$$

$$L\{e^{2t} \cdot t^2\} = \frac{2}{(s-2)^3}$$

$$\text{Ex.3/ } L\{e^{-3t} \cdot \sin 2t\}$$

$$L\{\sin 2t\} = F(s) = \frac{2}{s^2 + 4}$$

$$L\{e^{-3t} \cdot \sin 2t\} = \frac{2}{(s+3)^2 + 4} = \frac{2}{s^2 + 6s + 13}$$

$$\text{Ex.4/ } L\left\{\int_0^t \sin 2u \, du\right\}$$

$$L\{\sin 2u\} = \frac{2}{s^2 + 4}$$

$$L\left\{\int_0^t \sin 2u \, du\right\} = \frac{2}{s(s^2 + 4)}$$



$$\text{Ex. 5/ } L \left\{ \frac{\sin t}{t} \right\}$$

$$L \{ \sin t \} = \frac{1}{s^2 + 1}$$

$$L \left\{ \frac{\sin t}{t} \right\} = \int_s^\infty \frac{1}{u^2 + 1} du$$

$$= [\tan^{-1} u]_s^\infty = \tan^{-1} \infty - \tan^{-1} s \\ = \frac{\pi}{2} - \tan^{-1} s$$

$$\text{Ex. 6/ Find } L \{ (\sin t - \cos t)^2 \}$$

$$L \{ \sin^2 t - 2 \sin t \cos t + \cos^2 t \} = L \{ 1 - \sin 2t \}$$

$$= \frac{1}{s} + \frac{2}{s^2 + 4}$$

$$\text{Ex. 7/ If } L \{ f(t) \} = \frac{s^2 - s + 1}{(2s + 1)^2 (s - 1)} \text{, find } L \{ f(2t) \} ?$$

Sol:-

$$L \{ f(2t) \} = \frac{1}{2} f\left(\frac{s}{2}\right)$$

$$= \frac{1}{2} \frac{\left(\frac{s}{2}\right)^2 - \frac{s}{2} + 1}{\left(s + 1\right)^2 \left(\frac{s}{2} - 1\right)}$$



Applied Mathematics

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EX.8/ Find $L \left\{ e^{-3t} \int_0^t \frac{\sin 2t}{t} dt \right\}$ ^{3rd stage}

$$L \{ \sin 2t \} = \frac{2}{s^2 + 4}$$

$$L \left\{ \frac{\sin 2t}{t} \right\} = \int_s^\infty \frac{2}{u^2 + 4} du = \tan^{-1} \frac{u}{2} \Big|_s^\infty$$
$$= \frac{\pi}{2} - \tan^{-1} \frac{s}{2}$$

$$L \left\{ \int_0^t \frac{\sin 2t}{t} dt \right\} = \frac{\pi}{2s} - \frac{1}{s} \tan^{-1} \frac{s}{2}$$

$$L \left\{ e^{-3t} \int_0^t \frac{\sin 2t}{t} dt \right\} = \frac{\pi}{2(s+3)} - \frac{1}{s+3} \tan^{-1} \frac{s+3}{2}$$

H.W.3// Find laplace transform of the following:

(1) $\frac{e^{3t} \cos 4t}{t}$

(2) $5e^{2t} - t^3$

(3) $t^2 - 7 + \cosh 3t$



Unit step function:

$$\text{Theory: } \mathcal{L} [f(t) u(t-a)] = e^{-as} \mathcal{L} [f(t+a)]$$

where: $u(t-a)$ is unit step function

$$\text{Ex. (1)} \text{ Find } \mathcal{L} \{ \cos(t-1) u(t+1) \}$$

sol:-

$$f(t) = \cos(t-1) \quad \text{where } a=1$$

$$f(t+1) = \cos(t+1-1) = \cos t$$

$$\therefore \mathcal{L} \{ \cos(t-1) u(t-1) \} = e^{-s} \mathcal{L} \{ \cos t \}$$

$$= e^{-s} \frac{s}{s^2+1}$$

$$\text{Ex. (2)} \text{ Find } \mathcal{L} \{ t^2 u(t-2) \}$$

$$f(t) = t^2 \quad \text{where } a=2$$

$$f(t+2) = (t+2)^2$$

$$= t^2 + 4t + 4$$

$$\mathcal{L} \{ t^2 u(t-2) \} = e^{-2s} \left\{ \frac{2!}{s^3} + \frac{4}{s^2} + \frac{4}{s} \right\}$$



Ex.(3) Find $L\{(t^2-1)u(t-1)\}$

Sol:

$$f(t) = t^2 - 1$$

$$a = 1$$

$$f(t+1) = (t+1)^2 - 1$$

$$= t^2 + 2t$$

$$L\{(t^2-1)u(t-1)\} = e^{-s} \left[\frac{2!}{s^3} + \frac{2}{s^2} \right]$$

Ex.(4) Find $L\{(t^2-4t+5)u(t-3)\}$

Sol:

$$f(t) = t^2 - 4t + 5$$

$$a = 3$$

$$f(t+3) = (t+3)^2 - 4(t+3) + 5$$

$$= e^{-3s} L\{t^2 + 6t + 9 - 4t - 12 + 5\}$$

$$= e^{-3s} L\{t^2 + 2t + 2\}$$

$$= e^{-3s} \left[\frac{2!}{s^3} + \frac{2}{s^2} + \frac{2}{s} \right]$$