Lecture No. 3

Deflection of the beam

Introduction

When we consider designing beams based on rigidity consideration the deflection of the beam must be known at specific or critical location. Several methods are avaible for determining beam deflection. Although based on the same principles, they differ in technique and in their immediate objective.

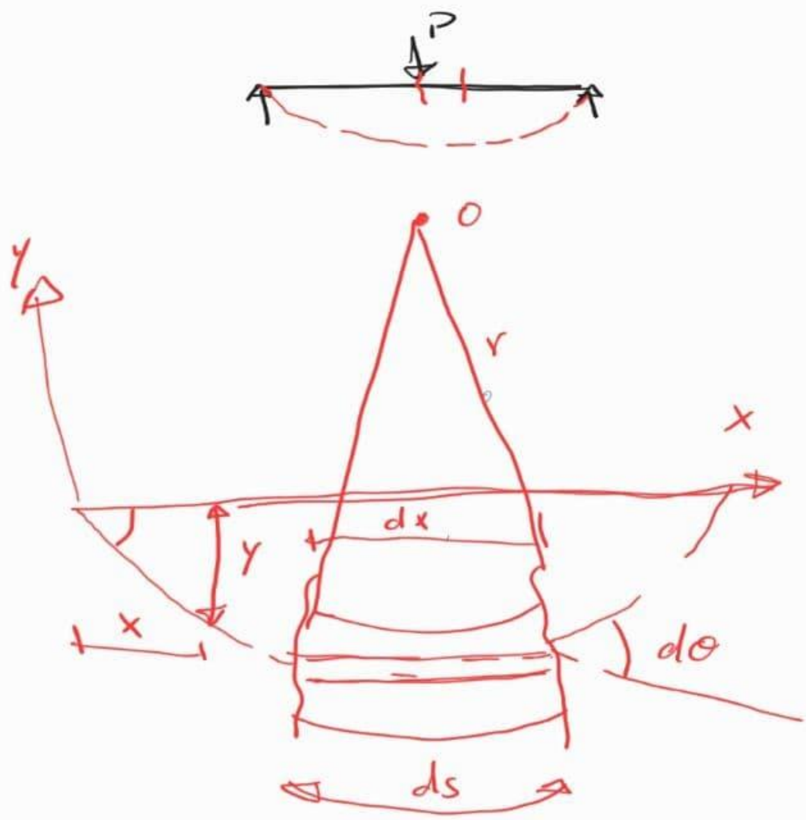
Methods of Determining Beam Deflections

Numerous methods are available for the determination of beam deflections. These methods include:

- Double-integration method
- Area-moment method
- Strain-energy method (Castigliano's Theorem)
- Conjugate-beam method
- Method of superposition

Double-integration method

The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve. *The double integration method can be used if the moment (M) has a single expression for the whole beam.*



$$\text{slope} : \tan\theta = \frac{dy}{dx}$$

since θ is small

$\Rightarrow \tan\theta = \theta$

$$\theta = \frac{dy}{dx} \quad (\text{slope})$$

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} \quad (\text{deflection})$$

The differential length of the strip ds can be expressed in terms of raduis of curvature

$$ds = r * d\theta$$

$$\Rightarrow \frac{1}{r} = \frac{d\theta}{ds} \simeq \frac{d\theta}{dx} \quad \text{——— (1)}$$

becuuse the deflection of the beam is very small

$(ds \simeq dx)$

From the basic law of bending

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{r}$$

$$\Rightarrow \frac{M}{I} = \frac{E}{r}$$

$$\frac{1}{r} = \frac{M}{EI} \quad \text{——— (2)}$$

equating eq. (1) & eq. (2)

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

since $\frac{d\theta}{dx} = \frac{d^2y}{dx^2}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\Rightarrow \boxed{EI \frac{d^2y}{dx^2} = M} \quad (4)$$

equation of bending

by diffentiation of eq. (4) we get the slope of the beam

$$\Rightarrow \boxed{EI \frac{dy}{dx} = \int M\,dx + C}$$

c = constant of integration

equation of slope (5)

another diffentiation of the equation (5), we get the deflection of beam

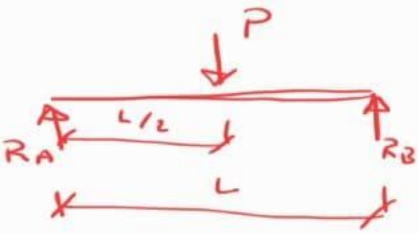
$$\boxed{EI\,y = \iint (M\,dx)\,dx + C_1 x + C_2}$$

C_2 = constant of integration

—— (6)

equation of deflection

Deflection of the simple Supported beam with concentrated load



* Reactions

$$\Sigma M_B = 0$$

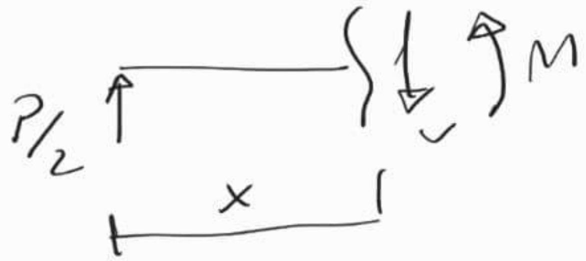
$$R_A(L) - P\left(\frac{L}{2}\right) = 0$$

$$\boxed{R_A = \frac{P}{2}}$$

$$\Sigma F_y = 0$$

$$\frac{P}{2} - P + R_B = 0 \Rightarrow \boxed{R_B = \frac{P}{2}}$$

* Bending moment equation



$$M = \frac{P}{2}x$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{P}{2}x \quad (B.M)$$

$$EI \frac{dy}{dx} = \frac{P}{4} x^2 + C_1 \text{ (slope)}$$

$$EI\, y = \frac{P}{12} x^3 + C_1 x + C_2$$

(deflection)

<u>Boundary condition</u>

at $x = 0 \Rightarrow y = 0$

$\Rightarrow C_2 = 0$

at $x = \frac{L}{2} \Rightarrow \frac{dy}{dx} = 0$

$$0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1$$

$$\Rightarrow \boxed{C_1 = \frac{-PL^2}{16}}$$

$\Rightarrow$ <u>The slope equation</u>

$$\boxed{EI \frac{dy}{dx} = \frac{P}{4} x^2 - \frac{PL^2}{16}}$$

⇒ The deflection equation

$$\boxed{EI\,y = \frac{Px^3}{12} - \frac{PL^2}{16}(x)}$$

The maximum slope occurs at A

$$\Rightarrow \boxed{EI\frac{dy}{dx} = -\frac{PL^2}{16}}$$

The maximum deflection occurs at centre of the beam $(x = \frac{L}{2})$

$$\Rightarrow EI\,y = \frac{P}{12}\left(\frac{L}{2}\right)^3 - \frac{PL^2}{16}\left(\frac{L}{2}\right)$$

$$\Rightarrow \boxed{EI\,y_{max} = -\frac{PL^3}{48}}$$

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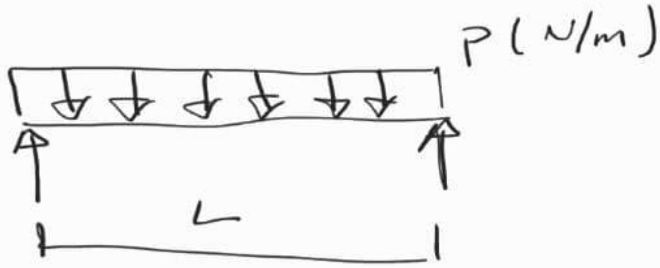
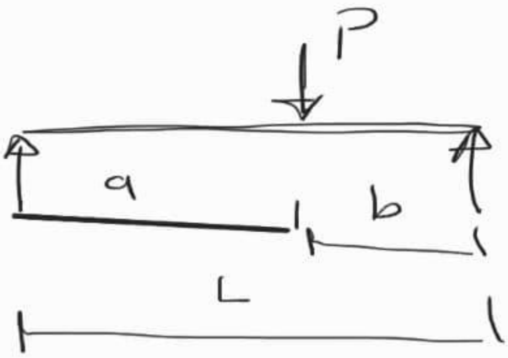
(1) Determination of the slope and deflection of-

(a)

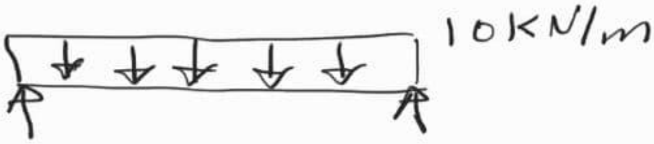


(b)

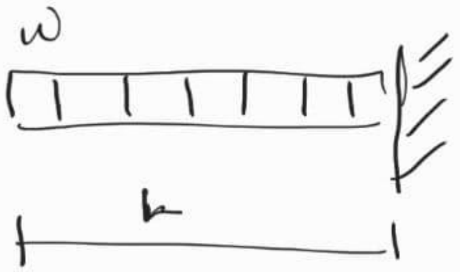


(c)

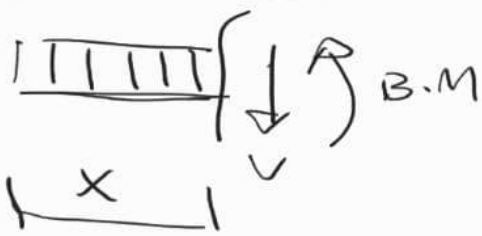
(D)



Deflection & slope of Cantilever beam with disturbuted load



Bending moment



$$M = -\frac{w x^2}{2}$$

$$\Rightarrow EI\,\bar{\bar{y}} = -\frac{w x^2}{2} \quad (B.M)$$

$$EI\bar{y} = -\frac{wx^3}{6} + C_1 \text{ (slope)}$$

$$EIy = -\frac{wx^4}{24} + C_1 x + C_2 \text{ (defl.)}$$

B.C

at $x = L \Rightarrow \bar{y} = 0$

$$0 = -\frac{wL^3}{6} + C_1$$

$$\boxed{C_1 = \frac{wL^3}{6}}$$

at $x = L \Rightarrow y = 0$

$$0 = -\frac{wL^4}{24} + \frac{wL^3}{6} L + C_2$$

$$\Rightarrow \boxed{C_2 = -\frac{wL^4}{8}}$$

$$\therefore \quad \boxed{EI\bar{y} = \frac{-wx^3}{6} + \frac{wL^3}{6}}$$

equation of slope

$$\boxed{EIy = \frac{-wx^4}{24} + \frac{wL^3}{6}x - \frac{wL^4}{8}}$$

equation of deflection

* maximum deflection occurs at $x=0$

* maximum slope occurs at $x=0$

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Determination of slope and deflection of

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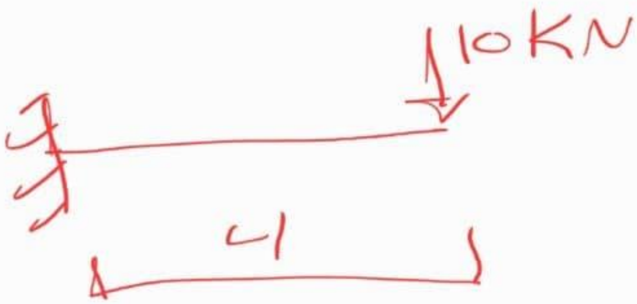

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