Lecture No. 6

Linear relation between E,G and

1. Strain

Definition:

Strain is a measure of deformation in a material due to applied stress. It is defined as the ratio of the change in dimension (length, angle, or volume) to the original dimension.

- **Formula:**

$$\epsilon = \frac{\Delta L}{L_0}$$

Where:

- ϵ: Strain (dimensionless)
- ΔL: Change in length
- L_0: Original length

Types of Strain:

1. **Normal Strain:**
   - Deformation along the axis of loading (tensile or compressive).
   - Example: Stretching a rubber band.
2. **Shear Strain:**
   - Deformation due to tangential forces causing angular distortion.
   - Example: Cutting with scissors.

Example:

A steel rod of length $2\,\mathrm{m}$ is stretched by $0.002\,\mathrm{m}$. Calculate the strain.

- **Solution:**

$$\epsilon = \frac{\Delta L}{L_0} = \frac{0.002}{2} = 0.001$$

2. Hooke's Law

Definition:

Hooke's Law states that for small deformations, the stress (σ) in a material is directly proportional to the strain (ϵ).

- **Formula:**

$$\sigma = E\epsilon$$

  Where:

  - σ: Stress (force per unit area, Pa or $\mathrm{N/m^2}$)
  - E: Young's modulus (modulus of elasticity, Pa)
  - ϵ: Strain

Key Points:

- Hooke's Law applies only to the **elastic region** of a material (where deformation is reversible).
- Beyond the elastic limit, materials exhibit plastic deformation, and Hooke's Law no longer applies

Example:

A force of $10,000\,\mathrm{N}$ is applied to a steel rod with a cross-sectional area of $0.01\,\mathrm{m^2}$. If Young's modulus for steel is $200\,\mathrm{GPa}$, calculate the strain.

- **Solution:**

$$\sigma = \frac{F}{A} = \frac{10,000}{0.01} = 1,000,000\,\mathrm{Pa}\,(1\,\mathrm{MPa})$$

$$\epsilon = \frac{\sigma}{E} = \frac{1,000,000}{200 \times 10^9} = 5 \times 10^{-6}$$

3. Poisson's Ratio (ν)

Definition:

Poisson's ratio is the ratio of transverse strain to axial strain when a material is subjected to uniaxial stress.

- **Formula:**

$$\nu = -\frac{\epsilon_{\text{transverse}}}{\epsilon_{\text{axial}}}$$

  Where:

  - $\epsilon_{\text{transverse}}$: Strain perpendicular to the applied force
  - ϵ_{axial}: Strain in the direction of the applied force

Key Points:

- Poisson's ratio ranges between 0 and 0.5 for most materials.
- For incompressible materials (e.g., rubber), $\nu = 0.5$.

The **bulk modulus (K)** is a material property that measures its resistance to uniform compression. It quantifies how much a material will compress under an applied external pressure. It is defined as the ratio of **volumetric stress** (pressure) to **volumetric strain** (relative change in volume).

Mathematical Definition

The bulk modulus is given by the formula:

$$K = -V\frac{\Delta P}{\Delta V}$$

Where:

- K: Bulk modulus (units: Pascals, Pa)
- V: Original volume of the material
- ΔP: Change in pressure (applied stress)
- ΔV: Change in volume (volumetric strain)

Volumetric Strain

Volumetric strain (ϵ_v) is the relative change in volume due to applied pressure:

$$\epsilon_v = \frac{\Delta V}{V}$$

Thus, the bulk modulus can also be written as:

$$K = -\frac{\Delta P}{\epsilon_v}$$

Key Points About Bulk Modulus

1. **Units:**
   - The SI unit of bulk modulus is **Pascals (Pa)** or **Gigapascals (GPa)**.
   - $1\,\mathrm{GPa} = 10^9\,\mathrm{Pa}$.
2. **Physical Meaning:**
   - A high bulk modulus means the material is **less compressible** (e.g., solids like steel).
   - A low bulk modulus means the material is **more compressible** (e.g., gases).

5. Linear Relationship Between E, G, and ν

For isotropic materials, the elastic constants E (Young's modulus), G (shear modulus), and ν (Poisson's ratio) are related as follows:

1. **Relationship between E, G, and ν:**

$$G = \frac{E}{2(1+\nu)}$$

   - This equation shows how the shear modulus G depends on Young's modulus E and Poisson's ratio ν.

2. **Relationship between E, G, and ν (alternative form):**

$$E = 2G(1+\nu)$$

   - This equation expresses Young's modulus E in terms of the shear modulus G and Poisson's ratio ν.

3. **Relationship between E, K (bulk modulus), and ν:**

$$E = 3K(1-2\nu)$$

Example 1: Calculating Shear Modulus (G)

- **Given:**
  - Young's modulus (E) for steel: $E = 200\,\text{GPa}$
  - Poisson's ratio (ν) for steel: $\nu = 0.3$
- **Find:**
  - Shear modulus (G) for steel.
- **Solution:**

  Using the relationship:

  $$G = \frac{E}{2(1+\nu)}$$

  Substitute the values:

  $$G = \frac{200\,\text{GPa}}{2(1+0.3)} = \frac{200}{2.6} \approx 76.92\,\text{GPa}$$

- **Answer:**

  The shear modulus of steel is approximately $76.92\,\text{GPa}$.

Example 2: Calculating Poisson's Ratio (ν)

- **Given:**
  - Young's modulus (E) for aluminum: $E = 70\,\mathrm{GPa}$
  - Shear modulus (G) for aluminum: $G = 26\,\mathrm{GPa}$
- **Find:**
  - Poisson's ratio (ν) for aluminum.
- **Solution:**

  Using the relationship:

$$G = \frac{E}{2(1+\nu)}$$

  Rearrange to solve for ν:

$$\nu = \frac{E}{2G} - 1$$

  Substitute the values:

$$\nu = \frac{70}{2 \times 26} - 1 = \frac{70}{52} - 1 \approx 1.346 - 1 = 0.346$$

Example 3: Calculating Young's Modulus (E)

- **Given:**
  - Shear modulus (G) for rubber: $G = 0.001\,\mathrm{GPa}$
  - Poisson's ratio (ν) for rubber: $\nu = 0.5$ (incompressible material)
- **Find:**
  - Young's modulus (E) for rubber.
- **Solution:**

  Using the relationship:

$$E = 2G(1+\nu)$$

  Substitute the values:

$$E = 2 \times 0.001 \times (1 + 0.5) = 0.002 \times 1.5 = 0.003\,\mathrm{GPa}$$

Example 4: Calculating Bulk Modulus (K)

- **Given:**
  - Young's modulus (E) for glass: $E = 70\,\mathrm{GPa}$
  - Poisson's ratio (ν) for glass: $\nu = 0.2$
- **Find:**
  - Bulk modulus (K) for glass.
- **Solution:**

  Using the relationship:

$$E = 3K(1 - 2\nu)$$

  Rearrange to solve for K:

$$K = \frac{E}{3(1 - 2\nu)}$$

  Substitute the values:

$$K = \frac{70}{3(1 - 2 \times 0.2)} = \frac{70}{3(1 - 0.4)} = \frac{70}{3 \times 0.6} = \frac{70}{1.8} \approx 38.89\,\mathrm{GPa}$$